

772

A
RATIONAL and PRACTICAL
TREATISE
OF
ARITHMETIC.

(In TWO PARTS.)

Containing

All that is necessary to be known in this art, in order to qualify a young person for trade, or an intended course of the mathematics.

THE WHOLE

Attempted in an easy, methodical, and consistent manner, and equally adapted to assist persons engaged in teaching, and the instruction of those, who have not the advantage of a master.

To which is added, in the manner of NOTES,

The reason and demonstration of every rule and operation, as they occur, on principles either purely arithmetical, or such as will easily be comprehended by a beginner.

12 E 29

By W. COCKIN,
WRITING-MASTER and ACCOMPTANT,
at the FREE-SCHOOL in Lancaster.

L O N D O N :

Printed for the AUTHOR;

And Sold by W. NICOLL, in *St. Paul's Church-Yard*; T. CASLON, opposite *Stationer's Hall*; WILSON and FELL, and HAWES, CLARK, and COLLINS, in *Paternoster-Row*; J. FLETCHER, in *Oxford*; T. MERRIL, in *Cambridge*; W. CHARNLEY, and T. SLACK, in *Newcastle*; J. GORE, in *Liverpool*; W. STUART, in *Preston*; A. ASHBURNER, and J. WILSON, in *Lancaster*; J. ASHBURNER, in *Kendal*; and W. SHEPHERD in *Whitehaven*.

MDCCLXVI.

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P R E F A C E.

PUBLICATIONS of this kind have become so frequent of late, that, agreeably to the usual methods of ushering them abroad, an apology for encreasing the number will naturally be expected. This is no other, than what is highly reasonable, and the following short account of my motives to this undertaking is dictated with the greatest sincerity. Having been commonly engaged in teaching this art for a considerable number of years past, and put occasionally upon examining the several * treatises hitherto published on the subject, I could not meet with any one (intended to instruct readers of the lower class) in which the materials were disposed in that methodical, consistent, and rational manner I thought they were capable of being thrown into. This to me was a sufficient inducement for a new attempt, if at the same time I had so good an opinion of my own abilities, as to hope it was in my power to remove these objections, by molding the materials over again in a more systematical; and intelligible method.—This too was the case, and a seasonable piece of leisure conspired with the above motives, and a natural love of order and consistency, to render the undertaking too tempting for me to resist.

4.

This

* In this survey I do but remember, at present, two modern authors of real abilities, who have kept close to a premeditated plan and manner of writing, and those were *Malcolm* and *Emerson*. But then they are by no means calculated in other respects for common use. *Malcolm*, purposing to give a complete view of every particular of the art, enters so deeply into it, and makes so much use of algebra, that it requires a tolerable skill in that branch to be able to read him; the principles of which he also intermixes with those of arithmetic, as a part of his plan. In *Emerson* is attempted a brevity in the method, which I think has rendered his book not properly fit for a mere novice; the rules are very few of them demonstrated, nor is it seemingly in other respects well calculated for an assistant to teachers, and the use of schools. However, in justice to these authors, it must be observed of the first, that his book will be a lasting monument of his learning and industry, and that the ingenious theory of numbers, given us in the end of the other's work, will intitle it to the perusal of all lovers of this art.

This honest confession contains virtually all, that I have to offer by way of apology for appearing in print; which if not thought circumstantial enough by some readers, 'tis hoped the following more ample display of it will give them all reasonable satisfaction on this head; in which I shall more particularly examine the merits of former publications, point out where they are defective, and which, in my opinion, is at present the best and most eligible method of laying this science before the young student, in reference to his inexperience, and to the occasions he may afterwards have to apply it, &c.

*A formal enumeration of names would be very tedious, and scarce charitable; therefore I shall only observe in general on the dark side (what will be granted, I dare say, by any person well acquainted with these subjects) that want of method is a fault treatises of this class are for the most part chargeable with, inasmuch as we find no settled manner of proceeding in them; no end aimed at, or necessary chain of dependance; this article introduced with a definition, and the next perhaps without; several things taught only by a scattered set of directions given in answering a few leading instances, which may be solved by concise and general canons; here an adequate rule, and there half of one, with a very confused page of science, Nota Benes, and ill-judged animadversion. Others fail greatly from delivering their subject in too refined a manner for pupils ignorant of even the first operations of numbers; throwing the substance of the art into the form of a lecture, or perplexing it with researches and particulars, they neither can comprehend, nor see the necessity of. Others again, in order to avoid this extreme, think, that to be plain is to be verbose, and hence imagine long illustrations, or the familiar style of dialogue the only way of letting their pupils into the mystery. Another class of writers (not wholly free from the above faults) have injudiciously left out some * rules, which ought either to be inserted in a book of arithmetic, or never more mentioned, and preserved others, which might very well have been omitted; have hurt the subject by a bad choice of examples, embarrassed the learner from the first with a parcel of definitions, which were never wanted, and after all flattered themselves perhaps, that supplying a set of new questions to the old rules, tho' nearly in the same words and order, was writing a new book.—In short, every one of the treatises*

* Position I should think an instance of this. See note page 309.

P R E F A C E.

v.

tises I have turned over (of the class intended to make an artist in the practical part) seemed to me chargeable with some one, or more of the consequences, which necessarily result from want of judgment, experience in teaching, a full and clear comprehension of the subject; from an obscure and embarrassed method of delivery, or from downright idleness. Add to this the constant omission of easy and striking demonstrations of the reasons of the rules, of so much satisfaction and improvement to every ingenious inquirer, and which (tho' promised in almost every title-page) have never yet been given, that I know of, upon ordinary arithmetical principles, or such, as may easily be understood by a learner.

Now, if this charge be well founded, the allowed utility of the subject makes it very much to be wished, the particular objections above specified and intimated were removed; and hence 'tis hoped, we have a sufficient apology for an attempt purposing this reformation, notwithstanding the great number already made.—And that the present one may lay some claim to the merit of having answered so desirable an end, the following plan has been studiously and carefully observed.

*As my chief design in this undertaking was not so much to * extend the science, as to make it useful and intelligible; to answer this end I have contented myself with the old materials, and only attempted to throw them into a more satisfactory and advantageous point of view.*

In order to this, I have endeavoured to introduce all the articles with clear and adequate definitions of their design, and after that, to give precise and general rules (at least so far as practicable) for the performing of what is intended by those definitions, together with such other hints
and

** It appears to me, that on the present notation all the methods of real utility for performing the simple operations of this art are already discovered, and that it would be ridiculous trifling to encounter with its principles all the difficulties, that might be overcome by them; more especially at a time, when we have the advantage of superior and more eligible methods of solution, so well known among us.*

This declaration is thrown in here, partly to obviate any objection the injudicious might make with regard to this particular, and partly to inform the young student, that where ordinary arithmetic, as taught in this book, will not easily answer his enquiries, 'tis the best way for him to add to its principles the more extensive and general ones of algebra and geometry.

*and explications interspersed occasionally, as I thought could not well be spared, if I intended to be understood. Next, each article has been illustrated by the solution of such a number of examples, as were judged necessary for this end, and selected in such a manner, as to take in the most material variety of instances which might fall within their limits. After this, * three classes of questions have been subjoined, with their answers only, for the severer trial of the learner, and ordered generally to rise one above another in difficulty, so as to suit most capacities and degrees of experience. Any contraction judged serviceable and instructing is next pointed out; and, lastly, there are added, in the places, which furnish the hints, Scholiums, containing remarks and other particulars, which, tho' generally more of a critical nature than real use, are not the less necessary in works of this kind, where curiosity, as well as improvement, is supposed often to excite people to their perusal. The whole is divided into two parts; one with respect to ordinary business, and the other, with respect to the particulars necessary to be known towards a farther progress in the mathematics, or a more extensive and curious view of the subject under consideration; and the several articles are carefully disposed so as to form a natural ascent in the difficulty of the praxis, or so, as to throw the greatest light one upon another, by their dependence, similitude, &c.*

Such is the method I have used in drawing up the preceptive part of the work, and I hope it will be found, agreeably to the intention above observed, not only to comprehend every thing necessary to be taught in one of these books towards forming a complete accomptant for business, but likewise all the arithmetical operations, and principles requisite to be known,

* As questions (or, which is the same thing, forms of questions) in arithmetic, by being frequently banded about from author to author, are now become a common-right, as it were, I think myself under no necessity of apologizing for the freedom I have taken in making this collection, except indeed I observe, that it is enriched with several questions from the very ingenious ones given us by Mr. *Clare*, in his *Introduction to trade and business*.

Those, who purchase this work chiefly for their own improvement, may perhaps think the number of questions here collected upon the whole too many: But when 'tis considered, that a great variety in this respect is a particular, which recommends these treatises more perhaps than any other, as helps to teach by, in schools &c. I hope the few additional pages, thus occasioned, as they have enabled me fully to include both plans in one, will not be deemed altogether superfluous.

known, before the young student enters upon other branches of mathematical learning.

Farther, for the sake of such, as are curious and inquisitive in these matters, I have in the form of notes, and without any assistance from algebra or references to Euclid, demonstrated all the rules in a manner I have not yet seen effected, and I flatter myself with such clearness and conviction, as to yield no small satisfaction and instruction to readers of this class. * And as this method of handling the subject (together with the plan of the Scholiums) afforded me an opportunity of inserting several things of a nature not very proper to be thrown into the text, without being distinguished from the subject-matter, and yet what might equally inform or entertain the reader, I have subjoined such remarks and other particulars, as I judged might answer this purpose, and which, if omitted there, would some of them, in all likelihood, have extended this preface to a still greater length.

As I have been very scrupulous in keeping the preceptive, and demonstrative parts separate from each other, so in the former care has been taken to have those particulars, which in effect contain the whole mystery, and in the end are all that need be attended to, distinguished from the rest by a different character; which I hope will be found of use to the student, as well as those teachers, who please to conduct their scholars manuscripts after this plan. And this circumstance, with the great variety of practical questions, collected in the classes all along, makes me hope the following performance will be found as well calculated for an assistant

to

* What is delivered under the word *Scholium* in most authors is so little different in general from what might otherwise have been called a *note*, or *remark*, that in a work, where a place is allotted for things of this nature, independent of the text, there is seemingly little occasion to croud it at all with any observations of this kind. But as there are some remarks, which are perhaps more worthy of attention (as belonging more immediately to the subject-matter, &c.) than others, and which in a treatise, where notes are not used, it would still have been very proper to insert, I have endeavoured to throw any thing of this kind into *scholiums*, as also what at the end of an article I judged necessary by way of conclusion, &c. However, (as one thing I have all along aimed at in this work has been to give the reader all the satisfaction I was able, with regard to the reason and intent of almost every step taken in it) if, after all, I am found departing from the above method, I am willing it should be concluded an oversight, or whatever else the reader pleases, provided that this confession may tend (in like manner) to prevent him from disturbing himself with attempting to find out a regularity and order in this particular, which it perhaps may not contain.

to teach by, as the private perusal of such persons, as have not the opportunity of going to school, or are desirous of entertaining themselves with the rationale of the art. And as in all this I have had an eye to what might be useful in academies and schools, I am farther in hopes to find it well received in these seminaries, and on the whole approved on for both the purposes, 'tis thus professedly intended to answer.

Perhaps some of my readers might have wished to have seen a greater variety of * contractions in particular cases, than what are here adopted, but I have purposely omitted a number of them on a thorough conviction, that the best way of forming an expert arithmetician is to give him such a general and rational account of the principal rules and operations, that he may be enabled so far to get beyond them, as to make them (begging indulgence in the phrase) a portion of his common sense, and of consequence what may be carried about him with as much ease. This once attained, nothing can be proposed to him by way of question, but some method will occur for the solution, and tho' perhaps not the nearest, yet as it will be the first and most evident, 'tis great odds, if on that account solely the answer required be not sooner determined by it, than if referred to others a little readier in themselves, but farther from recollection, on account of some step they contain not so clearly comprehended, &c. And if in any future employment a particular sort of question or two daily fall out, as he is from the above method of teaching very able, he may then apply himself to the discovery of the most ready way of answering them, as a thing worth his while; but 'till then, I look upon a number of contractions to be more burthensome and perplexing, than useful and instructing.

Into the latter end of the second part I have ventured to introduce the algorithm of ratios, &c. as I am persuaded this particular, so very necessary in reading any thing of a mathematical, and philosophical nature, is often ill understood in a practical way, and that a notion of it can never be too soon given to the Tyro.

But not to enlarge too far in this place on the reasons I had for such and such

* Those, who are desirous of being better acquainted with these things, may have ample satisfaction from the perusal of the 2d. vol. of *Wells's* arithmetic, in which, besides a collection of the most material contractions, &c. in other authors, are given several curious ones of his own.

such particulars in my method, I shall content myself with observing in general, that, as every day discovers to us, too much brevity produces confusion, too much explication disgust; and, as it would be unnecessarily tedious (notwithstanding 'tis practice and experience which makes an artist) to give every particular application, I have endeavoured to direct myself with all precaution betwixt these extremes; and farther, to have constantly in view the present state of this kind of learning, the supposed inexperience and curiosity of the reader, and every other particular, that I imagined would contribute towards a performance, which on a diligent perusal could not fail of making him a judicious and able arithmetician.

As to the merit of the execution, 'tis proper and becoming to submit it silently to the judgment of the public, whose candour, 'tis hoped, will incline them to put the most favourable construction on what is now designed intirely for their service. And as in any thing which relates to the sciences, I find the language, upon the whole, not so scrupulously attended to, as in other productions, 'tis hoped faults in the style, and other errors of a like pardonable nature, will not be too severely censured.

Thus much to my readers in general.—What now remains to add, is my humble thanks to those in particular, who have favoured me with their subscriptions, or otherwise encouraged the undertaking.

BURTON in KENDAL
WESTMORLAND
Dec. 23d. 1763.

W. Cockin.

1841
The first of the year was a very
cold one, and the weather was
very disagreeable. The snow
was very deep, and the wind
was very strong. The people
were very much distressed,
and the cattle were very
suffering. The people were
very much distressed, and the
cattle were very suffering.

The second of the year was a
very cold one, and the weather
was very disagreeable. The
snow was very deep, and the
wind was very strong. The
people were very much
distressed, and the cattle
were very suffering.

The third of the year was a
very cold one, and the weather
was very disagreeable. The
snow was very deep, and the
wind was very strong. The
people were very much
distressed, and the cattle
were very suffering.

The fourth of the year was a
very cold one, and the weather
was very disagreeable. The
snow was very deep, and the
wind was very strong. The
people were very much
distressed, and the cattle
were very suffering.

General Introduction.

AS books of this kind mostly contain several articles, which there is no absolute necessity every reader should make himself master of; and as they may fall into the hands of those, who have not the opportunity or inclination of reading the whole indiscriminately, or consulting persons acquainted with these things, in order to direct their studies, &c. I have sometimes thought, what directions *could* be given in a thing so much depending upon a variety of concurring circumstances, placed at the beginning of these treatises, might be exceeding useful and satisfactory.

In consequence of this I have ventured to offer what follows for the perusal of the young arithmetician before he begins his studies, which, with the explanation of a few mathematical terms he will frequently meet with in his progress, (especially in the latter part) is the chief intent of this introduction.

First then, I may observe in general, that though the reason of every rule is shewn in the notes as it occurs, and the saying, "Demonstrations are the best instructions," is become a common proverb; yet I would not advise any beginner to puzzle himself with them, till he has made himself perfectly master of the practical part, as far as he intends to go. But after that, I am persuaded he would find a great advantage from perusing them (either occasionally, or in order) and more especially, if he has any inclination to make a progress in the mathematics; as they will prepare him a little for the severity of reasoning used in these sciences. Though 'tis equally true on the other hand, that many an expert arithmetician knows very little of the theory, and would give, on examination, but a poor

account of the intent of such and such steps in the primary rules and operations he every day applies so justly and familiarly.

Another thing I would recomend is, the omission of all the contractions (except one or two, that will be occasionally pointed out) the first time going over, as also those *scholiums*, which contain something relative to the practice of the rule under consideration. Likewise, as a person is seldom completely master of the several rules, &c. of this art, 'till he has gone twice or thrice over them, I would advise him to strengthen his abilities in each essay by solving a different class of the practical questions (or part of one, as he thinks proper) all along, beginning with the first, or easiest.

As to the selecting of particular articles for the perusal of students, intended different ways, &c. I observe, that if he proposes his arithmetical knowledge chiefly as a preparative to an acquaintance with the mathematics, a good part of what relates to trade may be omitted, as of little use to him: Therefore I would direct his attention to what follows; (*viz.*) * The First four rules, the latter section of the † *Algorithmical praxis*, *Reduction*, the *Rule of Three*; a slight view of *Practice*, *Interest*, *Discount*, and *Alligation*, and the articles in the second part very particularly and without exception. But if he is intended solely for trade, and not willing to learn any thing, but what is directly pointed to that end, he must take the first part before him as it lies, and the *Doctrine of decimal fractions* in the second, with their application to *Interest* a little farther on. If again he is limited

* As it is a good deal of practice, which generally makes young people masters of any thing of this kind, I have laid down this plan partly in reference to that, and also to what might be useful to a person, who may have a turn for reducing the mathematical theories, he may afterwards be acquainted with, to real practice, &c. But if speculation be his chief aim, a knowledge of what follows may do very well, *viz.* The plan here referred to, as it stands to the end of the *Rules of Three*; *Vulgar and Decimal fractions*, the *Extraction of the square root*, and the *Algorithm of ratios*. Tho' after all, to be at a loss in answering such questions, as may daily fall out in our common affairs, for want of a sufficient knowledge of arithmetical rules, or an aptness in their management, must be something ridiculous in one, who informs us, he has read the *Principia*, and traced the foot-steps of *Newton* along all those surprizing and intricate labyrinths of mathematical reasoning, he had to travel through in order to get a sight of the regions of true philosophy, and a satisfactory, and rational knowledge of nature.

† As to the first part of this head, 'tis so tedious and unentertaining, that it would in any case be as well to omit it the first time going over.

limited in time, or for other reasons would wish to have an easier (tho' less perfect) system of accounts for business. the *First four rules*, *Reduction*, the *Golden rule*, *Practice*, *Interest*, and *Discount*, may serve his turn very well in an ordinary way, especially if he be ingenious in applying the *Rule of three* to the great variety of questions, which it may be made to answer.

From these hints I hope the young student will be better able to read this book to his wishes, than if left intirely to himself: But if he has the advantage of a master, 'tis fit I leave him to his care. And as I have accidentally mentioned this circumstance here, I shall take the opportunity of observing to those teachers, who chuse to instruct their pupils by the following plan, that they will find it a very easy one, as what is proper to be copied into a manuscript is printed in *italics, and the three classes of practical questions so ordered, that with a judicious management (in setting the reverse of some, &c.) they may be made to suit any capacity, and disappoint the intentions of those idle boys, who would now and then be guilty of a little plagiarism.

The terms in this work to be explained are as follow:

Definition is an explication of the meaning of terms of art, or particular titles, in order to obviate any doubt, that might arise of their import.

Proposition in general is something proposed to be done or demonstrated, and, according to the circumstances of it, is distinguished into the following kinds:

(1.) *Theorem*: This is a proposition, whose truth is to be demonstrated, in order to its being established and relied upon in any future practice or application.

(2.) *Lemma*: This is a proposition, whose truth is to be demonstrated in the same manner, and only differs from the above in the additional

* When what is thus pointed out falls in the middle of a sentence, it will sometimes want the article *a* (or *an*) prefixed, &c. but this will readily occur to any reader.

additional circumstance of being brought in as a preparative principle, in order to shorten the demonstration of some other.

(3.) *Problem*: This is a proposition of something to be done in a practical way by known and received rules, without any regard to their reason or invention.

(4.) *Corollary*: This is a proposition gained in consequence of some other immediately demonstrated, and whose truth is evident from the truth of the other.

Scholium is an additional explication, or remark upon what preceeds, and (in this book especially) contains for the most part some hint for a farther improvement of the article concluded, a criticism, apology, or other particular, on which no absolute stress is required to be laid, if it so happen that the reader does not find himself inclinable to examine it.

Demonstration implies the highest proof or certainty possible, acquired from a train of arguments, referring all along to such plain axioms (or self-evident truths) and other pre-established principles, as cannot be denied by any one, that considers them.

Proof (when mentioned) either only refers to the truth of an operation, without regarding whether its manner, or application be right or wrong; or the actual trying in a practical way, whether a conclusion gained from a certain process will answer, what it was intended to do or not, &c.

T A B L E

O F

C O N T E N T S.

PART THE FIRST.

N OTATION - - - - -	Page 2
Addition - - - - -	6
Subtraction - - - - -	12
Multiplication - - - - -	16
Division - - - - -	31
Algorithimical Praxis, Section first - - - - -	46
Section second. - - - - -	55
Reduction Simple - - - - -	63
Compound - - - - -	73
The Rule of Three Direct - - - - -	79
Inverse - - - - -	94
Practice - - - - -	97
Tare, Tret, &c. - - - - -	114
Profit and Loss - - - - -	118
Commission, Brokerage, &c. - - - - -	123
Purchasing of Stock - - - - -	126
Barter - - - - -	130
Fellowship Simple - - - - -	140
with Time - - - - -	147
Interest Simple - - - - -	153
Compound - - - - -	159
	Rebate

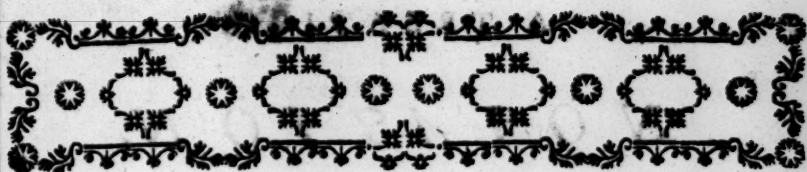
<i>Rebate or Discount</i>	- - - - -	161
<i>Equation of Payments</i>	- - - - -	164
<i>Exchange</i>	- - - - -	166
<i>Conjoined Proportion</i>	- - - - -	184
<i>Alligation Medial</i>	- - - - -	187
<i>Alternate</i>	- - - - -	189
<i>Alternation Partial</i>	- - - - -	192
<i>Total</i>	- - - - -	193

PART THE SECOND.

<i>INTRODUCTION, &c.</i>	- - - - -	Page 197
<i>To find a Greatest Common Measure</i>	- - - - -	200
<i>To find a Least Common Multiple</i>	- - - - -	202
<i>The Doctrine of Vulgar Fractions</i>	- - - - -	205
<i>Praxis (thereto)</i>	- - - - -	231
<i>The Doctrine of Decimal Fractions</i>	- - - - -	237
<i>Praxis (thereto)</i>	- - - - -	259
<i>Of Raising Powers</i>	- - - - -	267
<i>Of Extracting Roots</i>	- - - - -	268
<i>Progression Arithmetical</i>	- - - - -	287
<i>Geometrical</i>	- - - - -	296
<i>Permutation</i>	- - - - -	305
<i>The Rule of Contraries</i>	- - - - -	306
<i>Position Single</i>	- - - - -	310
<i>Double</i>	- - - - -	312
<i>Interest by Decimals</i>	- - - - -	323
<i>Algorithm of Ratios</i>	- - - - -	329
<i>Of the Composition, &c. of Ratios</i>	- - - - -	336
<i>Common Properties of Proportionality</i>	- - - - -	337
<i>Plural Proportion</i>	- - - - -	339

A COPPER PLATE.

A. R. A.



A

RATIONAL and PRACTICAL

TREATISE

OF

ARITHMETIC.

PART I.

Definition.

"**A**RITHMETIC is a science, which teaches the properties of
* numbers, and by them deduces precepts of computation from
certain † data, relative to the affairs of the busy part of man-
kind, and the enquiries of the curious."

Taking it for granted, that the student knows in some sort the present notation of numbers, in common with the letters and construction of his native language, the fundamental number of operations in arithmetic may then properly be reckoned † four (usually called the *Algorithm*) viz. *Addition, Subtraction, Multiplication, and Division*; from a right management and application of which every rule and process of calculation is deduced.

Before we enter upon these heads, it may be proper to lay before the young student a more perfect and comprehensive notation, than he may perhaps have occasionally formed for himself, as follows.

B

NOTATION.

DEMONSTRATIONS, NOTES, &c.

* *Number* is the name of that idea, under which things being considered are said to be one or many.

As to the question whether unity be included in the word number, we may observe, that it is, or is not, just as 'tis defined.---In the definition above unity is included; hence what is here meant by it cannot be misunderstood by any disputant, be he ever so inclinable to trifle.

† Requisites or conditions given.

‡ In strictness there are but two primary operations, from which the rest are drawn, i. e. addition, and subtraction, as will be visible farther on. For multiplication is but a compendious method of addition, and division of subtraction.

NOTATION.

Definition.

“*NOTATION teaches how to describe or denote any number by certain signs or characters, called figures, or digits.*”

All the figures now in use are the following ten;

One,	two,	three,	four,	five,	six,	seven,	eight,	nine,	* nothing.
1,	2,	3,	4,	5,	6,	7,	8,	9,	0.

And in order to express any number, however large, by their means, and without the help of new characters, the † inventors of them have happily made use of the following device.

Besides the *simple* value of the figures, as above noted, they have given them a ‡ *local* one, and according to the following law;

VIZ.

In a combination of figures, reckoning from right to left, they agreed, that the figure in the first place should represent its primitive simple value; that in the second ten times its simple value; that in the third a hundred times its simple value, or ten times what it would represent in the second place: And so on, giving each place ten times the value of that immediately preceding it.

Having established this, it will appear, that the local value of figures in all places under nine (which are sufficient in common affairs) will be rightly denominated, as in the following table, commonly called, *Numeration table*.

Numeration.

* The necessity for this figure 0 (commonly called a cypher) will easily appear from what follows about the local value of figures; for with it places are filled up, so that other figures may fall into those they ought to be in, to give the intended expression.

† 'Tis said the Arabians first made use of this method, and that they had it from the Indians.

‡ Derived from the *place* they stand in,

Numeration Table.

Units	9	8	7	6	5	4	3	2	1
Tens	9	8	7	6	5	4	3	2	1
Hundreds	9	8	7	6	5	4	3	2	1
Thousands	9	8	7	6	5	4	3	2	1
Tens of thousands	9	8	7	6	5	4	3	2	1
Hundreds of thous.	9	8	7	6	5	4	3	2	1
Millions	9	8	7	6	5	4	3	2	1
Tens of millions	9	8	7	6	5	4	3	2	1
Hundreds of millions	9	8	7	6	5	4	3	2	1

This being premised, we can easily see the reason of the two following rules, which answer all the questions, that can be proposed under this article.

1. "To write numbers."

Rule.

"Write down the figures in the same order their values are expressed in, beginning at the left hand, and writing towards the right, remembering to supply those places (of the natural order above) with cyphers, which are omitted in the question."

Examples.

1. Write down fifty four thousand three hundred and twenty six?

Answer 54326.

2. Write down four millions three thousand and thirty four?

Answer 4003034.

2. "To read any number."

Rule.

"To the simple value of each figure join the name of its place, beginning at the left hand and reading towards the right."

Examples.

1. Required to read all the numbers in the above table?

B 2

Answer.

Answer. { Nine.
 Ninety eight.
 Nine hundred and eighty seven.
 Nine thousand eight hundred and seventy six.
 Ninety eight thousand seven hundred and sixty five.
 Nine hund. and eighty seven thous. fix hund. and fifty four.
 Nine mill. eight hund. and seventy fix thous. five hund. and forty three.
 Ninety eight mill. seven hund. and sixty five thous. four hund. and thirty two.
 Nine hund. and eighty seven mill. fix hund. and fifty four thous. three hund. and twenty one.

2. Required to read these numbers — — { 6204 ?
 300806 ?
 73020019 ?

Anf. { Six thousand two hundred and four.
 Three hundred thousand eight hundred and six.
 Seventy three millions twenty thousand and nineteen.

Scholium.

Towards rendering the article of reading figures still more easy, we may observe in the above table of numeration, that every third figure has the name of hundreds. This gives rise to a very common as well as a very good method of placing a point betwixt every class or period of three, and pronouncing them in parcels (as it were), calling the first period so many *units*, the second *thousands*, the next *millions*, &c. as in the following table.

Mill.			Thousf.			• Units.		
C of mill.			C of thousf.			Hund. (C)		
X of mill.	Mill.		X of thousf.	Thousf.		Tens (X)	Units	
9	8	7	6	5	4	3	2	1

Farther it may be observed towards discovering “an easy method of numerating very large numbers,” that the seventh place is millions, and that the thirteenth (if carried on) would be † millions of millions, the nineteenth millions of millions of millions, and so on. Therefore if we “make a point over the seventh figure from the first,

two

* C and X are marks for the words they stand against.

† Some authors instead of millions of millions, say billions; for millions of millions of millions, trillions; for millions of millions of millions of millions, quadrillions, &c. But for any great occasion there is for either method, 'tis not material whether be followed,

Rule.

Thus the following number, pointed as you see, will be read—

three thousand six hundred and twenty four millions of millions of millions, six hundred and thirty eight thousand two hundred and fifty seven millions of millions, five hundred and forty thousand two hundred and eighty five millions, three hundred and twenty one thousand four hundred and fifty seven.

Practical Questions.

1. Write down three hundred and forty eight?
2. Write down two thousand four hundred and ninety?
3. Write down seven hundred and forty two thousand five hundred and six?
4. Write down fifty millions fourteen thousand and eleven?
5. Turn the following number into words — — 6923?
6. Do. — — — — — — — — 50634?
7. Do. — — — — — — — — 900208?
8. Do. — — — — — — — — 11020370?

1. Write down six thousand and eighty three millions seven hundred and sixty thousand four hundred and fifty two?
2. Write down five hundred thousand and four millions five hundred and sixty two thousand and eighty?
3. Turn the following number into words — 3604125708?
4. Do. — — — — — 10983706005?

1. Write down a million and half in South-Sea-Bonds?
2. Write down three score and fifteen thousand four hundred sheep?
3. Write down eleven thousand eleven hundred and eleven?
4. Write down a hundred and four score and five thousand; which is the number destroyed by an angel in the camp of the Assyrians, mentioned in Scripture.

ADDITION.

ADDITION.

Definition.

"**ADDITION** is the finding one number, or sum, equal to two, or more numbers or sums."

* Preparative to this, 'tis expected, that the student can find the sum of any two digits; add a digit to any number proposed, and tell how many tens are contained in a given number. This being premised we have the following rule for adding † *abstract* whole numbers.

† Rule.

"Place the numbers to be added under one another in such a manner, that units will fall under units, tens under tens, hundreds under hundreds, &c. Draw a line under them, and then beginning at the lowest figure in the units place, add it to the next above, and their sum to the next above that, and so on till the whole line is added together. This done, if the sum is under ten, set it down below the line it came from: If above ten, the odds only must be set down, and for every ten that is in it one (or an unit) must be carried to the next perpendicular line (or line of tens).—In this manner proceed from line to line (adding, setting down the odds, and carrying one for every ten) till the whole is completed."

The following example and illustration will make the method very plain.

Example.

	8635	
	2194	
	7421	
	5063	
	2196	
	1245	
	26754	Sum.

Required to add the adjoining numbers? {

Illustration.

* If this be not the case, the first requisite may be learned from the adjoining table, whose use is this. Take the greater of the two digits, whose sum is sought, in the upper line, and the other on the right hand column, and in the meeting of these lines stands the sum. The second requisite is so near a-kin to the first, as to easily follow; and the third is soon known from the nature of notation. For whatever the number be, write it down, and the figure in the tens place shews the number of tens, and that in the units the odds.

† Such numbers as have no denomination annexed to them.

‡ As to the reason of this rule, 'tis easily apprehended from what was said in notation

1	2	3	4	5	6	7	8	9	0
2	3	4	5	6	7	8	9	10	1
4	5	6	7	8	9	10	11	2	
6	7	8	9	10	11	12	3		
A	8	9	10	11	12	13	4		
table	10	11	12	13	14	5			
shewing	12	13	14	15	6				
the sum of	14	15	16	7					
any two digits.	16	17	8						
		18	9						

Illustration.

The numbers being rightly placed as above, I begin with 5 (the lowest figure in the units or first line) and say, 5 and 6 make 11, and 3 makes 14, and 1 makes 15, and 4 makes 19, and 5 makes 24. Now as there are 2 tens in 24, and 4 units or odds, I set down the 4, and carry 2 (for the tens) to 4 (the lowest figure in the next line) and go on saying, 2 carried and 4 make 6, and 9 makes 15, and 6 makes 21, and 2 makes 23, and 9 makes 32, and 3 makes 35. Hence I set down 5 (the odds in 35) and carry 3 (for the tens) to the next line saying, 3 carried and 2 make 5, and 1 makes 6, and 0 makes (still) 6, and 4 makes 10, and 1 makes 11, and 6 makes 17. Hence, as before, I set down 7, and carry one (to the lowest figure in the next line) saying, 1 carried and 1 make 2, and 2 makes 4, and 5 makes 9, and 7 makes 16, and 2 makes 18, and 8 makes 26. Therefore, as usual, I set down the odd 6, and carry 2; but, as there is now no other line to carry it to, it must consequently be placed at the left of the rest by itself.

The inspection of the following examples (which the reader may easily enlarge himself, if he thinks proper) will still make the subject plainer.

Examples.

246034	5762041	562163
298765	37637	21964
47321	638126	563
58653	5219	21
64218	5986	18536
5376	321	4340
9821	514265	279
340	2538	83
730528 Sum.	6966133 Sum.	607949 Sum.

'Tis usual to point out "*a method of proving (as 'tis called) the truth of an addition,*" which is thus,

* "*Draw a line with your pen below the highest number, and suppose it*

notation. For from that it appears in placing numbers, that if units, &c. were not set under one another, we should effectually alter their value. And 'tis plain that the method of carrying one for every ten from an inferior line to a superior is right, as an unit in the latter case is of the same value as ten in the former. Therefore this method of incorporating the excesses of lines (if the phrase be proper) with their next superior, answers neatly and exactly the intent of addition.

* the reason of this method of proving the truth of the operation is evident; for the sum of the parts must be equal to the whole.---On this principal in long additions,

it cut off. Then add all the numbers together, over again, except this highest, and set the sum under the number to be proved. Next add this last found number and the top line together, and if it is the same, as that found by the first addition, 'tis very probable 'tis right; if not, all that can be said is, that some part of the operation is wrong, but which cannot be discovered without a new essay."

872463

365498

216473

5642

59376

1519452 Sum.

646989

1519452 Proof.

See the adjoining example.

It only remains now, to the finishing of this particular, that we add "a few

Practical Questions."

CLASS I.

1. The distance between London and Royston is 33 miles; from thence to Cambridge 10; thence to Newmarket 10; thence to Bury 10; and from thence to Norwich 32 miles: How many miles on this road is it from London to Norwich.

Answer 95 miles.

2. An old man's age was required, and he answered, I have 5 sons and 3 daughters; betwixt the birth of each of my sons were 2 years, betwixt my last son and first daughter 4 years; and four years a-piece betwixt the rest of my daughters; in my 20th. year was my first son born, and that is the age of my youngest daughter. What is the father's age.

Answer 60 years.

3. How many days are there in the twelve calendar months; January having 31 days, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, and December 31 days.

Answer 365 days.

CLASS II.

1. There are three numbers, the first 215, the second 519, and the third as much as the other two. What is the sum of them all?

Answer 1468.

How many days are there from the 19th. day of April 1762 to the 27th. day of November 1764, both days exclusive?

Answer 953 days.

CLASS III.

From the creation of the world, to the beginning of the deluge are accounted 1656 years; and from the beginning of the deluge to the

additions, 'tis sometimes convenient to divide the column into several portions, adding them separately, and then the result into one total, by which means a good deal of the tediousness of reckoning is avoided.

the birth of Arphaxad, Helvicus reckons 2 years; and thence to Terah 220 years; thence to the birth of Abraham 70 years; thence to the promise given he reckons 75 years; and thence to the going out of Egypt 430 years; and from that going out to the temple of Solomon 480 years; and thence to the birth of Christ 1015 years; and he supposes, that from thence to the beginning of the common Christian or Dionysian *Æra* 2 years elapsed; and thence to the present year, wherein this was transcribed from Weston's arithmetic, we reckon 1764 years: According to the chronology of Helvicus, therefore, how long is it since the creation of the world.

Answer 5714 years.

What goes before being attained, we should next shew the manner of adding * *applicate* numbers, in which are contained different denominations (as in monies, weights, measures, &c.) but as some of these are broke in such a manner, that their addition would seem to require the knowledge of division, or at least more experience, than the learner is yet supposed to have, we shall † limit the subject at present to the addition of English money, expecting that he is perfectly acquainted, by unavoidable daily experience, with the method of ‡ turning any number of farthings, pence, and shillings into their higher denominations; if so, the article is easily effected, as follows.

“To add money.”

§ Rule.

“Place the pounds, shillings, pence and farthings exactly one under another.”

* *Applicate* numbers are those, which are applied to things (as 500 pounds) and if they are simple, they are added as abstract whole numbers.

† Tho’ this handling things in particular, and multiplying rules seems not to have so much of good method in it as some might expect, yet we are persuaded from other reasons, besides those expressed in the above paragraph, that the arithmetical management of weight, measures, &c. will be to the full as well effected by itself.---This is done at the end of division in a part called, *The Algorithmical Praxis*, in which some other articles of a miscellaneous nature, usually inserted in the four first rules, are largely handled.

		£. s.				s. d.	
† Those that are not perfect in this may make themselves so by perusing the adjoining tables.	20	shill.	make	1	0	20	pence make 1 8
	30	shill.	make	1	10	30	pence make 2 6
	40	shill.	make	2	0	40	pence make 3 4
	50	shill.	make	2	10	50	pence make 4 2
	60	shill.	make	3	0	60	pence make 5 0
	70	shill.	make	3	10	70	pence make 5 10
	80	shill.	make	4	0	80	pence make 6 8
	90	shill.	make	4	10	90	pence make 7 6
	100	shill.	make	5	0	100	pence make 8 4
	110	shill.	make	5	10	110	pence make 9 2
	120	shill.	make	6	0	120	pence make 10 0
	130	shill.	make	6	10	140	pence make 20 0

§ The reason of this rule (if a reason need be given for what is so easy to conceive)

another. Then beginning with the farthings (if there be any) add them (as before), and for every four or penny they contain, carry one to the pence, and set down the odds. Add the pence in the same manner and carry one for every shilling, or 12 pence, to the shillings, and set down the odds. Next add the shillings, and carry one for every pound, or 20 shillings, to the pounds, placing down the odds, as before. Lastly, add the pounds, as in abstract numbers, carrying one for every ten, and you have the sum required.

The process will appear pretty plain from the adjoining example; and as the manner of proving the operation here is similar to what was shewn of abstract whole numbers, 'twill be needless to insist on it particularly.

£.	s.	d.	
326	10	$8\frac{1}{4}$	{
426	18	$10\frac{3}{4}$	
198	15	$11\frac{1}{2}$	
621	6	5	
982	17	$11\frac{1}{2}$	
427	19	$11\frac{3}{4}$	{
2984	9	$10\frac{3}{4}$	
2657	19	$2\frac{1}{4}$	
2984	9	$10\frac{3}{4}$	Sum, Proof.

Illustration.

Before we begin with particulars, it will be proper to observe, that as the pence and shillings generally contain a good deal of tens, 'tis the easiest way to add them by going first up the units line in each denomination) and to the sum it makes adding ten for every ten in the column, as you come down again, as is more fully shewn in the course of the following process.—Beginning with the farthings I say 3 and 2 make 5, and 2 makes 7, and 3 makes 10, and 1 makes 11; 11 farthings make 2 pence 3 farthings; 10 I set down $\frac{3}{4}$, and carry 2 to the pence, saying as I go on (up the units side) 3 and 1 make 4, and 5 makes 9, and 1 makes 10, and 8 makes 18; then, pointing at the tens, as often as there are any, down again, I say, and 10 makes 28, and 10 makes 38, and 10 makes 48, and 10 makes 58;—58 pence make 4s. 10d.; therefore I set down 10, and

ceive) is evident from what was said in addition of abstract numbers; as, on a like consideration it appears, that 1 in the pence must be equal in value to 4 in the farthings; 1 in the shillings equal to 12 in the pence; and 1 in the pounds equal to 20 in the shillings. Consequently carrying, as directed, does no other, than provide a method of digesting the money arising from each column properly in the scale of denominations.

Several people in adding the pence and shillings make a little point with their pen on the right hand of the figures, as they go up, as soon as they have added to or above 12 and 20 respectively, going on with the overplus to the next figure or figures, and deducting when necessary, in the same manner, till they are at the top; then what odds they have remaining they put down, and carry one for every point, thus made, to the next superior denomination.

and carry 4 to the shillings, proceeding thus (upwards)—4 and 9 make 13, and 7 makes 20, and 6 makes 26, and 5 makes 31, and 8 makes 39, and (returning) 10 makes 49, and 10 makes 59, and 10 makes 69, and 10 makes 79, and 10 makes 89—89 shillings make £.4 9s. therefore I set down the 9 shillings, and carry 4 to the pounds; the manner of adding which being already taught, 'tis needless to repeat it.

An example or two more for inspection.

£.	s.	d.	£.	s.	d.	£.	s.	d.
264	15	8 $\frac{3}{4}$	240	17	6 $\frac{1}{2}$	62	10	3 $\frac{1}{4}$
198	16	10 $\frac{1}{2}$	196	11	8 $\frac{3}{4}$	19	19	6 $\frac{1}{2}$
42	17	6 $\frac{3}{4}$	65	—	6 $\frac{3}{4}$	64	12	11 $\frac{1}{4}$
451	10	11 $\frac{1}{2}$	214	17	10	9	2	3 $\frac{3}{4}$
382	12	—	641	5	3 $\frac{3}{4}$	5	6	10
643	9	10	493	16	2	3	7	6 $\frac{1}{4}$
24	10	8 $\frac{3}{4}$	123	10	5 $\frac{1}{4}$	46	13	8 $\frac{1}{2}$
2008	13	8 $\frac{1}{4}$ Sum.	1975	19	4 Sum.	211	13	1 $\frac{1}{2}$ Sum.
1743	17	11 $\frac{1}{2}$	1735	1	9 $\frac{1}{2}$	149	2	10 $\frac{1}{3}$
2008	13	8 $\frac{1}{4}$ Proof.	1975	19	4 Proof.	211	13	1 $\frac{1}{2}$ Proof.

The following are a few “*practical questions*,” under this article.

CLASS I.

1. Add £. 230-17-10 $\frac{1}{4}$, l. 175-12-11 $\frac{3}{4}$, l. 52-5-6, l. 9-0-8 $\frac{3}{4}$, and l. 506-13-7 $\frac{1}{2}$ together? Answer £. 974-10-7 $\frac{3}{4}$.
2. Add £. 46204-16-10 $\frac{1}{4}$, l. 39264-14-6 $\frac{1}{2}$, l. 42-17-8, l. 9-10, l. 470-16-4 $\frac{1}{4}$, l. 0-13-4 $\frac{1}{2}$, l. 126-17-8 $\frac{1}{4}$, l. 5-17, l. 3420-18-4, l. 37-0-6 $\frac{1}{2}$ and l. 226-14-8 together? Answer £. 89810-17-0 $\frac{3}{4}$.
3. A corn-factor buys several quarters of oats, for 46-7-6, thirty eight quarters of beans, for £. 100, twelve quarters of peas, which cost £. 16-16, eighty eight quarters of barley, for £. 73-8, sixteen ditto of wheat, for £. 56-9-10, and six quarters of rye, for £. 4-1-6. The water carriage of all comes to £. 13-2-7, his riding charges to £. 1-13, and if he clears eighteen guineas by the bargain, what do his bills of parcels amount to? Answer £. 330-16-5.

CLASS II.

1. A person dying left his widow the use of £. 5000, To a charity he bequeathed £. 846-10; to each of his three nephews £. 1230, to each of his four nieces £. 1050; to twenty poor house-keepers five guineas each, and 200 guineas to his executor. What must he have died possessed of. Answer £. 14051-10.

2. A gentleman lent his friend at different times these several sums, viz. £. 142-10-8, l. 6-17-10, l. 64-11-8, and threescore and fourteen pounds, half a guinea and a moidore. How much did he lend in all.

£. 289-17-8.

CLASS III.

The collector of cash has been out with bills, and gives account, that A paid him £. 13, and half a crown; B. 2-13, C. l. 0-14 and a groat; D. l. 1-9-8½, E. l. 11-0-6¼, F. l. 0-17, and a tetter (i. e. 6 pence;) G. l. 0-12-2, H. a pound and half a guinea; I. a moidore and 13 shillings; K. 2 broad-pieces of 23 shillings each, and a jacobus of 25 shillings and a shilling; L. nine pounds and a mark; M. l. 12-12; N. a bank-note of 15 pounds; and O. three crown-pieces and an angel (i. e. 10 shillings;) What cash has he in his charge?

Answer £. 76-2-6¼.

SUBTRACTION.

Definition.

"**SUBTRACTION** is the taking one number or sum out of another, or finding the difference of any two numbers or sums."

Premising, that the student * knows the difference of any two digits; the sum of a digit added to any other number; and what remains, when a digit is taken out of any number, subtraction of abstract whole numbers is effected by the following

+ Rule.

"Place the less number (called the Subtractor) properly under the greater (called the Subtrahend) as in addition. Draw a line, and then beginning with the right-hand figure of the subtractor, take it out of the figure above it in the subtrahend if the higher be bigger, and put down the difference; but if the higher is less, you must borrow ten to it, and then put down

* If not, the perusal of the adjoining table is necessary; in which finding the Subtractor on the right hand column, and against it under the Subtrahend, found in the highest line, is the difference required.

† From the nature of notation the reason of this rule cannot be hard to conceive. For first, if all the digits in the Subtractor were less than the corresponding ones in the Subtrahend, the case would be exceeding obvious. And where we find it not so, and borrow ten to the figure above (thereby conceiving the Subtrahend so much bigger, than it really is, as the power of the ten makes according to its place) 'tis evident that this excess is effectually rectified by carrying one to the next figure in the Subtractor (thereby conceiving the Subtractor likewise as much larger, than it really is, as the power of the one makes in its place) which is plainly equal

1	2	3	4	5	6	7	8	9	0
0	1	2	3	4	5	6	7	8	1
0	1	2	3	4	5	6	7	2	
0	1	2	3	4	5	6	3		
A	0	1	2	3	4	5	4		
table	0	1	2	3	4	5			
for finding	0	1	2	3	6				
the difference	0	1	2	7					
of any two digits.	0	1	8						
				0	9				

down the difference betwixt the lower figure and the higher so encreased. This done, proceed to the finding the difference betwixt the next figure of the subtrahend and its correspondent one in the subtrahend, according to the directions above, remembering, that if you borrowed ten, the operation immediately before, the subtrahend, in that case, must be encreased (or imagined encreased) by one (1).—Proceed thus figure by figure to the end, and the number under the line resulting from the process is the difference required (and commonly called the Remainder).

The illustration of the following example will make the method plain.

Example.

Required the difference of the two adjoining numbers?	{	625735 Subtrahend.
		468572 Subtractor.
		Anf. <u>157163</u> Remainder.

Illustration.

The numbers being placed properly, as above, I begin at the right hand figure of the subtrahend, and say 2 from 5 above and 3 remains, which (3) I put down; then going on, I say, 7 from 3 I cannot, but 10 borrowed (as the rule directs) to 3 makes 13; then 7 from 13 leaves 6, which I put down. Now as I borrowed 10 before, I must carry one to the next figure (in the subtrahend) 5, which makes it 6, then I say 6 from 7 and 1 remains; this put down, I proceed thus, 8 from 5 I cannot, but 8 from 15 (ten being borrowed) and 7 remains. This set down, I carry one to the next figure 6, and say 6 and 1 make 7,—7 from 2 I cannot, but 7 from 12 (borrowing) and 5 remains. This put down, I proceed to the last figure saying 4 and 1 (carried) make 5,—5 from 6 and 1 remains; this I put down also, and the operation is done.

The perusal of the following examples (which the learner may easily enlarge himself, as he thinks fit) will more amply illustrate the whole.

Examples.

From 2637804	3762162	78213606
Take 2376982	826541	27821890
Rem. <u>260822</u>	<u>2935621</u>	<u>50391716</u>

The

equal to that of ten in the place immediately preceeding, where it was borrowed. And if both Subtrahend and Subtrahend have been conceived encreased by the same number, their difference in that case must needs be the same, as if they had not been augmented at all. Hence the rule is right.

* "The common method of proving the work is, by adding the remainder to the subtrahend, and if the result is equal to the subtrahend, the operation is said to be right."

362763027

98276142

264486885 Remainder.

See the adjoining example.

362763027 Proof.

As a finishing of this article we shall add a few "practical questions."

CLASS I.

1. How long is it since the Spanish invasion this present year 1764, it being in the year 1588? Answer 176 years.

2. Rome was built, according to Varro, in the year of the Julian period 3960; Christ was born in the year of the same period 4714: How many years before Christ's birth was Rome built, according to this computation? Answer 754 years.

CLASS II.

A grant by the Crown, anno domini 1237, was forfeited 137 years before the revolution in 1688: How long did the same subsist?

Answer 314 years.

CLASS III.

Two brothers were born, one in the year 1751, and the other in 1763; two sisters are 18, and 23 years old respectively: What is the difference of the difference of the ages of the two sexes?

Answer 7 years.

† "To subtract money."

‡ Rule.

"Place the sums under one another as in addition. Then beginning with the farthings in the Subtraher (if there be any) take them out of those above, if those above be more, and put down the remainder; if not borrow 4 farthings to those in the Subtrahend, and take the farthings in the

* 'Tis very plain that the difference of two numbers, added to the less, must make it equal to the greater. But still the proof is but presumptive; for there may be a mistake in both operations of such a nature as to balance one another; and moreover the subtraction may be right and the addition faulty; hence a review of the work is often necessary.

† We shall continue the plan of leaving out the management of weights, measures, &c. till we come to the above-mentioned Algorithmical praxis.

‡ The reason of this appears very clearly from what was said in abstract numbers; if we can only conceive farther, that 4 in the farthings is equal to 1 in the pence, 12 in the pence equal to one in the shillings, and 20 in the shillings equal to one in the pounds.

§ Some take the figures to be subtracted out of what they borrow, and to the difference add the figures, which were too little in the Subtrahend, for the remainder.

the Subtractor out of the sum, and set down what remains.—Proceed next to the pence in the Subtractor, remembering, if you borrowed (in the farthings) before, to carry one to them, and take the sum out of the pence above, if the higher exceeds; but if not, borrow 12d. to them, and then subtract: This done, set down what remains, and go on to the shillings and pounds in the same manner, only observing to borrow (where wanted) 20 in the first mentioned denomination and 10 in the other, and to carry one to the next figure or denomination for it.”

An Example.

Required the difference of the two adjoining sums?—The proof is gained in a manner similar to that in abstract whole numbers.

£.	s.	d.	
246	10	$3\frac{1}{4}$	Subtrahend.
128	12	$5\frac{1}{2}$	Subtractor.
Ans. 117	17	$9\frac{3}{4}$	Remainder.
246	10	$3\frac{1}{4}$	Proof.

Illustration.

Beginning with the farthings, I say, 2 from 1 I cannot, but 4 borrowed to one makes 5. Then 2 from 5 and 3 remains: This I put down and proceed to the pence (remembering to carry) saying, 1 and 5 make 6, 6 from 3 I cannot, but 6 from 15 (borrowing 12) and 9 remains. This put down, I go on saying, 1 (carried) to 12 makes 13, 13 from ten I cannot, but 13 from 30 (borrowing 20) and there remains 17. This I put down, and carry one to the pounds, in which I proceed as in the subtraction of abstract whole numbers.

More examples for inspection.

	£.	s.	d.		£.	s.	d.		£.	s.	d.
From	264	8	4 $\frac{1}{4}$	2792	—	6 $\frac{1}{4}$	296	10	— $\frac{1}{2}$		
Take	136	11	3 $\frac{1}{2}$	1276	12	8	37	12	5 $\frac{1}{2}$		
Rem.	127	17	— $\frac{3}{4}$	1515	7	10 $\frac{1}{4}$	258	17	6 $\frac{1}{2}$		
Proof.	264	8	4 $\frac{1}{4}$	2792	—	6 $\frac{1}{4}$	296	10	— $\frac{1}{2}$		

Agreeably to our plan, we shall conclude this article with “a practical question or two.”

CLASS I.

1. What sum added to £. 276-7-4 $\frac{1}{4}$, will make £. 427-19-1?

Answer 151-11-8 $\frac{3}{4}$.

2. Suppose a person, worth £. 5000 a year, spends but £. 2764-10-6, what does he spare per annum?

Answer £. 2235-9-6.

CLASS

CLASS II.

What is the difference between £. 6000, and the following sums, viz. £. 2762-10-8, l. 56-10, l. 145-0-6 $\frac{1}{2}$, and l. 4444-4-4?

Answer, (1) £. 3237-9-4, (2) l. 5943-10, (3) l. 5854-19-5 $\frac{1}{2}$, (4) l. 1555-15-8.

CLASS III.

A horse in his furniture is worth £. 37-5; out of it, 14 guineas: How much does the price of the furniture exceed that of the horse?

Answer £. 7-17.

MULTIPLICATION.

Definition.

"**M**ULTIPLICATION is the taking any sum or number, as often as is expressed by a number proposed; or the finding a sum or number, any number of times bigger than a given one."

Preparative to this, the student should know the product of any digit by any digit: (* Found by adding one digit to itself as often, as there are digits in the other.) With this knowledge multiplication of abstract whole numbers is in general effected by the † two following rules.

1. "To multiply any abstract whole number by a single digit."

† Rule,

"Under the units place of the number to be multiplied (called the Multiplicand) place the multiplying digit (called the Multiplier.) Draw a line, and then (beginning at the right hand) find the product of the Multiplier, and every figure in the Multiplicand singly one after another

* Or better attained thus, by committing the adjoining table to memory: In which find the greater of the two digits in the upper line, and under it against the less, taken on the right side column, is the product sought.

† These two rules might indeed have been included in one, but as it is very convenient to have a distinction of this kind in division, it was judged as proper to handle both articles in a like manner.

‡ What was offered for the reason of addition of abstract whole numbers, with the inspection of the adjoining operation (which plainly shews multiplication to be but a compendious method of addition), is sufficient to evince the truth of the rule

§ The Multiplier and Multiplicand are also called *Factors*, in common without any distinction.

1	2	3	4	5	6	7	8	9	1
4	6	8	10	12	14	16	18	2	
9	12	15	18	21	24	27	3		
A	16	20	24	28	32	36	4		
table of	25	30	35	40	45	5			
multiplica-	36	42	48	54	6				
tion of the	49	56	63	7					
simple digits.	64	72	8						
	81	9							

580634
580634
580634
580634

2322536

other, setting down the odds, and carrying one for every 10 (as in addition) to the next product, till the whole be completed."

An example will make this obvious.

$$\begin{array}{r} \text{Required to multiply } 580634 \text{ by } 4? \left\{ \begin{array}{l} 580634 \text{ Multiplicand,} \\ 4 \text{ Multiplier} \\ \hline \text{Ans. } 2322536 \text{ Product.} \end{array} \right. \end{array}$$

Illustration.

The figures placed as directed, I say 4 times 4 make 16, set down 6 and carry one,—4 times 3 make 12 and 1 (carried) makes 13, set down 3 and carry one,—4 times 6 make 24, and 1 makes 25, set down 5 and carry 2,—4 times 0 make nothing but 2 (carried) makes 2, set down 2,—4 times 8 make 32, set down 2 and carry 3,—4 times 5 make 20, and 3 makes 23, set down 3, and also the 2 (as there are no more figures) on the left hand.

More examples for inspection (easily enlarged with others of the same kind).

$$\begin{array}{r} 2604376 \\ 5 \\ \hline 13021880 \end{array}$$

$$\begin{array}{r} 8209046 \\ 7 \\ \hline 57463322 \end{array}$$

$$\begin{array}{r} 21098340 \\ 9 \\ \hline 189885060 \end{array}$$

2. "To multiply one number by any other."

** Rule.*

"Place the Multiplier orderly under the Multiplicand: Draw a line, and then multiply the whole Multiplicand by every figure in the Multiplier, as before, with this farther observation (*viz.*) that whatever figure you are multiplying with, the first figure in the product (or that which comes from the units place in the Multiplicand) must be set under it, and all the rest removed to the left hand in the same manner, and also, that whenever cyphers occur in the multiplier, they must be passed over by placing others under them. These lines added together give the Product."

D

An

* As to the proof of the rule; I think nothing more need be observed, that 'tis as easy to see from what was said in Notation, and the demonstration of the last rule, that the products of the Multiplicand, by the simple digits of the Multiplier, must be placed orderly under one another, each a figure more to the left hand, than that preceding it, in order to be added, as that the sum of 5000, 300, 20, and

An example will make this plainer.

Multiply - - - - - 2639854 Multiplicand.
By - - - - - 3062 Multiplier.

$$\begin{array}{r} 5279708 \\ 15839124 \\ 79195620 \\ \hline \end{array}$$

Answer - - - - - 8083232948 Product.

Illustration.

The numbers properly placed, as above, I begin and multiply the whole Multiplicand over by 2; then I go over it again with the 6, remembering to place the first figure (4) under it, and to give all the succeeding figures the same remove. This done I come to the cypher, under which I place another, and pass on to the 3, which I multiply in the same manner (as the 2 and 6) into the multiplicand, setting the first figure resulting under it, &c.—These parts of the multiplication thus finished, I draw a line under them, and add them for the product required.

More examples for inspection,

$$\begin{array}{r} \begin{array}{c} 3 \\ 7 \times 3 \\ 3 \end{array} \quad \begin{array}{r} 857698 \\ 7680 \\ \hline 68615840 \\ 5146188 \\ 6003886 \\ \hline 6587120640 \end{array} \end{array}$$

$$\begin{array}{r} \begin{array}{c} 0 \\ 0 \times 5 \\ 0 \end{array} \quad \begin{array}{r} 786804768 \\ 806009 \\ \hline 7081242912 \\ 472082860800 \\ 62944381440 \\ \hline 634171724250912 \end{array} \end{array}$$

and 4. make up the multiplicand in the following example, which is subjoined with intent to make the matter still plainer, if possible.

$$\begin{array}{r} 135243 \\ 5324 \\ \hline 540972 \text{ --- 4 times the Multiplicand.} \\ 270486 \text{ --- 20 times Do.} \\ 405729 \text{ --- 300 times Do.} \\ 676215 \text{ --- 5000 times Do.} \\ \hline 720033732 \text{ Sum. --- 5324 times Do.---OR the answer.} \end{array}$$

I am sensible a stricter demonstration than this might have been given (See *Malcolm* page 37, 38. and 39) but it would have been tedious, and perhaps to many not so satisfactory as this.

$$\begin{array}{c} 1 \\ 7 \times 4 \\ 1 \end{array}$$

$$\begin{array}{r} 50710984 \\ 4050607 \\ \hline 354976888 \\ 3042659040 \\ 2535549200 \\ 2028439360 \\ \hline 205410266767288 \end{array}$$

* "The usual way of proving multiplication (though † not always certain) is by casting out the nines (as 'tis called) thus. Make a cross,
D 2 as

* In order to the demonstration of this, it will be necessary to premise this

Lemma.

"The figure, that stands in any place of a number, taken in its simple value, is equal to what will remain, after 9 is taken out of the complete value, as often as possible, i. e. after all the 9s contained in it are taken away. Thus, if the 9s in 8000, 500, and 30 be taken away, there will remain 8, 5, and 3 respectively."

Demonstration.

For the complete value (if above 10) always expresses a certain number of tens, which may be considered, as so many nines added to so many ones, and 'tis plain from Notation, that if the digit be removed to the next inferior place, we shall know the number of these aggregates (being equal to its value there), consequently if the 9s be thrown out of this aggregate, there will remain as many units, as are expressed by the figure removed, as above directed. Now (except this digit be in the unit's place) the units remaining expressed by the digit will themselves contain a number of nines added to so many ones, equal to its value in the next inferior place; consequently the next inferior place will express the units remaining, when the 9s are again thrown out. Now as in this method we never alter the value of this digit, but only remove it to an inferior place each operation, 'tis plain (as at last it will always fall into the units place), it is itself the true remainder, after the 9s in its complete value (where it first stood) are deducted.

Corollary.

This being granted, if out of the sum of all the simple digits in a compound number, the 9s that are contained in it, be thrown away, the remainder will be the same, as when the 9s are thrown out of the number represented by its compound value. For it is plain there can be no more 9s in any number, than what are in the several parts, and the sum of the excesses of 9 in the same parts.

Now if to the above Lemma and Corollary, we may be allowed to add the following truths (easy to conceive without a formal demonstration) their application below will easily evince the whole, viz. 1. That if any number of 9s be multiplied into any digit or number, the product will be an exact number of 9s (without any overplus). 2d. That if any digit or number be multiplied into any number of 9s, the product will be an exact number of 9s. 3d. That if any number be broke into two parts, and any other number multiplied into these parts, the sum of the products resulting will be equal to the product of the number to be multiplied and the given number unbroke.---Now suppose the multiplier in any example broke into two parts, i. e. the the number of 9s it contains, and the odds; then (by 1) multiplying

as in the margin, and then, beginning at either end of the Multiplicand, add the digits together, till they make 9 or above, and out of the sum throw away 9; add what remains to the next figure, and if the sum is 9, or above, reject 9, and proceed in the same manner with the overplus to the next figure; remembering, that if added to this next figure it be short of 9, you must take in the figure or figures immediately following, till the sum be 9 or above; and then out of it expunge 9. In the same manner proceed through the whole line, and what remains at the end, place on the left hand side of the cross. In the same manner throw out the 9s in the Multiplier, and set the result to the right of the cross. Next multiply these two figures together, and after having * cast the 9s out of the product, place what remains above the cross. Lastly, cast the 9s out of the Product, and place the digit, that remains, at the foot of the cross, which if the same, as that at the head of it, 'tis very probable your work is right, especially if the example to be proved has been carefully wrought."

X

Illustration in the last example.

Beginning with 4 in the Multiplicand, I say 4 and 8 make 12; 9 out of 12 leaves 3,—3 and 1 (passing the 9 as it must be thrown away)

multiplying these 9s into the whole multiplicand, the product must be all 9s. Next (to finish the multiplication) the multiplicand must be multiplied by the odds in the multiplier: But the 9s in the multiplicand thus multiplied will be 9s still (by 2); consequently there will be no odds at all resulting from the whole operation, but what come from the odds in the multiplier multiplied into the odds in the multiplicand. And as this little portion is all that is wanted to make the complete product (by 3) if the 9s be struck out of it, the remainder will be equal to what remains, when the 9s are struck out of the whole.---Hence, as this very thing is what we try in the above rule, it is shewn to be right.

As this method of casting out 9s may be applied to all simple arithmetical operations, we shall here just point out its application to the two already past, i. e. Addition and Subtraction, not doubting but the reason will readily appear from what is said above.

1. For Addition. Beginning at any part cast the 9s out of all the digits, to be added, and preserve the result; do the same with the sum; and if they be both alike, 'tis probable you are right; if not, certainly wrong---except indeed you have made a mistake in the tryal.

2 For Subtraction. Expunge the 9s out of the subtrahend, subtractor, and remainder: Then subtract the excess figure of the subtractor from that of the subtrahend (adding 9 to the latter if needful) and the remainder should be equal to the excess figure of the difference sought, if the operation be right.

† The certainty is destroyed, by reason that the same result will come from the 9s, from any transposition of the same digits, as is evident; hence it may appear to be right when it is not so. But as there must be two or more mistakes in an operation of such a kind in this case, as to balance each other, the great chance of its not falling out so, renders the method, in a careful hand, very probable. However as the tryal is very easy, and always answers, when 'tis right, the method is no despisable piece of knowledge in an accomptant.---As soon as division is learned, the student will there find another method of proof by its means.

* The 9s are easily cast out of the product by writing it down, and casting the 9s out of the digits (if there be two) as in other numbers.

away) make 4, and 7 makes 11; 9 out of 11 leaves 2,—2 and 5 make 7,—this 7 being what remains from the whole line, I place to the left of the cross. Next I go over the Multiplier, saying 7 and 6 make 13; 9 out of 13 leaves 4,—4 and 5 make 9, which rejected, the next figure 4 (being the last) $\overset{1}{X}^4$ is what remains from the whole line, which I put to the right of the cross. Next I say 7 times 4 make 28, out of which expunging the 9s, there remains 1, which I place at the head of the cross. Lastly I go over the Product, saying, 8 and 8 make 16; 9 from 16 leaves 7,—7 and 2 make 9,—7 and 6 make 13; 9 from 13 leaves 4,—4 and 7 make 11; 9 from 11 leaves 2,—2 and 6 make 8, and 6 makes 14; 9 from 14 leaves 5,—5 and 2 make 7, and 1 makes 8, and 4 makes 12; 9 from 12 leaves 3,—3 and 5 make 8, and 2 makes 10; 9 from 10 leaves 1, which (1) placed at the foot of the cross, and agreeing with the figure at the top, renders the truth of the whole greatly to be depended on.—For farther practice the learner may inspect the proofs adjoining to the examples already gone past.

It now remains that we add a few “*practical questions*,” for the farther exercise of the learner in this article of multiplication.

CLASS I.

1. Multiply 78094 by 7563? - - - - Answer - - 590624922.
2. Multiply 257827 by 19725? - - - - Answer - - 5085637575.
3. Multiply 378460873 by 73090? - - Answer 27661705207570.
4. Multiply 35234 by 52424? - - - - Answer - - 1847107216.

CLASS II.

1. Multiply 57498 by 60008? - - - - Answer - - 3450339984.
2. Multiply 127614 by 300402? - - - - Answer - 38335500828.
3. Multiply 106432 by 11605? - - - - Answer - - 1235143360.
4. Multiply 102030405 by 504030201?
Answer 51426405540261405.

CLASS III.

1. Multiply 134653170 by 281976453?
Answer 37969023261806010.
2. Multiply 2708630425 by 206008604?
Answer 558001172606176700.
3. Multiply 987654321 by 123456789?
Answer 121932631112635269.
4. Multiply 981237645 by 546732189?
Answer 536474205580054905.

In multiplying by more than one digit, there are often methods in particular cases, which abridge the work (called * "*Contractions*"); and before we enter upon Multiplication of money, it will not be amiss to lay down a few of the most useful, for the perusal of the curious reader as follows.

C A S E 1.

"*Tis plain from what has been already taught, that the product of any number by 10, 100, 1000, &c. is gained by annexing the cyphers, in the multiplier, to the end of the multiplicand.*"

C A S E 2.

"*To multiply by any digit with an unit on the left hand (as 12, 13, &c.) so as to bring the whole operation into one line.*"

† Rule.

"*Multiply each figure in the multiplicand by the digit in the multiplier, adding to each figure in the product, after the first is set down, a figure in the multiplicand beginning with the first.*"

Examples.

$$\begin{array}{r} 6729004 \\ 19 \\ \hline 127851076 \end{array}$$

$$\begin{array}{r} 12345 \\ 12 \\ \hline 148140 \end{array}$$

C A S E

* Perhaps it would hurt the progress of many learners to engage with these contractions the first or second time going over, except the 6th. which I would recommend to every ones consideration.

† The reason is easy to see, by observing what is added in the work at large, from which method most of these contractions have been discovered.

$$\left. \begin{array}{r} 6729004 \\ 19 \\ \hline 60561036 \\ 6729004 \\ \hline 127851076 \end{array} \right\} \text{The work at large.}$$

Table of twelves.

2 times 12 make	24
3 times 12 make	36
4 times 12 make	48
5 times 12 make	60
6 times 12 make	72
7 times 12 make	84
8 times 12 make	96
9 times 12 make	108

When the multiplier consists of a digit with 2, 3, or more units on the left hand, the student may easily discover similar rules to abridge them by, from an examination of the work at large.

As the next example by 12 is a very useful one, it perhaps had best be done by multiplying by 12 all at once, which will be an easy task when once the above table is got by heart.

CASE 3.

"To multiply by any number, in which a cypher is inserted betwixt an unit on the left hand, and any other digit on the right, as 101, 102, 103, &c."

* Rule.

"Multiply by the units digit, and, when you have found two figures in the product, take in each time a figure in the multiplicand, beginning with the first."

Examples.

$$\begin{array}{r} 4321 \\ 106 \\ \hline 458026 \end{array}$$

$$\begin{array}{r} 904150 \\ 109 \\ \hline 65852350 \end{array}$$

CASE 4.

"To multiply by any digit repeated, as 11, 111, 1111, &c. 22, 222, 2222, &c. 33, 333, 3333, &c."

† Rule.

"Multiply by the repeating figure, and out of the product make the total product thus. Begin at the place of units, and first take one figure, then two, then three, &c. (setting down the sums under or over 10s, and carrying the 10s) repeating the operation still from the first place, as often as the number of places in the multiplier; and if there is not another figure to take in at every operation, you must continue to take in the whole multiplicand, so oft, till your number of operations be finished. Then begin at the second place, and third place, &c. successively; taking in from each,

as

* An inspection of the adjoining example, wrought at large, is a sufficient proof. After the same manner may other similar rules be made for any number of cyphers between, and this particular one amongst the rest may be discovered, viz. That "when the number of cyphers is equal to the number of figures in the multiplicand, the product is had from multiplying by the two extreme digits, and setting their products at the end of each other in their proper order; remembering only, if the product of the multiplier in the units place into the last figure of the multiplicand does not make two figures, in that case to put a cypher between the two combined numbers."

$$\begin{array}{r} 4321 \\ 106 \\ \hline 25926 \\ 43210 \\ \hline 458026 \end{array}$$

$$\begin{array}{r} 8739 \\ 444 \\ \hline 34956 \\ 34956 \\ 34956 \\ \hline 3880116 \end{array}$$

† This rule is taken *verbatim* from *Malcolm*, and the reason of it is evident from viewing what is added in the work at large.

as many on the left, as the number of places of the multiplier, as long as you can find as many; and when they fail take in all that remains, till the last figure is taken in alone; (minding always to carry the tens from every operation to the next)."

Examples.

$$\begin{array}{r} 8739 \\ 444 \\ \hline \text{Prod. by 4} - 34956 \\ \hline \text{Answer } 3880116 \end{array}$$

$$\begin{array}{r} 4538797 \\ 7777777 \\ \hline \text{Prod. by 7} - - 31771579 \\ \hline \text{Answer } 35301750914269 \end{array}$$

C A S E 5.

"To multiply by a number consisting of nines, as 99, 999, 9999, &c."

* *Rule.*

"Put down as many cyphers to the right hand of the multiplicand, as there are nines in the multiplier, and from the result subtract the multiplicand, and what remains is the product required."

Examples.

Multiply 2378 by 999? and 37568 by 999999?

$$\begin{array}{r} 2378000 \\ 2378 \text{ Subtracted.} \\ \hline 2375622 \text{ Answer.} \end{array}$$

$$\begin{array}{r} 37568000000 \\ 37568 \text{ Subtracted.} \\ \hline 37568062432 \text{ Answer.} \end{array}$$

C A S E 6.

"If a multiplier is equal to the product of any two digits (as 56 is equal to 8 times 7) its product into any number may be differently found thus."

* *Rule*

* The reason is evident, for by adding the cyphers we multiply by a number, which is just one more (see case 1) than the given multiplier (i. e. in the first example 1000 instead of 999). Hence if out of this product we take the multiplicand, what remains must be the true product.

This method may be easily extended to other cases of this sort, 998, 9997, &c. by multiplying the multiplicand by the next greater number consisting of a digit and cyphers, and then out of the product taking so many times the multiplicand, as the multiplier is short of that number.

Farther, as soon as division is learned, this reasoning will be found to afford an easy method of multiplying by any repeating digit. For suppose the digit always 9, and from that find a product as above; and then by taking a ninth part of this product, and multiplying it by the repeating digit, the true product is acquired.

Also from the above case, and the help of division, we have an easy method of multiplying by 5; for tis but adding a cypher to the end of the multiplicand, and taking half of the result for the product.

* Rule.

"Multiply by one of the digits, and its product by the other, and the last product is the required one of the given factor."

Example. Multiply 3641 by 56?

$$\begin{array}{r}
 3641 \\
 \underline{8} \\
 29128 \\
 \underline{7} \\
 \text{Answer. } 203896
 \end{array}
 \quad \text{Thus, OR thus} \quad
 \begin{array}{r}
 3641 \\
 \underline{7} \\
 25487 \\
 \underline{8} \\
 \text{Answer. } 203896
 \end{array}$$

Scholium.

Several other contractions of the general rule would follow from hence, but as the cases would be almost without end, the hints contained in the subjoined examples (chiefly taken from *Malcolm*) 'tis hoped will be sufficient for the instruction of any student in this place.

Example (1.) 57684
63

$$\begin{array}{r}
 173052 \\
 346104 \\
 \hline
 3634092
 \end{array}$$

(2) 46213
9832

$$\begin{array}{r}
 92426 \\
 138639 \\
 369704 \\
 415917 \\
 \hline
 454366216
 \end{array}$$

(3.) 648
76

$$\begin{array}{r}
 3888 \\
 4536 \\
 \hline
 49248
 \end{array}$$

(4.) 974
862

$$\begin{array}{r}
 1948 \\
 5844 \\
 7792 \\
 \hline
 839588
 \end{array}$$

(5.) 5689
286

$$\begin{array}{r}
 11378 \\
 45512 \\
 34134 \\
 \hline
 1627054
 \end{array}$$

(6.) 4956
168

$$\begin{array}{r}
 39648 \\
 79296 \\
 \hline
 832608
 \end{array}$$

(7.) 5648
459

$$\begin{array}{r}
 50832 \\
 254160 \\
 \hline
 2592432
 \end{array}$$

E

(8.)

* This method is sometimes called *continual multiplication*, and the reason is obvious. For 7 times the product of 8, multiplied into the given number, make 56 times that given number as plainly, as 7 times 8 make 56, which we hope will not need a formal demonstration.

This method is equally true, if the factor was broke into 3 or more numbers (generally

(8.) 724689 545	(9.) 5697487 96488	(10.) 5742135 52575
3623445	45579896	28710675....
32611005	273479376	143553375..
394955505	546958752	430660125
	549739125656	301892747625

The operation of these examples is thus :

In the (1), after multiplying by 3, I double this product for 6. In the (2), after multiplying by 2 and 3, as usual, I quadruple the first line for the 8, and treble the second for the 9. In the (3), after multiplying by 6, I add that product to the multiplicand for the next, or 7 line. In the (4), I multiply first by 2, then this product by 3 (for 6), then add these two lines (the first figure of the one to the first of the other and so on) for 8. In the (5), I multiply by 2, this product by 4 (for the 8) and then subtract the first line from the second (for the 6), or multiply the first line by 3. In the (6), I multiply by 8, and double this product (for 16). In the (7), I multiply by 9, and the product by 5 (because 5 times 9 make 45) placing the results properly i. e. the right-hand figure of every product under the figure (if single), or right-hand figure (if double), it comes from. In the (8), I multiply for the last 5, and this product by 9, for the 45, only placing the results, as you see. In the (9), I first multiply by 8, then as 6 times 8 make 48, I multiply the product of 8 by 6, (placing it properly as you see) for the product by 48. And as twice 48 is 96, I multiply the 48 line by 2.—These added together give the answer. In the (10), as 75 is equal to 3 times 25, and 25 equal to 5 times 5, I multiply in a reverse order, beginning with the left hand 5: This product I multiply by 5, and place as you see (for the part 25 following) which multiplied by 3, and placed properly, is the product of the part 75, consequently their sum is the answer.

“ Multiplication of money.”

This is a very useful rule, and answers all those questions in business where the price of one thing is given to find the price of any number of the same kind: For, tis obvious, the price of one multiplied by the number must needs be the price of them all. This being

(generally called *component parts*) and multiplied continually in any order, as 2, 4, and 7 in the above example; for twice 4 make 8, and 7 times 8 make 56.—Also 4 times 7 make 28, and twice 28 make 56.—Also twice 7 make 14, and 4 times 14 make 56. &c. Farther, if continual multipliers cannot be found to make the exact factor, by coming pretty near it, the excess or defect may be rectified by adding or subtracting, to, or from the result, so many times the multiplicand as they fall short or over.—Instances of the use of this method will be seen in Multiplication of money following.

being premised, it remains only to shew the manner of this multiplication, which is usually divided into two cases, as follows.

CASE 1.

"To multiply by any simple digit, or the number 10."

** Rule.*

"Place the number under the price, and then multiply the farthings, pence, shillings, and pounds, one after another in their order, observing to carry from an inferior denomination to a superior, as in addition of money, and to set down the odds."

The inspection of the following examples will be sufficient Illustration.

1. Multiply £.476 10 8 $\frac{1}{4}$ by 5?

$$\begin{array}{r} 2382 \quad 13 \quad 5\frac{1}{4} \text{ Answer.} \\ \hline \end{array}$$

2. If one stone of goods cost £.4 †13 8 $\frac{1}{4}$, what will 6 stone cost at that rate?

$$\begin{array}{r} 28 \quad 2 \quad 1\frac{1}{2} \text{ Answer.} \\ \hline \end{array}$$

3. If one yard of cloth cost £.— 15 8, what will 8 yards cost?

$$\begin{array}{r} 6 \quad 5 \quad 4 \text{ Answer.} \\ \hline \end{array}$$

4. If 1 C. wt. of sugar cost £.2 3 10 what will 10 cost?

$$\begin{array}{r} 21 \quad 18 \quad 4 \text{ Answer.} \\ \hline \end{array}$$

CASE 2.

"To multiply by any number above 10."

‡ Rule.

"1. If the number is under a hundred, find its Components (i. e. break it into two digits, whose product is equal to it) if possible, as mentioned

E 2

in

* This needs no explanation, after what has been said of the conformity between addition and multiplication in general.

† 6 times 13 may be got by taking the sum of 6 times 3, and 6 times 10, or by setting it down and actually multiplying it, thus -

‡ This is all evident from what is said in note to case 6. of the contractions, and the certainty of the sum of the parts being equal to the whole.

$$\left\{ \begin{array}{r} 13 \\ 6 \\ \hline 78 \\ 4 \text{ carried.} \\ \hline 82 \end{array} \right.$$

in case 6th. of the contractions: If not, take two others whose product is pretty near, as mentioned in the note of the same case. Then multiply by these digits continually (as directed in the above case) and if the product be above or below the truth, finish it by deducting, or adding to it the excess or defect.

2. When the number is above 100 and under 1000 it will be the most convenient, always to get the price of 100 (i. e. by multiplying by 10 and 10) and then from the parts already gained in the operation, the price of any number of hundreds, tens, and units may be found, which are wanting to make up the whole, by multiplying the price of 1, 10, or 100 by the corresponding digits mentioned in the question. Then these parts added give the answer.

3. When 'tis above 1000 (and under 10000) get the price of 1000 (by multiplying by 10, 10, and 10) and then out of these parts get what remains, in a manner similar to that just mentioned."

&c.

The perusal of the following examples will be ample illustration.

1. If 1 pound of tea cost 4s. 5d. what will 24 pound cost?

$$\begin{array}{r} s. \quad d. \\ 4 \quad 5 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 1 \quad 6 \quad 6 \text{ The price of 6.} \\ \hline 4 \end{array}$$

Answer 5 6 — The price of 4 times 6 or 24.

O R thus.

$$\begin{array}{r} s. \quad d. \\ 4 \quad 5 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 1 \quad 15 \quad 4 \text{ The price of 8.} \\ \hline 3 \end{array}$$

Answer 5 6 — The price of 3 times 8 or 24.

2. If 1 ell cost 8 s. 11 d. what will 72 cost?

$$\begin{array}{r} 3 \quad 11 \quad 4 \text{ The price of 8.} \\ \hline 9 \end{array}$$

Answer 32 2 — The price of 9 times 8 or 72.

3. If

3. If 1 stone of goods cost £.1-13-5 what will 34 cost?

$$\text{£. } 1 \quad 13 \quad 5 \times^* 2$$

$$\begin{array}{r} \text{The price of 8} \quad 13 \quad 7 \quad 4 \\ \hline 4 \end{array}$$

$$\begin{array}{r} \text{Added} \quad 53 \quad 9 \quad 4 \quad \text{The price of } 32 \\ \hline 3 \quad 6 \quad 10 \quad \text{The price of } 2 \end{array}$$

$$\begin{array}{r} \text{Answer} \quad 56 \quad 16 \quad 2 \quad \text{The price of } 34 \\ \hline \end{array}$$

OR thus.

$$\text{£. } 1 \quad 13 \quad 5$$

$$\begin{array}{r} \text{The price of 7} \quad 11 \quad 13 \quad 11 \\ \hline 5 \end{array}$$

$$\begin{array}{r} \text{Subtracted} \quad 58 \quad 9 \quad 7 \quad \text{The price of } 35 \\ \hline 1 \quad 13 \quad 5 \quad \text{The price of } 1 \end{array}$$

$$\begin{array}{r} \text{Answer} \quad 56 \quad 16 \quad 2 \quad \text{The price of } 34 \\ \hline \end{array}$$

4. If 1 pound of sugar cost 5d. $\frac{1}{2}$ what will 112 pound cost?

$$\begin{array}{r} d. \\ 5 \frac{1}{2} \times 2 \\ \hline 10 \end{array}$$

$$\begin{array}{r} \text{The price of 10} \quad 4 \quad 7 \\ \hline 10 \end{array}$$

$$\begin{array}{r} \text{Added} \left\{ \begin{array}{l} 2 \quad 5 \quad 10 \quad \text{The price of } 100 \\ 4 \quad 7 \quad \text{The price of } 10 \\ 6 \quad 11 \quad \text{The price of } 2 \end{array} \right. \\ \hline \end{array}$$

$$\begin{array}{r} \text{Answer} \quad 2 \quad 11 \quad 4 \quad \text{The price of } 112 \\ \hline \end{array}$$

5. If

* This mark (X) is the sign of multiplication, implying that the sum or number going before it, is to be multiplied by the number that follows. Marks or signs for all the operations of arithmetic will be explained in due time.

5. If 1 yard of cloth cost 5s. 7d. what will 468 yards cost?

The price of	1			s.	d.
				5	7 × 8
					10

The price of	10				
		2	15	10 × 6	
					10

The price of	100				
		27	18	4	
					4

Added	{	111	13	4	The price of	400
		16	15	—	The price of	60
		2	4	8	The price of	8
Answer		130	13	—	The price of	468

6. If 1 pound of goods cost 9s. 7d. what will 3476 pound cost?

The price of	1			s.	d.
				9	7 × 6
					10

The price of	10				
		4	15	10 × 7	
					10

The price of	100				
		47	18	4 × 4	
					10

The price of	1000				
		479	3	4	
					3

Added	{	1437	10	—	The price of	3000
		191	13	4	The price of	400
		33	10	10	The price of	70
		2	17	6	The price of	6
Answer		1665	11	8	The price of	3476

The best method of proving questions of this kind (if thought material) is by doing them different ways.

Agreeable to our plan, we shall conclude this subject with "a few

Practical Questions."

CLASS I.

- | | | | | | | |
|----|-------------------------------------|---------|---------|----|----|------|
| | | | | £. | s. | d. |
| 1. | 8 C. of sugar at £. 2-4-4 per C. | - - - - | come to | 17 | 14 | 8 |
| 2. | 7 bags of hops at £. 3-17-9 per bag | - - - | come to | 27 | 4 | 3 |
| | | | | 3. | 12 | pair |

	£.	s.	d.
3. 12 pair of stockings at 9s. 8d. per pair - - - come to	5	16	—
4. 11 ounces of silver at 5s. 5d. $\frac{1}{2}$ per ounce - - come to	2	19	9 $\frac{1}{2}$
5. 35 firkins of butter at 15s. 3d. $\frac{1}{2}$ per firkin - come to	26	15	2 $\frac{1}{2}$
6. 42 C. of tallow at 34s. 6d. per C. - - - - - come to	72	9	—

CLASS II.

1. 17 ells of holland at 7s. 8d. $\frac{1}{2}$ per ell - - - come to	6	11	— $\frac{1}{2}$
2. 43 dozen of candles at 6s. 4d. per dozen - come to	13	12	4
3. 97 C. of cheese at £. 1-5-3 per C. - - - - - come to	122	9	3
4. 37 yards of tabby at 9s. 7d. per yard - - - come to	17	14	7
5. 105 gallons of rum at 7s. 6d. per gallon - - come to	39	7	6
6. 735 ells of damask at 4s. 5d. per ell - - - - - come to	162	6	3

CLASS III.

1. 4628 pounds of bohea tea at 8s. 9d. per pound come to	2024	15	—
2. 8954 pounds of fine snuff at 3s. 5d. per pound come to	1529	12	10
3. 3021 C. of currants at £. 2-13-6 per C. - - come to	8081	3	6
4. 1205 ounces of plate at 6s. 9d. per ounce - come to	406	13	9

DIVISION.

Definition.

“**DIVISION** is the finding, how oft one number is contained in another; or the dividing a number (or sum) into any assigned number of parts.

* The preparative to this is an easy consequence of the knowledge of the multiplication table, i. e. the finding, how many times a digit is contained in a given number, not above 81 (the product of the greatest digit 9 by itself) and what remains.—This premised, we have the two following rules for the division of abstract whole numbers.

1 “To divide any abstract whole number by a single digit.”

† Rule.

“Draw a line under the given number (called the Dividend) and place the dividing digit (called the Divisor) on the left hand of it. Then ask,

* If there be occasion, this preparative may be learned from the multiplication table thus. Suppose it was required to know how oft 8 is contained in 59. Look in the table under 8 for the given number. or the next less, as 56, and against it on the right hand side is 7, the answer, and 3 the difference between 56 and 59 is the remainder. Use the same method in other instances.

† The reason of this method will not be hard to conceive on an attentive review of the work in the example which follows. For in the first place, we suppose the dividend 600000, and ask how oft the divisor 4 is contained in it, in round numbers

ask, how oft the divisor is contained in the first figure of the dividend, or (if that be less, than the divisor) in the two first: Place the answer to this properly under the figure or figures it came from. Next, suppose the remainder (if any) wrote before the succeeding figure in the dividend; and ask again how oft the divisor is contained in the number represented by this supposition. Write the answer down, and the remainder (if any) imagine placed, as above directed, before the next figure. With the number thus supposed, proceed in the same manner, as above-mentioned; which process continued, till all the figures in the dividend have been used, gives the answer under the line (called the Quotient), and the overplus (from the last figure) is the † Remainder, by which term 'tis usually denominated. Farther, this remainder wrote above a small line, with the divisor under it, is commonly placed at the end of the quotient, as part of it, and understood to represent as much towards another time going, as the remainder approaches towards the divisor."

An

numbers. The answer to which is 100000 times, and 200000 to spare; to this we add 50000 (the power of the next digit respecting its place) which makes 250000. Then we enquire, how oft 4 is contained in the sum, which (in round numbers) we see is 60000 times, and 10000 to spare. This we add again to the power of the next digit, and proceed in the same manner; which will plainly in time exhaust the whole dividend, and the sum of all these quotients must evidently be the answer required. But that this affixing of cyphers all along is needless will appear from the subjoined work at large, designed to make the praxis still more clear.

$$\begin{array}{r} 4) 653783 \text{ ----- } 100000. \text{ The first quotient.} \\ \underline{400000} \end{array}$$

$$\begin{array}{r} 4) 253783 \text{ Remains. ----- } 60000. \text{ 2d. Do.} \\ \underline{240000} \end{array}$$

$$\begin{array}{r} 4) 13783 \text{ Remains. ----- } 3000. \text{ 3d. Do.} \\ \underline{12000} \end{array}$$

$$\begin{array}{r} 4) 1783 \text{ Remains. ----- } 400. \text{ 4th. Do.} \\ \underline{1600} \end{array}$$

$$\begin{array}{r} 4) 183 \text{ Remains. ----- } 40. \text{ 5th. Do.} \\ \underline{160} \end{array}$$

$$\begin{array}{r} 4) 23 \text{ Remains. ----- } 5. \text{ 6th. Do.} \\ \underline{20} \end{array}$$

$$\begin{array}{r} \underline{\quad\quad\quad} 3 \text{ Remains at the last. } 163445 \text{ The sum of all the quotients or answer.} \end{array}$$

That division is a short method of subtraction will readily occur to the reader from the above process: For if the divisor had been subtracted from the dividend (in order to exhaust it) and also from what remained, till a remainder was found less than it, the number of these operations would have been 163445, the same as the quotient found above.

† When there is no remainder to a division, the quotient is evidently the absolute

An example will make it easy.

Divide 653783 by 4?	Dividend. Divisor 4) 653783 (3 Remainder. Quotient 163445½.
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Illustration.

I first ask how oft 4 in 6, and the answer is once, and 2 over; I therefore put down one, and suppose the two, that remains, before the next figure 5, which will represent 25. Then I ask, how oft 4 in 25, the answer is 6 times and one over: Hence I put the 6 down, and suppose the 1 before the next figure 3, which will make 13: Then I say, how oft 4 in 13, and the answer is 3 times; this I put down also; and in short, proceed in the same easy manner to the last figure, from which I find 3 remains, which I place over the divisor, as directed, at the end of the quotient,

F

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absolute and perfect answer to the question; but where there is a remainder, we must consider the nature of the thing a little farther, in order to ascertain the quotient more intelligibly. And here tis plain, that if the remainder had been (by a fault in the process) exactly equal to the divisor, we should have said rightly that it would go another time. If then the remainder be one fourth of the divisor, will it not be proper to say, it will go one fourth of a time more; if a half, half a time more; three parts out of four, three fourths of a time more? &c. This granted (which is exceeding evident) let the remainder be what it will, it must go universally so much towards another time, as the remainder approaches towards the divisor. Now the best way of denoting these expressions is, as directed, and are understood as taught in the rule, i. e. The lower number (properly called the *Denominator*) shews into how many parts *one time going* is divided, and the higher (also properly called the *Numerator*) shews how many of these parts the example under consideration approaches towards going another time more, than what is already pointed out in the result of the operation. Thus in the question immediately following; 4 being the divisor, and 3 the remainder, we say it goes so many times (as is contained in 163445) and three fourths of a time more. And if a divisor was 365, and the remainder 28, we should read its value twenty eight three hundred and sixty fifths of a time, &c.

Farther; as these expressions $\frac{3}{4}$, $\frac{28}{365}$ and the like (aptly called *Fractions*, or parts of a number) must frequently occur in the course of practice [In all shapes, as divisors, multipliers, &c.] and in many cases, where they must not be neglected (as is frequently done in ordinary business, where they are judged of little value, &c.) 'tis necessary to have an *Algorithm* for their management, and this is what is taught under the article of VULGAR FRACTIONS.---DECIMAL FRACTIONS are for the same end, but only manage them in an easier, and more expeditious manner; tho' not in many cases with the same accuracy.

Two or three more examples for inspection (which may be easily encreased to any number if thought proper).

$$\begin{array}{r} 5) 21487619 \\ \hline 4297523\frac{1}{2} \end{array}$$

$$\begin{array}{r} 6) 314287621 \\ \hline 52381270\frac{1}{6} \end{array}$$

$$\begin{array}{r} 7) 916482317 \\ \hline 130926045\frac{2}{7} \end{array}$$

$$\begin{array}{r} 8) 20604087 \\ \hline 2575510\frac{7}{8} \end{array}$$

$$\begin{array}{r} 9) 26376030 \\ \hline 2930670 \end{array}$$

2. To divide by any number consisting of two or more digits."

* Rule.

"Place the divisor to the left of the dividend, as before; then ask how oft the divisor may be had in the same number of figures, as it contains itself, taken in the dividend, or one more, if wanted: Draw a perpendicular line, at the end of the dividend, and behind it place the answer for the first figure in the quotient. Then multiply this figure into the divisor, and put the product under the already named figures in the dividend. Next draw a line, and subtract this product from the figures above of the dividend; to this remainder bring down the next figure unused of the dividend (making a point under it in order to know how you proceed). Then ask again how oft your divisor may be had in this augmented remainder, and place the answer in the quotient after the other figure, as before. Multiply this quotient figure into the divisor, and place the product under the last remainder; subtract it out of the number above, and to the result bring down the next figure.—Proceed in this manner till the whole dividend is used, and then place the remainder over the divisor at the end of the quotient, as before, for the answer.

The above is strictly the method of dividing here intended, but as in large divisors, it is often something difficult to know how oft they may be had in the numbers of the different steps of the operation; and as the first figure is mostly of a considerably greater value than all the rest, one may give a tolerable guess, by asking how oft it may be had in the first or two first of the dividend: For the answer, made less by one, and sometimes two, is generally the quotient figure wanted. However a tryal or two (which the

* This rule on examination will be found to be the very same, as the last, excepting that by reason of the largeness of the divisors, you are directed to multiply, and subtract manually, in the places where you before could do them mentally. Add to this, that bringing down a figure to the remainder is the same, as placing the remainder before the figure, and the agreement of this rule with the last will appear plainly; consequently the proof of the one is the proof of the other.—Some people here, in order to spare the trouble of writing, subtract the figures of the product as they find them, and only put down the remainders.

the best are forced to make sometimes) will set one right, and this method with a little practice will be found the most convenient.—'Twill be almost needless to observe, that a remainder is never to be as big or bigger than the divisor, for in that case 'twill go a time more in the preceeding operation: And also, that when you have brought down a figure to a remainder, and it happens to be less than the divisor, a cypher must be put in the quotient (as it goes no times) and another figure brought down after it, in order to get the next digit by asking, as before, &c."

An example will make this very clear.

$$365 \overline{) 13534953023} \quad (37082063 \frac{28}{365}$$

1095.....

2584

2555

2995

2920

753

730

2302

2190

1123

1095

28

Illustration.

I begin with asking, how oft 3 (the first figure in the divisor) in 13 (the two first figures in the dividend) and the answer is 4 times. I try 3 (i. e. a time less on a loose piece of paper) and on multiplying it into the divisor (365) I find it will succeed. Next I subtract the product (1095) from (1353) the figures above it in the dividend, and to the remainder (258) bring down the digit 4 (making a point under it, to shew it has been used). This done, I ask, how oft 3 in 25 (the two first figures of the augmented remainder 2584) and the answer is 8 times. I try 7, then multiplying and subtracting, as before, I find it will do, and that there remains 29. This augmented with the next 9 makes 299, which is less than the divisor; therefore it goes no times (or the divisor cannot at all be had in it, as it is) and accordingly I put a cypher in the quotient. This being the case, I bring down, to what I have, the next digit 5, and find, that it will now go 8 times, which I put in the quotient, and from

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the

the last remainder take its product into the divisor, as before, for a new remainder to be augmented with the next figure in the dividend for a new operation.—Thus I proceed through the whole dividend, and find at last the quotient, and remainder to be as in the example.

More examples for inspection.

$$87) 123456789 (1419043 \frac{1}{87}$$

$$\begin{array}{r}
 \underline{364} \\
 87 \overline{) 123456789} \\
 \underline{348} \\
 165 \\
 \underline{87} \\
 786 \\
 \underline{783} \\
 378 \\
 \underline{348} \\
 309 \\
 \underline{261} \\
 48
 \end{array}$$

$$\begin{array}{r}
 1419043 \\
 87 \\
 \hline
 9933301 \\
 11352344 \\
 \hline
 123456789
 \end{array}$$

Proof by multiplication.

$$6 \overset{\circ}{X} 4 \text{ Proof by casting out the 9s.}$$

$$123456789 - - - - - \text{Proof by addition.}$$

$$918) 928765483 (1011727 \frac{27}{918}$$

$$\begin{array}{r}
 1076 \\
 918 \\
 \hline
 1585 \\
 918 \\
 \hline
 6674 \\
 6426 \\
 \hline
 2488 \\
 1836 \\
 \hline
 6523 \\
 6426 \\
 \hline
 97
 \end{array}$$

$$\begin{array}{r}
 1011727 \\
 918 \\
 \hline
 8093823 \\
 1011736 \\
 \hline
 9105543 \\
 928765483
 \end{array}$$

$$7 \overset{\circ}{X} 1$$

2345

$$2345) 345678901 \text{ (} 147411 \frac{106}{2345} \text{)}$$

$$\begin{array}{r} 2345 \\ \hline \end{array}$$

$$\begin{array}{r} 11117 \\ \hline \end{array}$$

$$\begin{array}{r} 9380 \\ \hline \end{array}$$

$$\begin{array}{r} 17378 \\ \hline \end{array}$$

$$\begin{array}{r} 16415 \\ \hline \end{array}$$

$$\begin{array}{r} 9639 \\ \hline \end{array}$$

$$\begin{array}{r} 9380 \\ \hline \end{array}$$

$$\begin{array}{r} 2590 \\ \hline \end{array}$$

$$\begin{array}{r} 2345 \\ \hline \end{array}$$

$$\begin{array}{r} 2451 \\ \hline \end{array}$$

$$\begin{array}{r} 2345 \\ \hline \end{array}$$

$$\begin{array}{r} 106 \\ \hline \end{array}$$

$$\begin{array}{r} 147411 \\ \hline \end{array}$$

$$\begin{array}{r} 2345 \\ \hline \end{array}$$

$$\begin{array}{r} 737061 \\ \hline \end{array}$$

$$\begin{array}{r} 589644 \\ \hline \end{array}$$

$$\begin{array}{r} 442234 \\ \hline \end{array}$$

$$\begin{array}{r} 294822 \\ \hline \end{array}$$

$$\begin{array}{r} 345678901 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ 5 \times 7 \\ 7 \end{array}$$

$$4072495) 3215862471628 \text{ (} 789654 \frac{504898}{28507465} \text{)}$$

$$\begin{array}{r} 36511597 \\ \hline \end{array}$$

$$\begin{array}{r} 32579960 \\ \hline \end{array}$$

$$\begin{array}{r} 39316371 \\ \hline \end{array}$$

$$\begin{array}{r} 36652455 \\ \hline \end{array}$$

$$\begin{array}{r} 26639166 \\ \hline \end{array}$$

$$\begin{array}{r} 24434970 \\ \hline \end{array}$$

$$\begin{array}{r} 22041962 \\ \hline \end{array}$$

$$\begin{array}{r} 20362475 \\ \hline \end{array}$$

$$\begin{array}{r} 16794878 \\ \hline \end{array}$$

$$\begin{array}{r} 16289980 \\ \hline \end{array}$$

$$\begin{array}{r} 504898 \\ \hline \end{array}$$

* " There are three different ways commonly used to prove division. The

* The first of these is very evident: For if the quotient be the number of times the divisor is contained in the dividend, the product of these two with the remainder must be equal to the dividend. The 2d. will not be hard to conceive on reflecting, that the lines directed to be added are the products of the divisor into every quotient figure singly, rightly placed, as to value, &c. Therefore (taking in the remainder) the sum of the parts must be equal to the whole. The 3d. follows easily from what is said above, and is demonstrated in effect. In the proof of multiplication.

The (1.) and * most frequent is, by multiplying the divisor and quotient together, and to their product adding the remainder (usually taken in a figure at a time in the beginning of every line in the process). This sum, if the work be right, will be the same as the dividend. The (2d.) is, by adding together all the products of the quotient digits multiplied into the divisor, in order as they stand in the work, and the remainder; which sum should be also the same, as the dividend. The (3d.) is by casting out the nines thus: Throw the 9s out of the divisor, and quotient, and place the results on different sides of a cross (as in the proof of multiplication). Then cast the 9s out of their product, and carry what leaves to the remainder; out of which having expunged the 9s. place the result at the top of the cross. Lastly, the 9s cast out of the dividend should leave the same digit to be placed at the bottom."

For illustration, we need only refer to the forgoing examples, especially the first, which is proved all the ways; the asterisms (*) shewing the lines to be added in the second method of proof, and the perpendiculars their order.

For the farther exercise of the learner, we shall add "a few"

Practical Questions."

CLASS I.

1. Divide 350465 by 483? - - - - - Quot. $725 \frac{295}{483}$
2. Divide 3211473 by 27? - - - - - Quot. $118943 \frac{12}{27}$
3. Divide 1406373 by 108? - - - - - Quot. $13021 \frac{105}{108}$
4. Divide 4376850 by 96? - - - - - Quot. $45592 \frac{18}{96}$
5. Divide 72683164 by 3710? - - - - - Quot. $19591 \frac{554}{3710}$
6. Divide 47198714 by 6357? - - - - - Quot. $7424 \frac{4346}{6357}$

CLASS II.

1. Divide 6069944827 by 8376? - - - - Quot. $724683 \frac{19}{8376}$
2. Divide 521047321 by 31709? - - - - Quot. $16432 \frac{5033}{31709}$
3. Divide 4637064283 by 57606? - - - Quot. $80496 \frac{11707}{57606}$
4. Divide 293839455936 by 8405? - - - Quot. $34960078 \frac{346}{8405}$

CLASS III.

1. Divide 3846739204 by 483064? - - Quot. $7963 \frac{100872}{483064}$
2. Divide 352107193214 by 210472? - Quot. $1672940 \frac{168534}{210472}$

3. Di-

* Most frequent indeed, while learning in schools (as it exercises the pupil in the very useful rule of multiplication) but to an artist I should think the joint concurrence of the other two (which are run over in an instant) by far the most eligible, except he might prefer a careful review.

Having now gone through the four first rules and their proofs (at least in the theoretical or abstract part) we shall just mention, as a final end to these operations, that as addition, and subtraction, as also multiplication, and division, of consequence mutually undo one another, so they, by very obvious applications, must interchangeably prove one another.

3. Divide 40377982057 by 35016? - - Quot. 1153129 $\frac{16993}{35016}$
 4. Divide 558001172606176724 by 2708630425? Quot. 206008604 $\frac{24}{270}$ &c.

In division, as well as in multiplication, there are several "*Contractions*" made use of; the most material of which are, as follows.

CASE 1.

"When the divisor is one, with cyphers annexed, as 10, 100, 1000, &c. 'tis obvious that if we cut off (by a perpendicular line) so many figures to the right hand of the dividend, as there are cyphers in the divisor, the figures on the left hand of the separation, will be the quotient, and those on the right hand the remainder."

CASE 2.

"To divide by any number with cyphers annexed."

* Rule.

"Cut the cyphers off from the divisor, and the same number of digits from the right hand of your dividend, then divide the remaining figures by each other, as usual, and the quotient is the answer, and what remains wrote before the figures cut off, is the true remainder."

Examples.

$$\begin{array}{r} 7 \overline{) 000} 269 \overline{) 762} \\ \underline{38 \frac{3762}{000}} \text{ Quot.} \end{array}$$

$$\begin{array}{r} 43 \overline{) 00} 637 \overline{) 82} \left(14 \frac{3582}{43000} \text{ Quot.} \right. \\ \underline{43} \\ 207 \\ \underline{172} \\ 35 \end{array}$$

CASE 3.

"An easy method of dividing by any number nearly equal to 10, 100, 1000, &c."

† Rule.

"Suppose the dividend to be divided by the 10, 100, 1000, &c. which, the divisor approaches, by cutting off figures to the right hand, as mentioned

* The reason is not hard to conceive: For the cutting off the same figures to the right hand of each is dividing each of them by 10, 100, 1000, &c. (from case 1st.) and 'tis evident, that as often as the whole divisor is contained in the whole dividend, so often must any part of the divisor be contained in a like part of the dividend.---This method is only to avoid a needless repetition of cyphers, which would happen in the common way, as the reader may see by working an example both ways.

† To demonstrate this method: Having divided by 1000 (in the first example which

tioned case 3rd. Then multiply the figures on the left hand of the separation, by the number the given divisor is short of the assumed one, placing the product in order under the dividend. Next multiply the part of the last product, which falls to the left of the separation (if any) by the same digit, placing its product properly under the other, as before; and if any part of it falls to the left of the separation, that must be multiplied again, &c. in the same manner. This process repeat till no figure falls on the left of the separation. Then add up the right hand side, and carry one for every time the real divisor is contained in the sum, to the next figure on the left hand; the sum of which is the true quotient, and what stands on the other side the true remainder."

Examples.

Divide $425\overline{)648}$ by 997, $\begin{array}{r} 1275 \\ 3 \end{array}$ $426\overline{)997}$ Quot.	$2810\overline{)46}$ by 98, $\begin{array}{r} 5620 \\ 112 \\ 2 \end{array}$ $2867\overline{)98}$ Quot.	and $2375\overline{)634}$ by 999? $\begin{array}{r} 2375 \\ 2 \end{array}$ $2378\overline{)999}$ Quot.
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CASE 4.

"When the divisor is equal to the product of two or more numbers, the operation may be differently performed thus."

* Rule.

"Divide the dividend by any of the numbers (or component parts) and that quotient by another, and so on, and the last quotient is the answer sought. But to find the true remainder (as if you had divided as usual) you must multiply the quotient by the given divisor, and subtract the product

which follows) if 425 was the right quotient, we know that its product multiplied into the divisor 997, and the remainder 648 added, would make the whole dividend. But 425 multiplied into 1000 with the remainder will only make this, consequently, we are 3 times 425, or 1275 (besides the remainder 648) yet short of exhausting the dividend, (as the factor 997 is 3 short of 1000). Therefore this 1275 must be looked on as a new dividend, to be divided by the same divisor (997). Hence cutting off three figures (i. e. placing them as you see) I find the quotient to be one, and the remainder 275. Now the quotient one multiplied into 1000 (with the remainder 275) will be the dividend, but one multiplied into 997 the right divisor, must necessarily be 3 short (i. e. 3 times 1). Therefore if this 3 be wrote under the other remainders, and they added as directed, it will give the true remainder to the question, and the carriage (if any) added to the figures on the other side of the separation (being the quotients at the different operations) must needs be the true quotient.---The same reasoning cannot but hold good in other instances.---It appears from this that when the divisors are all 9's the work is done very readily, as there is no multiplication.

* This follows from case 6th. in multiplication, of which it is only the converse, for

duct from the given dividend, and the result is the remainder.—OR, Multiply the last remainder by the preceding divisor (or last but one) and to the product add the preceding remainder; this sum multiply by the next preceding divisor, and to the product add the next preceding remainder, and so on, till you have gone through all the divisors and remainders to the first."

Examples.

Divide 18473 by 32, and 64865 by 144?

	For true remainder by Rule 1.	By Rule 2d.
(1.) 4) 18473	577	Div. 4
8) 4618 $\frac{1}{4}$	32	Mult. 2 the last rem.
Quot. 577 $\frac{2}{8}$	1154	8
	1731	Add 1 the first rem.
Answer 577 $\frac{9}{32}$	18464	Quot. into Div. 9 True remainder.
	18473	Dividend.
	18464	
	9 True remainder.	

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(2).

for an eighth part of a quarter of any thing is evidently the same as the thirty second part of the whole, &c.---As to the methods for finding the true remainders, the first is obvious. For if the product of any quotient and divisor, added to the remainder, be equal to the dividend; their product, taken from the dividend, must leave the remainder. As to the second, we must observe, that every unit in the first quotient (7207 in the 2d. example) may be looked on as containing 9 of the units in the given dividend (of which aggregates as it were the required remainder is to be) consequently every unit, that remains from it, will contain the same. Hence this remainder must be multiplied by 9 (the divisor) in order to find the units it contains of the given dividend. Again, every unit in the next quotient 1801 will contain four of the preceding ones (or 36 of the first i. e. 9 times 4). Therefore what remains from it must be multiplied by 36 (or which is the same thing 9 and 4 continually) &c. Now, in the rule, instead of finding the values of the remainders separately, they are reduced from the bottom upwards step by step to one another, and the remaining units of the same class taken in as they occur. This will be evident on an attentive examination.

The finding these remainders is indeed something tedious; but in cases, where they need not be much regarded, this method of dividing will be of good use; instances of which immediately follow in division of money.

For true remainder by rule 2d.

$$\begin{array}{r}
 (2.) \quad 9) 64865 \\
 \hline
 4) 7207\frac{2}{5} \\
 \hline
 4) 1801\frac{3}{4} \\
 \hline
 450\frac{5}{4} \\
 \hline
 \end{array}$$

Answer $450\frac{65}{144}$ Mult. $\frac{4}{1}$ the last rem.Add $\frac{4}{3}$ the second rem.Mult. $\frac{7}{9}$ the first div.Add $\frac{63}{2}$ the first rem. $\frac{65}{1}$ True remainder.*"Division of money."*

This is the converse of multiplication of money, and solves those questions in business, where the price of a number of articles is given to find the price of one. For 'tis plain the whole value of any number of things, divided by the number must give the value of one. This granted, we proceed to the method, which is resolved into two cases, as follows:

C A S E 1.

"To divide money by a single digit (or, 10, 11, 12, &c. if the student can manage them as one)."

** Rule.*

"Divide the pounds, shillings, pence and farthings one after another, as usual, placing the quotients under their respective denominations for answers, only observing, that what remains from any denomination must be turned (mentally if you can) into the next inferior, and added to it before you divide. Remainders are commonly placed at the end in curved lines, thus (."

The following examples will illustrate the whole.

1. Divide £. 2382-13-5 $\frac{1}{2}$ by 5?

$$\begin{array}{r}
 5) 2382 \quad 13 \quad 5\frac{1}{2} \quad (\frac{1}{4} \text{ Remainder.} \\
 \hline
 476 \quad 10 \quad 8\frac{1}{4} \text{ Answer.} \\
 \hline
 \end{array}$$

2. If

* The contrary procedure of this rule at every step to multiplication of money sufficiently evinces its truth, if there were no other light of viewing it in.

2. If 6 stone of goods cost £. 28-2-1½ what is the price of one stone?

$$\begin{array}{r} 6 \overline{) 28 \quad 2 \quad 1\frac{1}{2}} \\ \underline{4 \quad 13 \quad 8\frac{1}{4}} \end{array} \text{ Answer.}$$

3. If 8 yards of cloth cost £. 6-5-4 what is that per yard?

$$\begin{array}{r} 8 \overline{) 6 \quad 5 \quad 4} \\ \underline{15 \quad 8} \end{array} \text{ Answer.}$$

4. If 10 C. wt. of sugar cost £. 21-18-4 what will one cost?

$$\begin{array}{r} 10 \overline{) 21 \quad 18 \quad 4} \\ \underline{2 \quad 3 \quad 10} \end{array} \text{ Answer.}$$

CASE 2.

“To divide by any number above a digit, or those used as digits.”

* Rule.

“If you can find the component digits of the divisor, divide (as in the last case) first with one, and then what results with the other (or others) in order, agreeably to what is said in case 1. of the contractions: The last quotient is the answer, and the true remainder (if thought proper) may be discovered in the same manner as there directed.—But if you cannot break it into components, divide with it as it is (by case 2d. of abstract division) one denomination after another, remembering to multiply what remains by the number of parts of the next inferior denomination, that make one of those, you were dividing, and to add (or take in, as the remainders were in the proof of division by multiplication) to the product the parts in the dividend of that denomination.”

On perusing the following examples the whole will appear very plain.

1. Divide £. 434-17-8½ by 24?

$$\begin{array}{r} 6 \overline{) 434 \quad 17 \quad 8\frac{1}{2}} \\ \underline{4 \overline{) 72 \quad 9 \quad 7\frac{1}{4}} \quad (3 \text{ farthings rem.}} \\ \underline{18 \quad 2 \quad 4\frac{3}{4}} \quad (1 \text{ farthing rem.} \end{array}$$

G 2

2. If

* After what has been said in the last case of this, and the case in contractions referred to, the reason of the first part of the rule is evident: And the second part will be found no way different from the first case hereof, excepting that you are directed actually to multiply, &c. instead of doing it in your mind.

2. If 35 C. wt. cost £.47-10 what is that per C?

$$\begin{array}{r}
 7) \quad 47 \quad 10 \\
 \underline{5) \quad 6 \quad 15 \quad 8\frac{1}{2}} \quad (2 \text{ far. rem.} \\
 \text{Answer} \quad \underline{1 \quad 7 \quad 1\frac{1}{2}} \quad (4 \text{ far. rem.}
 \end{array}
 \left\{ \begin{array}{l} \text{Hence (if worth while) the} \\ \text{true remainder is 4 times 7} \\ \text{(or 28) added to 2, i. e. 30} \\ \text{farthings or } 7d. \frac{1}{2}. \end{array} \right.$$

3. If 63 stone cost £.13-16-5 what will 1 cost?

$$\begin{array}{r}
 9) \quad 13 \quad 16 \quad 5 \\
 \underline{7) \quad 1 \quad 10 \quad 8\frac{1}{2}} \quad (2 \text{ far. rem.} \\
 \text{Answer} \quad \underline{4 \quad 4\frac{1}{2}} \quad (4 \text{ far. rem.}
 \end{array}
 \left\{ \begin{array}{l} \text{The true remainder is 4} \\ \text{this 9 added to 2, i. e. 38} \\ \text{farthings or } 9d. \frac{1}{2}. \end{array} \right.$$

OR thus.

$$\begin{array}{r}
 3) \quad 13 \quad 16 \quad 5 \\
 \underline{3) \quad 4 \quad 12 \quad 1\frac{1}{2}} \quad (2 \text{ far. rem.} \\
 \underline{7) \quad 1 \quad 10 \quad 8\frac{1}{2}} \quad (0 \text{ far. rem.} \\
 \text{Answer} \quad \underline{4 \quad 4\frac{1}{2}} \quad (4 \text{ far. rem.}
 \end{array}
 \left\{ \begin{array}{l} \text{The true remainder, thus.} \\ 4 \text{ times 3 make 12. 3 times} \\ 12 \text{ make 36, and 2 makes} \\ 38 \text{ as before.} \end{array} \right.$$

4. If 112 pound of soap cost £.2-13-8 what is that per pound?

$$\begin{array}{r}
 8) \quad 2 \quad 13 \quad 8 \\
 \underline{7) \quad 6 \quad 8\frac{1}{2}} \\
 \underline{2) \quad 11\frac{1}{2}} \\
 \text{Answer} \quad \text{---} \quad \underline{5\frac{3}{4}}
 \end{array}$$

5. If 19 acres of land cost £. 1126-10-6, what is that per acre?

$$\begin{array}{r} 19) 1126 \quad 10 \quad 6 (59l. \\ \underline{95} \\ 176 \\ \underline{171} \end{array}$$

Mult. by $\frac{5}{20}$ and take in 10s.

$$\begin{array}{r} 19) 110 (5s. \\ \underline{95} \end{array}$$

Mult. by $\frac{15}{12}$ and take in 6d.

$$\begin{array}{r} 19) 186 (9d. \\ \underline{171} \end{array}$$

Mult. by $\frac{15}{4}$

$$\begin{array}{r} 19) 60 (3f. \\ \underline{57} \end{array}$$

3 far. rem.

6. Divide £. 145-5-9½ by 57?

$$\begin{array}{r} 57) 145 \quad 5 \quad 9\frac{1}{2} (2l. \\ \underline{114} \\ 31 \\ \underline{20} \end{array}$$

57) 625 (10s.

$$\begin{array}{r} \underline{57} \\ 55 \\ \underline{12} \end{array}$$

57) 669 (11d.

$$\begin{array}{r} \underline{57} \\ 99 \\ \underline{57} \end{array}$$

$$\begin{array}{r} 42 \\ \underline{4} \end{array}$$

$$\begin{array}{r} 57) 171 (\frac{3}{4} \\ \underline{171} \end{array}$$

0 rem.

Hence the answer is, £. 59-5-9½.

Answer £. 2-10-11½.

These questions may be easily proved by multiplication (taking in the remainders properly).

“Practical questions” are all now, that we have to add to this article.

CLASS I.

1. Divide £. 1-16-8 into 5 parts? - - - - - Answer 7s. 4d.

2. What's the $\frac{1}{2}$ of £. 11-13-11? - - - - - Answer £. 1-13-5.

3. Gave £. 37-6-5½ for 10 pieces of cloth, what is that per piece?
Answer £. 3-14-7½ and $\frac{1}{4}$ rem.

4. If 18 hogheads cost £. 168-10-9½ what will 1 cost?
Answer £. 9-7-3, and 3d. $\frac{1}{2}$ rem.

5. If 42 bushel of barley cost £. 15-18-11½ what will 1 cost?
Answer. 7s. 7d. and 5d. $\frac{1}{4}$ rem.

6. If 56 C. of sugar cost £. 121-16 what is that per C.?
Answer £. 2-3-6.

CLASS

CLASS II.

1. Divide £.4-8-1 $\frac{1}{2}$ into 9 parts? - - - - - Answer 9s. 9d. $\frac{1}{2}$.
2. Spent £.257-2-5 in 12 months, what is that per month?
Answer £.21-8-6 $\frac{1}{4}$.
3. Gave £.53-1-8 $\frac{1}{4}$ for 11 C. of hops; what is that per C.?
Answer £.4-16-6, and 2d. $\frac{1}{4}$ to spare.
4. The cloathing of 35 charity-boys comes to £.57-3-7 $\frac{1}{4}$, what is the expence of each?
Answer £.1-12-8, and 3d. $\frac{1}{4}$ rem.
5. A prize of £.7257-3-11 $\frac{1}{2}$ is to be equally divided amongst 500 sailors, what is each man's share? Answer £.14-10-3 $\frac{1}{4}$, and 8s. 6d. $\frac{1}{2}$ rem.
6. If 34 stone of goods cost £.56-16-2, what is that per stone?
Answer £.1-13-5.

CLASS III.

1. If 112 pound of currents cost £.3-10-6, what is that per lb?
Answer 7d. $\frac{1}{2}$, and 6d. rem.
2. A trader cleared £.1150-6-8 equally in 17 years, how much did he lay by in a year?
Answer £.67-13-4.
3. If 43 dozen of candles cost £.13-13-1 $\frac{3}{4}$, what is that per dozen?
Answer 6s. 4d. and 9d. $\frac{3}{4}$ rem.
4. The inlisting of 150 men into his Majesty's service cost £.678-17-6, what is that per man?
Answer. £.4-10-6, and 2s. 6d. rem.

Algorithmical Praxis.

SECTION I.

AS it has been thought * convenient to defer the management of weights, measures, &c. to a place by themselves, the first part of this head is allotted to that purpose. But before we proceed to particulars, it will be very proper to insert tables of the usual division of each species, to have recourse to occasionally, as follows.

I. *Troy Weight*

$$\left\{ \begin{array}{l} 24 \text{ grains (grs) make - - - - - 1 \text{ pennyweight.} \\ 20 \text{ pennyweights (pwt) make - - - 1 \text{ ounce.} \\ 12 \text{ ounces (oz) make - - - - - 1 \text{ pound (lb).} \end{array} \right\}$$

By this are weighed gold, silver, jewels, electuaries, bread, and liquors in general.

II. *Avoir-*

* The reasons for this separate management are these, 1st. because the knowledge of weights and measures is not absolutely necessary to all arithmeticians alike, 2dly, because it is a little tedious and discouraging to beginners, and 3dly, because to add some species, we must be supposed able to subtract or divide pretty large numbers, which is preposterous.

II. *Avoirdupoise weight.*

4 quarters (<i>qrs</i>) make	1 dram.
16 drams (<i>drs</i>) make	1 ounce.
16 ounces (<i>oz</i>) make	1 pound.*
14 pounds (<i>lb</i>) make	1 stone.
28 pounds (<i>lb</i>) make	1 quarter of an C.wt.
4 quarters (<i>qrs</i>) make	1 C. weight.
20 hundred weight (<i>C</i>) make	1 tun (<i>T</i>).

This scale is commonly divided into two kinds, called *Greater* and *Lesser*. The greater contains tons, hundreds, quarters, and pounds; the lesser, stones, pounds, ounces, drams, and quarter-drams. —By the first is weighed any thing of a coarse drossy nature, as all sorts of grocery and chandlers ware, and all sorts of metals, but gold and silver. The second is chiefly used for silk.

III. *Apothecaries weight.*

20 grains (<i>grs</i>) make	1 scruple.
3 scruples (<i>ʒ</i>) make	1 dram.
8 drams (<i>ʒ</i>) make	1 ounce.
12 ounces (<i>ʒ</i>) make	1 pound (<i>lb</i>). †

Apothecaries mix their medicines by this rule, but buy and sell their drugs by avoirdupoise weight,

VI. *Long measure.*

3 barley-cornes (<i>bar</i>) make	1 inch.
12 inches (<i>in</i>) make	1 foot.
3 feet (<i>fe</i>) make	1 yard.
6 feet (<i>fe</i>) make	1 fathom (<i>fa</i>).
5½ yards (<i>yds</i>) make	1 rod, pole, or perch.
40 poles (<i>po</i>) or 220 yards make	1 furlong.
8 furlongs (<i>fur</i>) make	1 mile.
3 miles (<i>mi</i>) make	1 league. (<i>lea</i>).

By this the length or breadth of any thing is taken, and the distance of one thing or place from another.

V. *Wine*

* One pound avoirdupoise is equal to 14oz. 11dwt. 15½grs. Troy.

† The apothecaries pound, and the pound Troy are equal, only differently divided.

V. *Wine measure.*

2 pints (<i>pts</i>) make	1 quart.
4 quarts (<i>qts</i>) make	1 gallon.
10 gallons (<i>gal</i>) make	1 anchor of brandy.
18 gallons (<i>gal</i>) make	1 runlet (<i>run</i>).
31½ gallons (<i>gal</i>) make	1 half-hogshead.
42 gallons (<i>gal</i>) make	1 tierce (<i>tie</i>).
63 gallons (<i>gal</i>) make	1 hogshead.
2 hogheads (<i>hhd</i>) make	1 pipe or but
2 pipes (<i>pi</i>) or 4 <i>hhd</i> s make	1 tun.

All brandies, spirits, perry, cyder, mead, vinegar, honey, and oil, are measured by this measure.

VI. *Ale and Beer measure.*

2 pints (<i>pts</i>) make	1 quart.
4 quarts (<i>qts</i>) make	1 gallon.
8 gallons (<i>gal</i>) make	1 firkin of ale (<i>A fir</i>).
9 gallons (<i>gal</i>) make	1 firkin of beer (<i>B fir</i>).
4 firkins or 2 kilderkins (<i>kil</i>) make	1 barrel.
1½ barrels (<i>bar</i>) or 54 gallons	1 hoghead of beer.
3 barrels or 2 hogheads (<i>hhd</i>)	1 butt.

VII. *Dry measure.*

2 pints (<i>pts</i>) make	1 quart.
2 quarts (<i>qts</i>) make	1 pottle.
2 pottles (<i>pot</i>) make	1 gallon.
2 gallons (<i>gal</i>) make	1 peck.
4 pecks (<i>pk</i>) make	1 bushel.
2 bushels (<i>bu</i>) make	1 strike (<i>str</i>).
4 bushels (<i>bu</i>) make	1 coomb.
2 coombs (<i>coo</i>) or 8 bushels make	1 quarter.
4 quarters (<i>qr</i>) make	1 chaldron (<i>cha</i>).
5 quarters (<i>qr</i>) make	1 wey.
2 weys (<i>wey</i>) make	1 last (<i>last</i>).

VIII. *Time.*

60 seconds (<i>ll</i>) make	1 minute.
60 minutes (<i>l</i>) make	1 hour.
24 hours (<i>h</i>) make	1 day.
7 days (<i>d</i>) make	1 week.
4 weeks (<i>w</i>) make	1 month (<i>m</i>).
12 months 1 day, or 365 days make	1 year (<i>y</i>).

"To add weights, measures, &c."

* Rule.

"Add as usual, remembering to carry one from each denomination to the next superior, for every number in that denomination, which makes one in the superior."

In the perusal of the following examples, the reader will perhaps be something surpris'd, now and then, to find different successions of denominations in one and the same species of weights, measures, &c. But as this is a thing, not fixt at present, as in money, the examples are varied in conformity to what may happen from different customs, &c. in the course of business. And not to give him too much trouble, he will find at the head of each denomination, what he is to carry for, agreeably to the tables.

Examples.

I. Troy weight.

(10)	(20)	(24)
oz.	dwt.	grs.
24	13	10
19	11	15
7	4	14
21	13	16
11	5	23
8	4	12
14	7	10
15	10	11
122	11	15

(10)	(20)	(24)
oz.	dwt.	grs.
105	10	5
217	6	8
360	3	6
195	11	7
217	2	9
196	2	8
321	7	5
172	8	11
1785	11	11

II. Avoirdupoise weight.

(20)	(4)	(28)
℥.	C.	℥rs. lb.
7	17	3 6
5	19	1 12
9	7	1 —
7	7	2 12
2	15	9 20
9	17	3 21
43	5	— 15

(16)	(16)
lb.	oz. dss.
4	10 6
7	14 12
5	7 4
9	14 15
4	5 7
6	11 14
39	— 10

(16)	(4)
lb.	oz. grs.
4	13 1
7	12 2
9	11 3
7	4 2
4	13 1
5	3 2
39	10 3

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III. Apothecary.

* None of these rules will need explanation, if what is already past be understood.

III. Apothecaries weight.

	(12)	(8)	(3)	(20)
lb.	3.	3.	9.	grs.
42	10	7	2	19
17	8	5	2	11
43	7	6	1	10
15	4	5	1	9
40	11	3	1	8
64	9	6	2	15
225	5	4	—	12

	(12)	(8)	(3)	(20)
lb.	3.	3.	9.	grs.
4	10	7	2	19
3	10	6	1	10
2	9	5	—	6
1	8	4	2	4
1	6	3	1	12
1	—	5	—	7
15	11	—	2	18

IV. Long measure.

	(12)	(3)
fe.	in.	bar.
27	9	2
35	10	1
17	2	2
35	11	1
2	6	—
46	7	2
165	11	2

	(220)	(3)	(12)
fur.	yds.	fe.	in.
2	160	1	9
3	27	2	4
6	103	—	11
2	84	1	2
2	120	2	7
3	50	—	5
20	107	—	2

	(3)	(8)	(40)
lea.	mi.	fur.	po.
72	2	1	37
27	1	7	21
35	2	5	14
79	2	6	30
3	—	3	7
19	1	7	25
239	—	—	14

V. Wine measure.

	(4)	(63)	(8)
Tuns	hhd.	gal.	pts.
7	2	19	4
24	1	27	6
12	2	56	4
42	—	37	7
15	3	60	2
102	3	12	7

	(18)	(4)
run.	gal.	qts.
27	17	2
35	15	3
56	14	1
97	10	3
52	15	—
79	2	1
350	3	2

	(42)	(4)
ti.	gal.	qts.
25	36	2
75	41	3
62	15	1
97	13	—
15	14	1
9	8	3
286	3	2

VI. Ale

VI. Ale and Beer measure.

(4) (8)			(4) (9)			(54) (4)		
A.B.	fir.	gal.	B.B.	fir.	gal.	hhds.	gal.	qts.
25	2	7	22	2	7	37	52	1
18	3	6	27	1	8	20	27	3
4	2	5	37	3	4	9	50	2
19	3	7	2	—	5	16	30	—
21	1	5	27	2	3	7	14	2
6	2	4	6	1	4	19	42	—
97	1	2	124	—	4	110	1	—

VII. Dry measure.

(8) (4) (2)				(4) (8) (2)			
qu.	bu.	pk.	gal.	cha.	qr.	bu.	pk.
24	6	2	1	174	3	6	3
17	3	2	—	241	1	7	2
20	7	1	1	296	2	4	3
15	5	—	1	171	1	5	—
19	4	2	—	97	—	5	2
21	2	1	1	471	2	4	3
119	5	2	—	1453	1	5	1

VIII. Time.

(4) (7) (24)				(24) (60) (60)			
m.	w.	d.	h.	d.	h.	'	"
10	3	6	20	37	19	52	17
9	2	3	19	16	23	14	50
12	1	5	23	27	19	37	12
6	3	—	14	64	3	—	54
3	2	6	—	8	—	28	8
15	1	4	22	13	21	7	19
59	—	—	2	168	15	20	40

'Tis not worth while to tire the reader with practical questions.

“ To subtract weights, measures, &c.”

Rule.

“ Subtract similar denominations from one another, as in money, borrowing when there is occasion the number, that you carry one for in the addition of that denomination.”

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Examples.

*Examples.*I. *Troy weight.*

	(12)	(20)	(24)
lb.	oz.	dwt.	grs.
24	9	14	12
19	7	15	20
5	1	18	16

	(20)	(24)
oz.	dwt.	grs.
756	12	15
478	11	22
278	—	17

	(12)	(20)	(24)
lb.	oz.	dwt.	grs.
96	11	15	3
12	3	19	20
84	7	15	7

II. *Avoirdupois weight.*

	(20)	(4)	(28)
Tons	C.	qr.	lb.
700	11	1	17
421	4	2	20
279	6	2	25

	(16)	(16)
lb.	oz.	dr.
46	13	10
27	15	14
18	13	12

	(20)	(4)	(28)
Tons	C.	qr.	lb.
37	19	2	3
6	13	3	25
31	5	2	6

III. *Apothecaries weight.*

	(8)	(3)	(20)
	℥	℥	grs.
3	3	0	
27	1	—	1
15	2	—	8
11	6	2	13

	(8)	(3)	(20)
	℥	℥	grs.
3	3	0	
3	1	2	4
1	—	—	7
2	1	1	17

	(12)	(8)	(3)
℥	℥	℥	grs.
3	3	0	
5	2	1	—
2	5	2	1
2	8	6	2

IV. *Long measure.*

	(3)	(12)	(3)
yds.	ft.	in.	bar.
37	2	1	1
19	2	10	2
17	2	2	2

	(8)	(40)
mi.	fur.	po.
52	1	27
35	7	35
16	1	32

	(3)	(8)	(40)
lea.	mi.	fur.	po.
271	1	7	—
150	—	3	27
121	1	3	13

V. *Wine measure.*

	(18)	(4)
run.	gal.	qts.
72	1	1
35	1	2
36	17	3

	(42)	(4)
tie.	gal.	qts.
25	27	1
19	35	2
5	33	3

	(63)	(4)
hhd.	gal.	qts.
75	57	1
57	59	3
17	60	2

VI. *At*

VI. Ale and Beer measure.

	(4)	(8)
<i>A.B. fir. gal.</i>		
25	1	1
21	1	5
3	3	4

	(4)	(9)
<i>B.B. fir. gal.</i>		
37	2	1
25	1	7
12	—	3

	(54)	(4)
<i>hhd. gal. qts.</i>		
27	27	1
19	50	2
7	30	3

VII. Dry measure.

	(8)	(4)
<i>qr. bu. pk.</i>		
72	1	2
35	2	3
36	6	3

	(32)	(4)
<i>cha. bu. pk.</i>		
79	3	—
54	7	2
24	27	2

	(8)	(4)
<i>qr. bu. pk.</i>		
65	2	1
57	2	3
7	7	2

VIII. Time.

	(4)	(7)	(24)	(60)	(60)
<i>m. w. d. h. / //</i>					
10	2	5	19	50	37
9	3	6	20	7	57
2	5	23	42	40	

	(24)	(60)	(60)
<i>d. h. / //</i>			
24	10	10	20
19	21	50	42
4	12	19	38

“To multiply weights, measures, &c.”

Rule.

“This is similar to multiplication of money, observing to carry properly, and when you cannot multiply the denominations in your mind, &c. to do them with the pen.”

As there is little occasion for this rule, except in Troy and Avoirdupoise weights, an instance or two of each will suffice for the nature of it in general.

I. Troy weight.

	lb.	oz.	pwt.	grs.
1. Multiply	3	4	9	12 by 7?
				7
	23	7	6	12 Answer.

2. What

2. What is the weight of 48 pieces of plate each weighing 8lb. 7oz. 18pwt. 16grs?

lb.	oz.	pwt.	grs.	
8	7	18	16	
			6	
51	11	12	—	
			8	
415	8	16	—	Answer.

II. Avoirdupoise weight.

1. Multiply 6 ^{C. grs. lb.} 2 17 by 8?

53	—	24	Answer.
----	---	----	---------

2. If 1 hoghead weighs 37 C. 3grs 18lb. what will 28 of the same kind weigh?

C.	grs.	lb.	
37	3	18	
		4	
151	2	16	
		7	
1061	2	—	Answer.

"To divide weights, measures, &c."

Rule.

"Divide one denomination after another, as in division of money, turning the remainders into the next inferior denominations, &c."

An example or two will suffice.

1. Divide 23lb. 7oz. 6pwt. 12grs. troy by 7?

lb.	oz.	pwt.	grs.	
23	7	6	12	
3	4	9	12	Answer.

2 If

2. If 48 bars of silver weigh 415 lb. 8 oz. 16 pwt. how much does one weigh?

$$\begin{array}{r}
 \text{lb.} \quad \text{oz.} \quad \text{pwt.} \\
 8) 415 \quad 8 \quad 16 \\
 \hline
 6) 51 \quad 11 \quad 12 \\
 \hline
 \text{lb.} \quad 8 \quad 7 \quad 18 \quad 12 \text{ Answer.} \\
 \hline
 \end{array}$$

3. Divide 27 C. 1 qr. 19 lb, avoirdupoise by 8?

$$\begin{array}{r}
 \text{C.} \quad \text{qrs.} \quad \text{lb.} \\
 8) 27 \quad 1 \quad 19 \\
 \hline
 3 \quad 1 \quad 19\frac{7}{8} \text{ Answer.} \\
 \hline
 \end{array}$$

4. If 42 bags of Spanish wool weigh 110 C. 1 qr. what is that per bag?

$$\begin{array}{r}
 \text{C.} \quad \text{qrs.} \\
 6) 110 \quad 1 \\
 \hline
 7) 18 \quad 1 \quad 14 \\
 \hline
 \text{C.} \quad 2 \quad 2 \quad 14 \text{ Answer.} \\
 \hline
 \end{array}$$

Algorithmical Praxis.

SECTION II.

HAVING in the practical questions, subjoined to the algorithm of whole numbers and money, chiefly pitched on such simple ones, as the rules were directly applicable to; and, as 'tis proper the student should be acquainted (before he proceeds any farther) with cases of a mixt nature, which require two or more rules in the solution; this second part is designed to contain the most material variety of that kind.

And here it will be proper once for all to observe, that in these complicated questions, no determinate rules can be given, the variety being so large; and that the application of the algorithm in the solution intirely depends upon the good sense and judgement of the student. Therefore it is necessary, he should be very perfect in the fundamental operations, and that all his attention should be paid to their special effects, and the intent of the question under consideration.

This

This premised, we shall give a few of the most useful cases at large, not doubting, but when the reader has considered these, he will be able of himself to answer others of the like nature, in the practical part annexed.

"Addition and subtraction apply'd to the balancing of accounts, &c."

Examples for inspection.

	£.	s.	d.
1. A borrowed of B, - - - - -	376	10	8
and A paid again at several times as under			
At one time - - - - -	27	10	6
At another - - - - -	10	12	9
At another - - - - -	56	17	10
At another - - - - -	5	—	—
At another - - - - -	127	6	6
At another - - - - -	32	15	3
	Sum paid	260	2 10
What is still due to B?	Answer	116	7 10

2. A gentleman let an estate at £. 376-10 a year, but was to allow out of it for repairs and assessments; now suppose the repairs be yearly £. 4-17-6, land-tax £. 17-18-5; poor-rates £. 3-14-8, and some other little assessments together £. 2-16-8, and that the farmer has paid them, what is the clear rent due to the landlord?

£.	s.	d.	
376	10	—	
4	17	6	
17	18	5	
3	14	8	
2	16	8	
29	7	3	
347	2	9	Answer,

3. A merchant delivered a bill to a chandler of £.67-10-8, and the chandler draws out his account against him as follows, what is the balance due to the merchant?

	£.	s.	d.
By cheese - - - - -	5	15	—
By best flour - - - - -	—	5	10
By oat meal - - - - -	—	1	—
By rice - - - - -	—	2	11½
By fine butter - - - - -	2	6	8
By soap - - - - -	—	13	4
	9	4	9½

	£.	s.	d.
The merchant's bill	67	10	8
The chandler's bill	9	4	9½

Answer 58 5 10½ Balance due to the merchant,

4. A trader failing was indebted to A, £.171-12-6, to B, £.34-9-9, to C, £.16-8-8, to D, £.44, to E, £.66-7-6, to F, £.11-2-3, to G, £.9-19, and to H, £.8-13-4. At the time of this disaster, he had by him in cash £.3-13-6, in commodities £.23-10, in household furniture £.113-8-6, in plate £.7-18-5, in a tenement £.56-15, in recoverable book-debts £.87-13-10. Supposing these things faithfully surrendered to his creditors, what will they then lose by him?

	£.	s.	d.
To A	171	12	6
B	34	9	9
C	16	8	8
D	44	—	—
E	66	7	6
F	11	2	3
G	9	19	—
H	8	13	4
Debts	362	13	—

	£.	s.	d.
Cash - - - - -	3	13	6
Commodities - - -	23	10	—
Furniture - - - -	113	8	6
Plate - - - - -	7	18	5
Tenement - - - -	56	15	—
Book-debts - - - -	87	13	10
Effects - - - - -	292	19	3

	£.	s.	d.
Debts - -	362	13	—
Effects -	292	19	3
Answer	69	13	9

"Addition and multiplication of money apply'd to casting up of bills, &c."

I. A Mercer's bill.

Bought of GEORGE BAILEY May 17th. 1764.

		s.	d.		£.	s.	d.
9 yards of filk	at	14	6	per yard	6	10	6
12 yards of flowered filk	at	16	8		10	—	—
16 yards of farfenet	at	6	9		5	8	—
10 yards of satin	at	9	6		4	15	—
15 yards of brocade	at	10	8		8	—	—
11 scarves	at	2	—	each	1	2	—
14 yards of Genoa-velvet	at	17	4	per yard	12	2	8
10 yards of lustring	at	5	2		2	11	8
					£.	50	9 10

II. A Grocer's bill.

Bought of THOMAS HARTLEY June 19th. 1764.

		s.	d.		s.	d.
8 lb. of raisins of the sun	- - - - -	at	0	5 per lb.	- - -	3 4
15 lb. of Malaga raisins	- - - - -	at	0	4½	- - -	5 7½
10 lb. of currants	- - - - -	at	0	6½	- - -	5 5
11 lb. of sugar	- - - - -	at	0	4½	- - -	4 1½
2 sugar loaves weight 15 lb.	- - - - -	at	0	9	- - -	11 3
13 lb. of rice	- - - - -	at	0	3	- - -	3 3
5 lb. of black pepper	- - - - -	at	1	6	- - -	7 6
10 oz. of cloves	- - - - -	at	0	10 per oz.	- - -	8 4
£. 2 8 10						

III. A Woollen-draper's bill.

Bought of THOMAS SIMMONDS July 18th. 1764.

	s.	d.
16 yards of drugget - - - - -	at 7	0 per yard
12 yards of broad-cloth - - - - -	at 15	0 - - - -
9 yards of black cloth - - - - -	at 16	5 - - - -
10 yards of shalloon - - - - -	at 1	8 - - - -
15 yards of serge - - - - -	at 1	10 - - - -
7 yards of fine Spanish black - - - - -	at 18	0 - - - -
16 yards of frize - - - - -	at 4	6 - - - -
12 yards of superfine scarlet - - - - -	at 18	0 - - - -

£. 44 17 11

IV. A

IV. *A Linen-draper's bill.*

Bought of JOHN CLAY August 21st. 1764.

	s.	d.
16 ells of dowlas - - - - -	at 2	2 per ell
18 ells of holland - - - - -	at 4	0 - - - -
12 ells of diaper - - - - -	at 1	0 - - - -
12 damask napkins - - - - -	at 2	0 each -
20 yards of printed linen - - - - -	at 2	4 per yard
10 yards of cambric - - - - -	at 12	6 - - - -
10 yards of muslin - - - - -	at 7	3 - - - -
14 yards of canvas - - - - -	at 3	4 - - - -

£. 21 13 6

"In the following bills the odds must be computed by the assistance of division of money."

Mr. CLAUDE COCKSON,

Bought of ROBERT FISHMONGER.

	£.	s.	d.
1764 27 March. 3 hundred of haberdine	at 7	10	6 each
1½ hundred of ling - -	at 8	12	6 - - -
4½ hundred of stockfish	at 4	10	6 - - -
4 kegs of sturgeon - -	at 0	16	10½ - -
6½ barrels of herrings -	at 3	10	2 - - -
95 dried salmon - - -	at 0	1	2 - - -

£. 87 11 11

JAMES BATEMAN Esq;

Bought of CLEMENT COFFEESSELLER.

	s.	d.
1764 10 Feb. 27½ lb. of Smyrna coffee -	at 5	8 per lb.
33 lb. of Mocha Do. - - -	at 5	4 - - -
26½ lb. of Imperial tea - -	at 25	- - - -
10½ lb. of best bohea - - -	at 14	6 - - -
13 lb. of royal green-tea -	at 18	- - - -
21 lb. of double refined sugar	at 1	0½ - - -

£. 70 6 1

"A complicated question or two in abstract numbers."

1. From the creation to the flood were 1656 years; thence to the building of Solomon's temple, 1336 years; thence to Mahomet, who lived 622 years after Christ, 1630 years: In what year of the world was Christ then born?

$$\begin{array}{rcl}
 \text{Add} & \left\{ \begin{array}{l} 1656 \\ 1336 \\ 1630 \end{array} \right. & \\
 \hline
 & 4622 & \text{Year of the world Mahomet lived in.} \\
 \text{Sub.} & 622 & \\
 \hline
 \text{Ans.} & 4000 & \text{Year of the world Christ was born in.} \\
 \hline
 \end{array}$$

2. The building of Solomon's temple was in the year of the world 3000; Troy was, by computation, built 443 years before the temple, and 260 years before London: Now Carthage was built 113 years before Rome, founded 744 years before Christ, born *Anno Mundi* 4000: is London or Carthage the antienter place, and how much?

$$\begin{array}{rcl}
 \text{Sub.} & \begin{array}{r} 3000 \\ 443 \end{array} & \\
 \hline
 & 2557 & \text{Year of the world Troy was built in.} \\
 \text{Add} & 260 & \\
 \hline
 & 2817 & \text{Year of the world London was built in.} \\
 \hline
 & 113 & \\
 \text{Add} & 744 & \\
 \hline
 & 857 & \text{Years Carthage was built before Christ.} \\
 \hline
 \text{Sub.} & \begin{array}{r} 4000 \\ 857 \end{array} & \\
 \hline
 & 3143 & \text{Year of the world Carthage was built in.} \\
 & 2817 & \\
 \hline
 \text{London built} & 326 & \text{years before Carthage, the answer.} \\
 \hline
 \end{array}$$

3. What

3. What number deducted from the 26th. part of 2262 will leave the 87th. of the same?

26) 2262 (87 — 26th part.

208

182

182

Sub.

87

26

61 Answer.

87) 2262 (26 — 87th. part.

174

522

522

4. The remainder of a division question is 423; the quotient 423; the divisor is the sum of both and 19 more; what then was the number to be divided?

423

423

19

865 Divisor.

865

423

2595

1730

3460

365895

423

366318 Answer.

“ *Practical Questions* ” of the same nature.

CLASS I.

1. A was born when B was 18 years of age: How old shall A be when B is 41? and what will be the age of B when A is 72?

Answer A 23. B 90 years.

2. K is 19 years older than L, who was 27 years of age in the *South-sea* year, 1720; How old is M in 1740, who in the year 1738, was within 24 years of being as old as both of them together?

Answer 87 years.

3. Received from my factor at Alicant, on account of sales of tin, to the value of £.197-12 sterling; of bees-wax to £.71-7-6, of stockings, to £.47-3-6, of tobacco, the net proceeds whereof were £.943-15-10, of cotton, to £.123-3-7, and of wheat, to the amount of £.116-5-6. He at the same time advises, that he has sent per order shipped for my account and risk, Alicant wines, to the value of 226-16-6, figs of £.157-11-3, fruit, ninety chests, cost £.104-6, and

and Spanish wool to the value of £.73-13-8, the commission of the whole consignment comes to £.71-18-11. The question is, which of us is to draw for the difference, and how much?

Answer £.865-1-7 is to be remitted by him.

CLASS II.

1. What number divided by 419844, will quote 9494, and leave just a third part of the divisor remaining? Answer 3986138884.

2. There is in 3 purses the sum of £.3312; in the first purse is £.1962-11-3, in the second a fifth of as much; what is in the third?

Answer £.856-18-6.

3. Five notable discoveries were made in 215 years time. viz. 1, the invention of the compass, 2, gunpowder, 3, printing; 4, the discovery of America, 5, truth in the reformation. The last was brought about Anno 1517; the third 77 years before; the second 42 years after the first; and the fourth 148 years after the second. The question is, in what year of Christ did each of these happen to be found?

Answer Compass Anno 1302. Fire-arms 1344. Printing 1440. America 1492.

CLASS III.

1. There are three numbers, the first 228, the 2d. three times as much but for 46, and the 3d. a fourth of as much and 35. Required their sum and continual product.

Answer Sum 958. Product 13382688.

2. My purse and money quoth Dick, are worth 12s. 8d. but the money is worth 7 of the purse: Pray what was then in it?

Answer 11s. 1d.

3. A father left among five sons an estate, consisting of £.500 in cash, with five bills of £.48-10-6 each, he ordered £.20 to be bestowed upon his burial, and his debts to be paid amounting to £.164, then his free estate to be divided in this manner, viz. the eldest son to have the third part, and the other four sons to have equal shares: What is the share of each son?

Answer £.186-4-2, to the eldest, and £.92-2-1 to each of the rest.

R E D U C .

REDUCTION.

Definition.

"THIS is, in general, the bringing money, weights, measures, &c. out of one denomination into another; and it naturally resolves itself into two kinds, called simple, and compound, as follows."

I. Simple Reduction.

Definition.

"This is the turning of money, weights, measures, &c. out of a higher denomination into a lower, or a lower into a higher, according to the scale of succession of denominations in the tables."

* *Rule.*

1. "To bring higher denominations into lower, multiply the highest by the number of the next inferior denomination, which makes one in it, and take in the odds in that inferior (as the remainders were in proving multiplication by division). This product multiply again by the next denomination, &c. till 'tis as low as desired.

2. To bring lower denominations into higher, divide by the number of that denomination, which makes one of the next superior, and that quotient in the same manner by the next superior, &c. till 'tis in the denomination required, remembering to write the remainders after the last quotient in their proper order."

The perusal of an example or two in each of the tables (which are to be consulted occasionally) will be more than sufficient illustration,

I. Money

* The reason of the first part of this has been in effect anticipated in division of money, it being as certain, that there are 20 times as many shillings in any number of pounds, as that there are 20 shillings in one pound, &c. The other part is obvious on a contrary consideration, nor can the procedure upon the whole fail of evidence in any other instance.

I. *Money.*

1. In £.1465-14-5 how many farthings?

$$\begin{array}{r}
 \text{£.1465} \quad 14 \quad 5 \\
 \quad \quad \quad 20 \\
 \hline
 29314 \text{ Shillings,} \\
 \quad \quad \quad 12 \\
 \hline
 351773 \text{ Pence,} \\
 \quad \quad \quad 4 \\
 \hline
 \text{Answer } 1407092 \text{ Farthings,}
 \end{array}$$

2. In 1407092 farthings how many pounds?

$$\begin{array}{r}
 4) 1407092 \\
 \hline
 12) 351773 \\
 \hline
 2|0) 2931|4 \quad 5 \\
 \hline
 \text{£. 1465} \quad 14 \quad 5 \text{ Answer.}
 \end{array}$$

3. In £.25-18-4½ and ¾ (i.e. three eights) of a farthing how many 8ths of a farthing?

$$\begin{array}{r}
 \text{£.25} \quad 18 \quad 4\frac{1}{2} \quad \frac{3}{4} \\
 \quad \quad \quad 20 \\
 \hline
 518 \\
 \quad \quad \quad 12 \\
 \hline
 6220 \\
 \quad \quad \quad 4 \\
 \hline
 24882 \\
 \quad \quad \quad 8 \\
 \hline
 199059 \text{ Answer,}
 \end{array}$$

4. In

* This example proves the former, and in the same manner may any of the practical examples be proved by reverting the enquiry.

4. In 199059 eights of a farthing how many pounds?

$$\begin{array}{r} 8) 199059 \\ \hline 4) 24882 \frac{3}{4} \\ \hline 12) 6220 \frac{1}{2} \\ \hline 2|0) 51|8 \quad 4 \end{array}$$

Answer £. 25 18 4½ and ¾ of a farthing.

Practical Questions.

CLASS I.

1. In 18s. 4d. ½ how many farthings? - - - Answer 882.
2. In 327601 farthings how many pounds? Answer £. 341-5-0¼.
3. In £. 9462-18, how many pence? - - - Answer 2271096.
4. In 26420 pence how many pounds? - - - Answer £. 110-1-8.

CLASS II.

1. In £. 360-10-10¾ how many farthings? Answer 346555.
2. In 2760426 farthings how many pounds? Answer £. 2875-8-10½.

CLASS III.

1. In 16s. 10d. ¼ how many half farthings? Answer 1618.
2. In 126420 quarter farthings how many pounds?
Answer £. 32-18-5¼.

II. *Troy Weight*

1. In 1390lb. 11oz. 18pwt. 19grs. how many grains?

$$\begin{array}{r} \text{lb.} \quad \text{oz.} \quad \text{pwt.} \quad \text{grs.} \\ 1390 \quad 11 \quad 18 \quad 19 \\ \hline 12 \\ \hline 16691 \text{ Ounces.} \\ \hline 20 \\ \hline 333838 \text{ Pennyweights,} \\ \hline 24 \\ \hline 1335361 \\ 667677 \\ \hline 8012131 \text{ Answer.} \end{array}$$

K

2. In

2. In 8012131 grains how many pounds?

$$\begin{array}{r}
 24) 8012131 \text{ (2|0) } 33383|8 \\
 \underline{72} \\
 81 \\
 \underline{72} \\
 9 \\
 \underline{0} \\
 1390 \text{ lb. } 11 \text{ oz. } 18 \text{ pwt. } 19 \text{ grs. Answer.} \\
 \hline
 \text{Ec. rem. } 19 \text{ grs.}
 \end{array}$$

Practical Questions.

CLASS I.

1. In 47lb. 10oz. how many grains? - - - - Answer 275520.
2. In 477472 grains how many pounds?
Answer 82lb. 10oz. 14pwt. 16grs.

CLASS II.

1. In 729lb. 9oz. 11pwt. 12grs. how many grains?
Answer 4203636.
2. In 22253 grains how many pounds?
Answer 3lb. 10oz. 7pwt. 5grs.

CLASS III.

How many grains in 7 ingots of silver, each containing 23lb. 5oz. 7pwt?
Answer 39389.

III. Avoirdupoise weight.

1. In 14T. 12C. 3Qrs. 8lb. how many pounds?

$$\begin{array}{r}
 \text{T. C. Qrs. lb.} \\
 14 \quad 12 \quad 3 \quad 8 \\
 20 \\
 \hline
 292 \text{ Hundreds.} \\
 4 \\
 \hline
 1171 \text{ Quarters.} \\
 28 \\
 \hline
 9376 \\
 2342 \\
 \hline
 32796 \text{ Answer.}
 \end{array}$$

2. In

28) 32796 (4) 1171

$$\begin{array}{r} 28 \quad \underline{\hspace{1cm}} \\ 2 \overline{) 29} \mid 2 \quad 3 \text{ } 2\text{rs.} \\ 47 \\ \text{£c. 8 rem. } 14 \text{ } T. 12 \text{ } C. 3 \text{ } 2\text{rs } 8 \text{ } lb. \text{ Answer.} \end{array}$$

CLASS I.

1. In 72 *T.* 15 *C.* 3 *Qrs* 18 *lb.* how many ounces? Answer 2608992.
2. In 78111 *lb.* how many tons? Answer 34 *T.* 17 *C.* 1 *Qr.* 19 *lb.*

CLASS II.

1. In 1748*T*. 16*C*. 12*r*. 20*lb*. 11*oz*. how many ounces?
Answer 62677771.
2. In 53777630 drams how many tons?
Answer 93*T*. 15*C*. 22*rs*. 12*lb* 13*oz*. 14*drs*.

CLASS III.

In 5 hogsheads each weighing 9 C. 3 Qrs. 11 lb. how many pounds?
Answer 5515.

IV. Apothecaries weight.

1. In 17 lb 93 53 03 18 grs. how many grains?

℥	3	3	3	grs.
17	9	5	—	18
12				

213 Ounces.

8

1709 Drams.

3

5127 Scruples.

20

102558 Answer.

K 2

2. In

2. In 102558 grains how many pounds?

$$\begin{array}{r} 2 \overline{) 102558} \end{array}$$

$$\begin{array}{r} 3 \overline{) 5127} \quad 18 \text{ grs.} \end{array}$$

$$\begin{array}{r} 8 \overline{) 1709} \quad - 9 \end{array}$$

$$\begin{array}{r} 12 \overline{) 213} \quad 53 \end{array}$$

$$\underline{17 \text{ lb } 9 \text{ } \frac{3}{4} \text{ } \frac{53}{64} \text{ } - 9 \text{ } \frac{18}{64} \text{ grs. Answer.}}$$

Practical Questions.

CLASS I.

1. In $1 \text{ } \frac{3}{4} \text{ } \frac{7}{8} \text{ } 2 \text{ } \frac{1}{2}$ how many grains? - - - - - Answer 940.

2. In $37 \text{ } \frac{6}{8}$ scruples how many pounds?
Answer $13 \text{ lb } - 3 \text{ } \frac{6}{8} \text{ } 1 \text{ } \frac{1}{2}$.

CLASS II.

1. In $12 \text{ lb } 1 \text{ } \frac{3}{4} \text{ } 2 \text{ } \frac{3}{4} - 9 \text{ } \frac{1}{2} \text{ gr.}$ how many grains? Answer 69721.

2. In 159022 grains how many pounds?
Answer $27 \text{ lb } 7 \text{ } \frac{3}{4} \text{ } 2 \text{ } \frac{3}{4} \text{ } 1 \text{ } \frac{1}{2} \text{ } 2 \text{ grs.}$

CLASS III.

In a composition weighing $13 \text{ } \frac{3}{4} \text{ } 6 \text{ } \frac{3}{4}$, how many *boluses*, each $33 \text{ } 2 \text{ } \frac{1}{2}$?
Answer 30.

V. Long measure.

1. In 115 yds. 2 ft. 8 in. 1 bar. how many barley-corns?

yd. ft. in. bar.

115 2 8 1

3

347 Feet.

12

4172 Inches

3

12517 Answer.

2. In

2. In 12517 barley-corns how many yards?

$$\begin{array}{r}
 3) 12517 \\
 \hline
 12) 4172 \text{ } 1 \text{ bar.} \\
 \hline
 3) 347 \text{ } 8 \text{ in.} \\
 \hline
 115 \text{ yds. } 2 \text{ ft. } 8 \text{ in. } 1 \text{ bar. Answer.} \\
 \hline
 \end{array}$$

Practical Questions.

CLASS I.

1. In 927 miles how many inches? - - - - - Answer 58734720.
 2. In 57 miles how many poles? - - - - - Answer 18240.

CLASS II.

1. In 50 leagues how many yards? - - - - - Answer 264000.
 2. In 36115200 barley-corns how many miles? Answer 190.

CLASS III.

How many barley-corns will reach round the world, supposing it to be (according to the best calculations) 25020 miles?
 Answer 4755801600.

VI. *Wine measure.*

1. In 2 Tun 3 hds. 17 gal. how many quarts?

$$\begin{array}{r}
 \text{Tun hds. gal.} \\
 2 \quad 3 \quad 17 \\
 4 \\
 \hline
 11 \text{ hogheads.} \\
 63 \\
 \hline
 40 \\
 67 \\
 \hline
 710 \text{ Gallons.} \\
 4 \\
 \hline
 2840 \text{ Answer.} \\
 \hline
 \end{array}$$

2. In

2. In 2840 quarts how many tuns?

$$\begin{array}{r}
 4) 2840 \\
 \hline
 63) 710 (4) 11 \\
 \underline{63} \\
 80 \\
 \underline{63} \\
 17
 \end{array}$$

2 Tun. 3 hhd. 17 gal. Answer.

Practical Questions.

CLASS I.

1. In 12 hogheads of wine how many pints? Answer 6048.
2. In 132240 pints how many tuns? Answer 65 Tun 2 hhd. 24 gal.

CLASS II.

1. In 14 Tun 1 pi. 1 hhd. 10 gal. how many quarts? Answer 14908.
2. In 993031 pints how many tuns?
Answer 492 Tuns 2 hhd. 18 gal. 3 qts. 1 pt.

CLASS III.

Suppose a ship's cargo consists of 96 pipes, 27 hogheads, 120 tierces, and 306 runlets, what is the quantity of the whole in gallons?
Answer 24345.

VII, Ale and Beer measure.

1. In 43 Butts 1 hhd. 52 gal. 3 qts. how many quarts of ale and beer?

$$\begin{array}{r}
 \text{Butts hhd. gal. qts.} \\
 43 \quad 1 \quad 52 \quad 3 \\
 \underline{2} \\
 87 \text{ Hogheads.} \\
 \underline{54} \\
 350 \\
 \underline{440} \\
 4750 \text{ Gallons.} \\
 \underline{4} \\
 19003 \text{ Answer.}
 \end{array}$$

2. In

2. In 19003 quarts how many butts?

$$4) 19003 \text{ (3)}$$

$$54) 4750 \text{ (2) } 87$$

$$\underline{432}$$

43 Butts 1 hhd. 52 gal. 3 qts. Answer.

$$\underline{432}$$

$$\underline{378}$$

$$52$$

Practical Questions.

CLASS I.

1. In 156 barrels of beer how many pints? Answer 44928.

2. In 26482 quarts how many barrels of ale?

Answer 206 bar. 3 fir. 4 gal. 2 qts.

CLASS II.

1. In 38 butts of ale how many gallons? Answer 3648.

2. In 4282 firkins how many butts? Answer 356 butts 2 bar. 2 fir.

CLASS III.

In 17 butts, 86 hogheads, 56 firkins, and 86 gallons of beer,
how many gallons? Answer 7070.

VIII. Dry measure.

1. In 26qr. 7bu. 3pks. how many quarts?

qr. bu. pks.

26 7 3

8

215 Bushels.

4

863 Pecks.

8

6904 Answer.

2. In

2. In 6904 quarts how many quarters?

$$\begin{array}{r} 8) 6904 \\ \hline \end{array}$$

$$\begin{array}{r} 4) 863 \\ \hline \end{array}$$

$$\begin{array}{r} 8) 215 \quad 3 \\ \hline \end{array}$$

26 qr. 7 bu. 3 pks. Answer.

Practical Questions.

CLASS I.

1. In 324 lafts how many bushels - - - - - Answer 25920.

2. In 3642 pecks how many coombs? Answer 227 co. 2 bu. 2 pks.

CLASS II.

1. In 14 lafts 7 qr. 6 bu. 2 gal. how many gallons? Answer 9458.

2. In 1087 bushels how many weys? Answer 27 w. oqr. 7 bu.

CLASS III.

A ship is freighted with 20 lafts, 76 quarters, and 276 bushels;
how many pecks are there in all? Answer 9936.

XI. Time.

1. In 6m. 3w. 5d. 21h. how many hours?

$$\begin{array}{r} m. \quad w. \quad d. \quad h. \\ 6 \quad 3 \quad 5 \quad 21 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \hline \end{array}$$

27 Weeks.

$$\begin{array}{r} 7 \\ \hline \end{array}$$

194 Days.

$$\begin{array}{r} 24 \\ \hline \end{array}$$

$$\begin{array}{r} 777 \\ \hline \end{array}$$

$$\begin{array}{r} 390 \\ \hline \end{array}$$

4677 Answer.

2. In 4677 hours how many months?

$$\begin{array}{r}
 24) 4677 \text{ (7) } 194 \\
 \underline{24} \\
 227 \\
 216 \\
 \underline{} \\
 117 \\
 96 \\
 \underline{} \\
 21
 \end{array}
 \quad
 \begin{array}{r}
 4) 27 5 \\
 \underline{} \\
 6m. 3w. 5d. 21h. \text{ Answer.}
 \end{array}$$

Practical Questions.

CLASS I.

1. In 25 years how many minutes? - - - Answer 13140000.
2. In 63724 hours how many years? - - - Answer 7y. 10od. 4h.

CLASS II.

1. In 5^w. 6d. 11h. how many seconds? - - Answer 3582000.
2. In 157634 minutes how many months?
Answer 39m. 0w. 2d. 16h. 2l.

CLASS III.

How many minutes are there in 50 tropical years, supposing every one to consist of 365d. 5h. 49l? Answer 26297450.

II. *Compound Reduction.

Definition.

"This is the turning money, weights, measures, &c. out of a given denomination into a different one (or other aggregate) that has no reference to it in a scale of succession, as the coins, &c. of different nations."

+ Rule.

"Reduce into some denomination, common to both, the whole given quantity to be changed, and one of those it is to be changed into (if necessary). Then is the quotient of the former divided by the latter the answer."

L

This

* These questions might have been deferred to the Golden Rule, or Exchange (the latter of which is a head particularly allotted to any enquiry of the kind relating to coins), but as they follow from the nature of reduction, &c. and may furnish the learner with useful hints, perhaps there is no fault in running over a few of them here, as usual.

† The reason is obvious from the nature of division, the definition of which is, to tell, how oft one sum or number is contained in another.

This will be very manifest from the perusal of the following

Examples.

1. In 420 quarter guineas how many moidores (of 27s. each)?

$$\begin{array}{r}
 4 \overline{) 420} \\
 \underline{105} \text{ Guineas.} \\
 21 \\
 \underline{105} \\
 210 \\
 \underline{210} \\
 27 \overline{) 2205} \text{ (81 Answer.} \\
 \underline{216} \\
 45 \\
 \underline{27} \\
 18 \text{ Shillings to spare.}
 \end{array}$$

2. A traveller would change £. 233-16-8 sterling for Venice ducats at 4s. 9d. $\frac{1}{2}$ each: How many ducats must he have?

£. 233 16 8	s. d.
20	4 9 $\frac{1}{2}$
4676	12
12	57
56120	2
2	115
115) 112240 (976 Answer.	
1035	
874	
805	
690	
690	

3. A cashier hath received 759 ducats, at 7*s.* 6*d.* per ducat; and 579 dollars at 4*s.* 8*d.* per dollar; which he would exchange for Flemish marks at 14*s.* 3*d.* per piece: How many ought he to have?

<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: right; padding-right: 10px;">s. d.</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">7 6</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">12</td> <td></td> </tr> <tr> <td colspan="2"> </td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">90</td> <td>Pence in a ducat.</td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">759</td> <td></td> </tr> <tr> <td colspan="2"> </td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">68310</td> <td>Pence in all the ducats.</td> </tr> </table> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: right; padding-right: 10px;">s. d.</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">14 3</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">12</td> <td></td> </tr> <tr> <td colspan="2"> </td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">171</td> <td>Pence in a mark.</td> </tr> </table>	s. d.		7 6		12				90	Pence in a ducat.	759				68310	Pence in all the ducats.	s. d.		14 3		12				171	Pence in a mark.	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: right; padding-right: 10px;">s. d.</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">4 8</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">12</td> <td></td> </tr> <tr> <td colspan="2"> </td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">56</td> <td>Pence in a dollar.</td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">579</td> <td></td> </tr> <tr> <td colspan="2"> </td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">3474</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">2895</td> <td></td> </tr> <tr> <td colspan="2"> </td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">32424</td> <td>Pence in all the dollars.</td> </tr> </table> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: right; padding-right: 10px;">68310</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">32424</td> <td></td> </tr> <tr> <td colspan="2"> </td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">171) 100734</td> <td>(589 Answer.</td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">855</td> <td></td> </tr> <tr> <td colspan="2"> </td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">1523</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">1368</td> <td></td> </tr> <tr> <td colspan="2"> </td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">1554</td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">1539</td> <td></td> </tr> <tr> <td colspan="2"> </td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">15</td> <td>Pence over.</td> </tr> </table>	s. d.		4 8		12				56	Pence in a dollar.	579				3474		2895				32424	Pence in all the dollars.	68310		32424				171) 100734	(589 Answer.	855				1523		1368				1554		1539				15	Pence over.
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4. In 364*lb.* 5*oz.* 15*pwt.* of silver how many bowls may be made, each bowl to weigh 4*lb.* 3*oz.* 6*pwt.*?

<i>lb. oz. pwt.</i>	<i>lb. oz. pwt.</i>
364 5 15	4 3 6
<u>12</u>	<u>12</u>
4373	51
<u>20</u>	<u>20</u>
1026) 87475 (85 the answer.	1026 Pennyweights.
<u>8208</u>	
5395	
<u>5130</u>	
210) 2615 Pennyweights over or 1lb. 10z. 5pwt.	
<u>12) 13 5</u>	
<u>1lb. 10z. 5pwt.</u>	

5. The filk mill at Derby contains 26586 wheels, and 97746 movements, which wind off, or throw 73726 yards of filk every time the great water wheel, which gives motion to all the rest, goes about, which is three times in a minute. The question is, how many yards of filk may be thrown by this machine in the compass of a year, deducting for sundays and great holidays 63 days, provided no part of it stands still, and that 10 hours be reckoned a days work?

365	73726
<u>63</u>	<u>3</u>
302 Days at work:	221178 Yards in a minute.
<u>10</u>	<u>181200</u>
3020 Hours Do	44235600
<u>60</u>	<u>221178</u>
181200 Minutes Do.	1769424
	<u>221178</u>
	<u>40077453600 Answer.</u>

6. A master of a ship bought for his sea use 554*C.* 3*Qrs.* 14*lb.* of beef, and ordered the same to be cut into half pound pieces, pound pieces, pound and half pieces, 2, 3, 4, 5 and 6 pound pieces, and of each an equal number. I desire you'll tell me the number of pieces of each?

	<i>C. Qrs. lb.</i>
$\frac{1}{2}$	554 3 14
1	4
$1\frac{1}{2}$	
2	2219
3	28
4	
5	17756
6	4439
23 <i>lb.</i>	23) 62146 (2702 Answer.
	46
	161 <i>Qrs.</i>

7. In 274 marks (each 13*s.* 4*d.*) and 87 nobles (each 6*s.* 8*d.*) how many pounds?

274
2
548 Nobles in 274 marks.
87
3) 635 Nobles in all.
£. 211 13 4 Answer.

8. In 376 ells Flemish (of 3 quarters each) how many ells English (of 5 quarters each)?

376
3
5) 1128
225 ells 3 <i>qrs.</i> Answer.

Practical Questions.

CLASS I.

1. In 4105 moidores each valued at 26s. 6d. how many dollars at 4s. 9d. per dollar? Answer 22901 $\frac{33}{4}$.
2. How many rix-dollars at 4s. 3d. $\frac{1}{4}$ each, may I receive for £. 1750-15-6 $\frac{1}{2}$ sterling? Answer 8198 $\frac{1}{2}$ $\frac{5}{8}$.
3. In £. 562-18 how many French crowns at 54d. $\frac{1}{8}$ per crown? Answer 2496.
4. Allowing the distance between York and London to be 204 miles, how many times will a coach wheel turn round, whose circumference is 6 yards, in going from one of these places to the other? Answer 59840 times.
5. In 47C. 3Qrs. 17lb. how many parcels each 26 $\frac{1}{2}$ pounds? Answer 202, and 12lb. over.
6. In £. 2391-19-4 $\frac{1}{2}$ how many Jamaica bitts, each 7d. $\frac{1}{2}$? Answer 76543.

CLASS II.

1. A rich nobleman has 5 villages, in every village 3 streets, in every street a dozen houses, in every house 5 rooms, in every room 2 bureaux, in every bureau 12 drawers, in every drawer 4 bags, in every bag 150 guineas; which he is going to exchange for three pound twelve shilling pieces, how many must he receive in all? Answer 3780000.
2. How many synodic months, of 29d. 12h. 45' a piece, are there contained in a tropical year consisting of 365d. 5h. 49' $\frac{1}{3}$? Answer 12, and 10d. 20h. 49' $\frac{1}{3}$ rem.
3. How many shillings, sixpences, 4 pences, 3 pences, 2 pences, pence, half-pence and farthings of each a like number will discharge a debt of £. 335-8-4? Answer 2800.

CLASS III.

1. A traveller would have an equal number of crowns at 5s. 6d. per crown; dollars at 4s. 5d. a piece; how many of each fort may he have for £. 309-8? Answer 624.
2. A gentleman having two old-fashioned tankards of silver, one weighing 1lb. and the other 1lb. 4oz. 3dwt. he has ordered his gold-smith to melt them, and with the metal to make him spoons of 2 ounces, cups of 4 $\frac{1}{2}$ ounces, salts of 1oz. 10dwt. and snuff-boxes of 1oz. 6dwt. and of each an equal number: It is required to find the number of each, allowing 5 pennyweights for waste in melting &c? Answer 3 of each fort.

3. There

3. There are 430218 pounds of gunpowder to be employed in an assault, or battery, by 6 pieces of cannon; the first shooteth 4 pounds, the second 6, the third 8, the fourth 10, the fifth 12, and the sixth 14. I demand how many shots they will make to make an equal number, and what quantity of powder each piece shooteth?

Answer. The number of shots are 7967, and what each piece shooteth as follows.

$$\begin{array}{l} \text{lb.} \\ \left\{ \begin{array}{ll} 1 \text{ piece shooteth} & 31868 \\ 2. \text{ Do.} & - - - - 47802 \\ 3. \text{ Do.} & - - - - 63736 \\ 4. \text{ Do.} & - - - - 79670 \\ 5. \text{ Do.} & - - - - 95604 \\ 6. \text{ Do.} & - - - - 111538 \end{array} \right. \end{array}$$

430218 Proof.

THE GOLDEN RULE of * Proportion,

O R

The Rule of Three Direct.

Definition.

"*THIS rule teaches by having three numbers given to find a fourth proportional, or one, that shall bear the same proportion to the third, that the second does to the first.*"

This is a concise account of it, but it may not be amiss to illustrate its intent, &c. a little farther.—As it is a clear and well known truth, that the quantity of goods bought is in proportion to the money laid out, that the space gone over by an uniform motion is in proportion

* It has been the custom of most writers, as soon as they entered upon this subject, to define what is meant by the term *Proportion* in a strict geometrical sense; but as 'tis a word of very common use, and the idea annexed to it easier to conceive, perhaps, than explain, 'tis thought to the full as judicious to hazard it as it stands, than puzzle the subject with what not a learner in an hundred would be the better for. And this was the more readily resolved upon, as 'tis presumed, that independent of the precise meaning of the word *Proportion*, and its deducible properties, the truth of the rule, as applied to ordinary enquiries, may be made very evident, by attending only to principles and operations, already gone past.

In a scholium at the latter end of Geometrical Progression the reader will meet with the strict idea of proportion here spoken of, and also the rule demonstrated on principles naturally resulting therefrom.

proportion to the time, and that in general the magnitude or quantity of any effect varies constantly in proportion to the varying part of the cause, it must be a very useful and common enquiry, having a concurring cause and effect given in one case, to determine from either of the other in different states, or suppositions, it's correspondent. Now the solution of this question apply'd to particular occurrences, &c. is what is proposed under this rule, and as there will be always three terms in the question given, to find a fourth (as above defined i. e. two conditional, and one, on which the demand lies) the rule is called *The rule of three*.—From it's excellency it is also named the *Golden rule*; and the term *Direct* is added in contradistinction to another branch of proportion (immediately following) called *Inverse*.

But to be a little more familiar, than this explanation (the formality of which in general terms could not be well avoided) we shall illustrate what is meant by an easy example. Suppose 4 yards of cloth cost 30 shillings, and it were required to find the price of 25 yards at that rate: Here 4 yards and 30 shillings are connected as cause and effect, and called the two conditional terms; 25 yards is another stage of the cause (as it were) and on which the demand lies, and it's proportional effect, when discovered, is the fourth term, or answer to the question.—This being premised, we have the following general rule for all questions of the like nature.

* Rule.

"Of the three terms of the question place that, which is of the same name with the required answer, in the middle; that in the first which is connected with it in the supposition, and that in the third, on which the demand lies.—This disposition of the terms is what is called Stating of the question.

O R,

It will be found perhaps a better method of stating, to refer the three terms to a form of words like these, "If (such a cause) give, produce, &c. (such an effect), what will (such other cause) give, produce, &c." placing the numeral terms in the parenthesis, so that it will be sense when read.

This

* We have shewn in multiplication of money, that the price of one multiplied by the quantity is the price of the whole. Also on the contrary in division of money it appears, that the price of the whole divided by the quantity gives the price of one. Now the rule under consideration solves the same questions with this difference and advantage, that it equally includes them both, and in the same manner finds the answer, let the price of any number be given or required. On these principles we will examine the directions given in this rule, and see if they be agreeable to reason. By perusing a question or two it will be found (in all cases of valuing goods, &c.) that the rule agrees with multiplication of money, in directing to multiply the price by the quantity, (i. e. the second and third terms together) and if the second term was the price of one individual, this would evidently be the answer, according to both rules, (as 1, the first number in the stating will not divide), but as it is commonly here the price of many, 'tis plain that on mul-

This done if your first and third number consist of different denominations make them both of the lowest name mentioned in either of them. Reduce likewise your second number into the lowest denomination expressed in it (and lower if you think proper). Next divide the product of the second and third terms so reduced, by the first, and the quotient is the answer, and of the same name you left your second number in, which when required may be easily turned into any other convenient denomination.

Farther it is to be observed, that when the second term is not reduced so low as to make, in that case, what remains in the division too trifling to be regarded, the remainder may be farther valued (as 'tis called) by multiplying it by the next inferior denomination (to that the second term was left in) and dividing the product by the old divisor. Repeat the process with the next denomination and remainder in the same manner, till it be exhausted to the desired degree of exactness, and the quotient or quotients thus gained, placed orderly after the first, is the answer required."

On perusing the following examples the rule will be found easier upon the whole, than it may at first appear, all the difficulty lying in separating the parts of complicated questions, where two or more statings are required; and in applying, where there is occasion, little operations of another kind, preparative to a stating, or after a proportion is wrought, &c. But as there can be no general directions given for the right management of these cases, all that remains after this hint is to collect a few of the most ordinary examples, and to leave the rest to the experience and judgment of the artist; not doubting but he will, after a careful examination of what follows, be able to conquer most difficulties of this nature, that may fall in his way.

M

Examples,

multiplied the second and third terms together, the result will be as much greater than the true answer required, as the price in the second term exceeds the price of one (or as the first term exceeds an unit). Consequently this divided by the number of times it exceeds the answer (i. e. the first term, which is what the rule directs in the next step) will give the true answer in the quotient.---With regard to abstract numbers, the rule must still be the same; for it will not affect them to suppose the conditional terms, goods and money (for the time) or any other thing connected, as cause and effect, &c. This being the case, the operations of the rule are shewn to be rightly adapted to their intent in general.

As to the reason of the method for valuing the remainder, it will readily occur, when 'tis observed, that after the first and third terms are reduced as directed, they may be safely looked upon, as abstract numbers for the time; hence the dividend will be a number of the same name the second term was left in, and consequently its division by the first must be managed in a similar manner to that used in the division of money, weights, &c. which is what the rule directs.

Examples.

1. Suppose 4 yards of cloth cost £.1-10 what is the price of 25 yards?

Stated thus

$$\begin{array}{rcccl} \text{yds.} & & \text{£.} & \text{s.} & \text{yds.} \\ 4 & \text{---} & 1 & 10 & \text{---} & 25 \end{array}$$

OR more commonly with words thus.

If 4 yards cost £.1 10 what will 25 yards cost?

$$\begin{array}{r} 20 \\ \hline 30 \end{array} \qquad \begin{array}{r} 30 \\ \hline 4) 750 \end{array}$$

Answer in shillings 187 2 remains.

$$\begin{array}{r} 12 \\ \hline 4) 24 \\ \hline 6 \text{ pence.} \end{array}$$

2|0) 187 6

£.9 7 6 The usual answer.

2. Suppose £.9-7-6 buy 25 yards what is the worth of 4 yards?

Stated thus.

* If 25 yards cost £.9 7 6 what will 4 cost?

$$\begin{array}{r} 20 \\ \hline 187 \\ 12 \\ \hline 2250 \\ 4 \\ \hline 25) 9000 (12) 360 \text{ Answer in pence.} \\ 75 \\ \hline 150 \\ 150 \\ \hline 0 \end{array}$$

2|0) 3|0

£.1 10 Proper Answer.

3. Bought

* This stating proves the former. Any of the four terms of every proportion may be found from the other three, which makes four cases, and, in this example, the remaining statings, to make up that number, are as follow,

- (3) If £.1-10 buy 4 yards what will £.9-7-6 buy? Answer 25 yards.
 (4) If £.9-7-6 buy 25 yards what will £.1-10 buy? Answer 4 yards.

3. Bought 3C. 2Qrs. 17lb. of fugar at $7d. \frac{1}{2}$ per lb. what does it come to?

Stating.

lb. d. C. Qrs. lb.
If 1 cost $7\frac{1}{2}$ what will 3 2 17 cost?

4
30
14
28
119
29

409 lbs. of the same name with the 1st. term.
30

4) 12270

12) 3067 $\frac{1}{2}$

2) 0 25 5 7

£. 12 15 $7\frac{1}{2}$ Answer.

4. Suppose a merchant buys 10 packs of cloth, each pack containing 12 pieces, and each piece $28\frac{1}{2}$ yards, what does it all come to at 3s. 4d. per yard?

10
12
120 pieces in all.
57 half yards in $28\frac{1}{2}$.
840
600
6840 half yards in all.

} Preparative work.

Stating.

If 2 half yards cost 3 ^{s.} 4 ^{d.} what will 6840 half yards cost?

$$\begin{array}{r} 12 \\ \hline 40 \end{array}$$

$$\begin{array}{r} 40 \\ \hline 2) 273600 \end{array}$$

$$12) 136800 \text{ Answer in pence.}$$

$$2|0) 1140|0$$

$$\underline{\underline{\pounds. 570}} \text{ Answer.}$$

5. A merchant on his journey buys 576 gallons of rum at 6s. 10½d. per gallon ready money, but having only an £. 87-10 bill by him, that he can spare, he borrows the rest of a friend; pray what sum does his friend furnish him with?

Stating.

If 1 gallon cost 6 ^{s.} 10½ ^{d.} what will 576 gallons cost?

$$\begin{array}{r} 12 \\ \hline 82 \\ 4 \\ \hline 330 \end{array}$$

$$\begin{array}{r} 330 \\ \hline 17280 \\ 1728 \\ \hline \end{array}$$

$$4) 190080 \text{ Value in farthings.}$$

$$12) 47520$$

$$2|0) 396|0$$

After work {
$$\begin{array}{r} \pounds. \\ 198 \\ 87 \quad 10 \\ \hline 110 \quad 10 \end{array} \text{ Answer.}$$

$$\underline{\underline{\pounds. 198}} \text{ Value in } \pounds.$$

6. What is the worth of 47lb. of tobacco, when 16½lb. cost 20 shillings?

Stating.

Stating.

If $16\frac{1}{2}lb.$ cost 20s. what will $47lb.$ cost?

$ \begin{array}{r} 32 \\ 12 \\ \hline 33) 384 \text{ (11d.} \\ \underline{33} \\ 54 \\ 33 \\ \underline{33} \\ 21 \\ 4 \\ \hline 33) 84 \text{ (}\frac{1}{2}\text{)} \\ \underline{66} \\ 18 \end{array} $	$ \begin{array}{r} 2 \\ \hline 94 \\ 20 \\ \hline 33) 1880 \text{ (56s or } \pounds.2-16. \\ \underline{165} \\ 230 \\ 198 \\ \hline 32 \text{ remains.} \\ \pounds.2-16-11\frac{1}{2}. \end{array} $
---	--

7. A debtor owing several persons in all $\pounds.1490-5-10$, compounds with and pays them as far as his effects will go, which amount to no more than $\pounds.931-8-7\frac{3}{4}$; how much do the creditors, by this composition receive per pound?

Stating.

If $\pounds.1490 \text{ } 5 \text{ } 10$ become $\pounds.931 \text{ } 8 \text{ } 7\frac{3}{4}$ what will $\pounds.1$ become?

$ \begin{array}{r} 20 \\ \hline 29805 \\ 12 \\ \hline 357670 \\ \hline \hline 3576700 \\ 1788350 \\ \hline 357670) 214602000 \text{ (4) } 600 \\ \underline{2146020} \\ 00 \end{array} $	$ \begin{array}{r} 20 \\ \hline 18628 \\ 12 \\ \hline 223543 \\ 4 \\ \hline 894175 \\ 240 \\ \hline 35767000 \\ 1788350 \\ \hline 12s. 6d. \text{ Answer.} \end{array} $
---	--

8. What

8. What must I give for 6 shares out of 32 of a ship, that is worth £.635-5?

If 32 shares cost £.635 5, what will 6 cost?

$$\begin{array}{r}
 \begin{array}{r}
 6 \\
 12 \\
 \hline
 32) 72 \text{ (2d.} \\
 64 \\
 \hline
 8 \\
 4 \\
 \hline
 32) 32 \text{ (}\frac{1}{4}\text{)}
 \end{array}
 \qquad
 \begin{array}{r}
 \begin{array}{r}
 20 \\
 \hline
 12705 \\
 6 \\
 \hline
 32) 76230 \text{ (2|0) } 238|2 \\
 64 \\
 \hline
 122 \\
 96 \\
 \hline
 263 \\
 256 \\
 \hline
 70 \\
 64 \\
 \hline
 6 \text{ remains.}
 \end{array}
 \end{array}
 \end{array}$$

£.119-2-2 $\frac{1}{4}$ Answer.

9. A merchant sent goods to a certain place, which cost him £.1640-10, which his factor sold there advancing £.24-14 per cent (or for every £.100) and returned the whole proceed in tobacco which cost him £.2-8-6 per C. weight; now I demand how many C. weight the merchant must receive?

1. Stating to find the proceed at the far end.

If £.100 gain £.24-14 what will £.1640 10 gain?

$$\begin{array}{r}
 \begin{array}{r}
 \text{£.} \quad \text{s.} \\
 1640 \quad 10 \\
 405 \quad 4 \\
 \hline
 2045 \quad 14 \text{ The whole proceed.}
 \end{array}
 \qquad
 \begin{array}{r}
 \begin{array}{r}
 20 \\
 \hline
 32810 \\
 494 \\
 \hline
 131240 \\
 295290 \\
 \hline
 131240
 \end{array}
 \end{array}
 \end{array}$$

$$2|000) 16208|140$$

$$2|0) 810|4$$

$$\text{£.405 } 5 \text{ gain.}$$

2. Stating

If £. 2 8 6 buy 1C. what will £. 2045 14 buy?

$$\begin{array}{r} 20 \\ - \\ 48 \\ 12 \\ \hline 582 \end{array}$$

$$\begin{array}{r} 20 \\ \hline 40914 \\ 12 \\ \hline 582) 490968 (843 C. \\ 4656 \end{array}$$

2536
2328

2088
1746

342 remains to value.

4

582) 1368 (22rs.
1164

204
28

1632
408

582) 5712 (9lb.
5238

474

3

$$\begin{array}{r}
 3 \\
 4 \\
 \hline
 12 \\
 63 \\
 \hline
 756 \\
 85 \\
 \hline
 671 \text{ gallons left to make his money on.}
 \end{array}$$

If 671 gallons cost £. 15 1 14 what will 1 cost?

$$\begin{array}{r}
 20 \\
 \hline
 3034 \\
 12 \\
 \hline
 36408 \\
 4
 \end{array}$$

$$\begin{array}{r}
 671 \overline{) 145632} \quad (4) \overline{217} \\
 1342 \\
 \hline
 1143
 \end{array}$$

$$\begin{array}{r}
 12 \overline{) 54 \frac{1}{4}} \\
 \hline
 5.4 \quad 6 \frac{1}{4} \quad \frac{25}{871} \text{ Anf,}
 \end{array}$$

$$\begin{array}{r}
 1143 \\
 671 \\
 \hline
 4722 \\
 4697 \\
 \hline
 25
 \end{array}$$

25

11. If 12 apples are worth 21 pears, and three pears cost a half-penny, what will be the price of fourcore and four apples?

1. If 3 pears cost $\frac{1}{2}d.$ what will 21 cost?

$$\begin{array}{r}
 2 \\
 \hline
 3 \overline{) 42} \\
 \hline
 4 \overline{) 14} \\
 \hline
 3 \frac{1}{2}d. \text{ The price of 21 pears or 12 apples.}
 \end{array}$$

3 $\frac{1}{2}d.$ The price of 21 pears or 12 apples.

Again, If 12 apples cost $3d. \frac{1}{2}$ what will 84 cost?

$$\begin{array}{r}
 4 \\
 \hline
 14 \\
 \hline
 12 \overline{) 1176} \\
 \hline
 4 \overline{) 98} \\
 \hline
 12 \overline{) 24 \frac{1}{2}}
 \end{array}$$

25. $od. \frac{1}{2}$ Answer,

12. A

12. A young hare starts 5 rods before a greyhound, and is not perceived by him, 'till she has been up 34 seconds; she scuds away at the rate of 12 miles an hour, and the dog on view, makes after her at the rate of 20: How long will it be before the dog come up with the hare?

1. If 1 hour run 12 miles what will 34'' run?

$$\begin{array}{r} 60 \\ \hline 60 \\ 60 \\ \hline 3600 \end{array} \quad \begin{array}{r} 1760 \\ \hline 21120 \\ 34 \\ \hline 84480 \\ 63360 \end{array}$$

36|00) 7180|80 $\begin{matrix} \text{yds.} & \text{qrs.} \\ 199 & 2 \end{matrix}$ The hare runs before she is seen.
36 27 2 The yards in 5 poles.

358 227 — What the hare was before the dog
324 when he started.

$$\begin{array}{r} 340 \\ 324 \\ \hline 1680 \end{array} \quad \begin{array}{r} 20 \\ 12 \\ \hline 8 \end{array}$$

8 Miles the dog gains in one hour,
4
36) 6720 (2qrs. nearly.

Hence this 2d. stating.

If 8 miles require 1 hour what will 227 yards require?

$$\begin{array}{r} 1760 \\ \hline 14080 \end{array} \quad \begin{array}{r} 60 \\ \hline 60 \\ 60 \\ \hline 3600 \end{array} \quad \begin{array}{r} 3600 \\ \hline 136200 \\ 681 \\ \hline 14080) 817200 (58\frac{56}{1408} \text{ Answer.} \\ 70400 \\ \hline 113200 \\ 112640 \\ \hline 560 \end{array}$$

N

The

The hints contained in the following "*method of contracting*" some particular cases may be of great use.

* If the two first terms, or first, and third terms in a stating can be divided by any common number without a remainder, then may the quotients be used in the stating instead of the given numbers, as in the following examples.

1. If 6 yards cost 12s. what will 30 yards cost?

$$\begin{array}{r} 30 \\ \hline 6 \overline{) 360} \end{array}$$

60 shill. common way.

OR thus neater by dividing the first and second by 6.

$$\begin{array}{r} 1 \text{ --- } 2s. \text{ --- } 30 \\ \hline 2 \end{array}$$

60 shill. as before.

OR thus by dividing the first and third by 6.

$$\begin{array}{r} 1 \text{ --- } 12s. \text{ --- } 5 \\ \hline 5 \end{array}$$

60 shill. as before.

2. If 12 yards cost £.5-6-8 what will 60 yards cost?

By dividing the first and third by 12 we have it

$$\begin{array}{r} 1 \text{ --- } £.5 \quad 6 \quad 8 \text{ --- } 5 \\ \hline 5 \\ \hline 26 \quad 13 \quad 4 \text{ Answer.} \end{array}$$

Scholium.

The great use and extent of this rule will be more fully shewn in the following articles, which in a general way require its assistance. And though several of them (as Profit and Loss, Commission, &c. Purchasing of Stocks, Barter, Single Fellowship, Exchange, &c.) might have been properly enough illustrated in this place, yet we think

* The reason is obvious, for if the divisor (or 1st. term) be made less, the dividend (or product of the other two) is lessened in the same proportion, consequently the quotient will be the same.

think it better to allow each a particular head, as this method renders them more *striking*, which is a circumstance of the greatest moment in works of this nature. Farther, as this rule can never be made too familiar to the student, the number of examples in each ought not to be objected to, which indeed on any other consideration might very reasonably have been the case.

Practical Questions.

CLASS I.

1. If 17 yards of cloth cost £.19-2-6 what will 35 yards cost at that rate? Answer £.39-7-6.
2. What will 23C. 3Qrs. 14lb. of tobacco come to at $15\frac{1}{2}$ per lb? Answer £.172-13-11.
3. A goldsmith bought a wedge of gold, which weighed 14lb. 3oz. 8pwt. for £.514-4, what did he pay per ounce? Answer £.3.
4. How many oranges may be bought for £.17-7 when 15 oranges are valued at $19\frac{1}{2}d$? Answer 3203 $\frac{1}{3}$.
5. If 5 gallons of brandy cost £.1-6-8 what will 63 gallons, or a hoghead cost at that rate? Answer £.16-16.
6. If a dozen ells of linen are valued at £.3-6, how much will 8 pieces, each piece containing 54 ells, amount to at the same rate? Answer £.118-16.
7. If $\frac{3}{4}$ of a yard of velvet cost 7s. 3d. how many yards will be got for £.13-15-6? Answer 28 $\frac{1}{2}$ yards.
8. If £.45 buy 15C. 3Qrs. 12lb. of mather, what will £.200 buy at that rate? Answer 70C. 1Qr. 25lb.
9. A grocer bought 4 hogheads of sugar, each weighing neat 6C. 2Qrs 14lb, which cost him £.2-8-6 per C. what is the value of the 4 hogheads? Answer £.64-5-3.
10. If 24lb. of raisins cost 6s. 6d. what will 18 fraills cost each weighing 2Qrs. 18lb? Answer £.18-0-9.
11. If 35 yards cost £.39-7-6 how many yards may be bought at that rate for £.19-2-6? Answer 17 yards.
12. If 18 $\frac{3}{4}$ yards of serge cost £.2-5 how many yards of the same may I have for 200 guineas? Answer 1750.
13. A draper bought 8 packs of cloth, each containing 4 parcels, each parcel 10 pieces, and each piece 26 yards, and gave after the rate of £.4-16 for 6 yards. I desire to know what the 8 packs stood him in? Answer £.6656.
14. A factor bought 84 pieces of stuffs which cost him in all £.537-12, at 5s. 4d. per yard: I demand how many yards there were in all, and how many ells English were contained in a piece of the same? Answer 2016 yards and $19\frac{1}{3}$ ells.

15. A draper laid out £. 120 in this manner *viz.* the $\frac{1}{3}$ d. of it in callacoës at 20s. 5d. per piece; $\frac{1}{3}$ d. in cambricks at 32s. 9d. per piece, and the rest in Holland-cloth at 5s. 9d. per ell. I desire to know how much he had of each sort?

Answer $39\frac{45}{113}$, $24\frac{168}{113}$, and $139\frac{2}{113}$ respectively.

16. A factor bought a certain quantity of serge and shalloon which cost him together £. 26-14-10; the quantity of serge he had was 48 yards, for which he gave after the rate of 3s. 4d. per yard, and for every 2 yards of serge, he had 5 of shalloon, I demand how many yards of shalloon he had, and what it stood him in per yard?

Answer 120 yards at 3s. $1\frac{1}{2}$ d. per yard.

17. A merchant sent goods to Spain to the value of £. 763-10, to have returns from thence the $\frac{1}{3}$ in Spanish tobacco at 7s. 9d. the pound, and the rest in wine at £. 14-12 the tun; how much of each of these goods must he receive for satisfaction?

Answer $656\frac{22}{93}$ lb. and $34\frac{252}{93}$ tun.

18. Suppose a footman walks on continually 24 miles a day, and, after he has been gone 6 days, is pursued by a horse-man who rides 36 miles a day; how many days, and how many miles must he ride to overtake the footmen?

Answer 432 miles in 12 days.

CLASS II.

1. Bought a box of superfine tea, which weighed 2Qrs. 7lb. and gave £. 73-12-7 $\frac{1}{2}$ for it, what is 5lb. of it worth?

Answer £. 5-16-10 $\frac{1}{2}$.

2. If an ingot of gold weighing 9lb. 9oz. 12pwt. be worth £. 411-12 what is that per grain?

Answer 7 farthings.

3. Bought a cask of wine for £. 62-8, how many gallons were there in the same when the gallon was valued at 5s. 4d?

Answer 234 gallons.

4. What must I pay for the carriage of 17C. 3Qrs. 11lb. at the rate of 7s. per C. weight?

Answer £. 6-4-11 $\frac{1}{4}$.

5. The rents of a whole parish amount to £. 1750, and a rate is granted of £. 32-16-3, what is that in the pound?

Answer $4\frac{1}{2}$ d. $\frac{6250}{7873}$.

6. If I buy 2C. 1Qr. 7lb. of coffee for £. 64-15, at how much must I sell it out per ounce, to gain £. 21-11-8 upon the whole?

Answer 5 pence.

7. A gentleman sold the lease of a parcel of houses worth £. 165-12 a year, for $12\frac{1}{2}$ years purchase; I demand what the purchase money will amount to?

Answer £. 2070

8. A bankrupt is indebted £. 2980-10, but all his effects amount but to £. 931-8-1 $\frac{1}{2}$, what have his creditors in the pound?

Answer 6s. $3\frac{1}{4}$ d.

9. Bought 7 yards of cloth for 17s. 8d. what must be given for 5 pieces each containing $27\frac{1}{2}$ yards?

Answer £. 17-7-0 $\frac{1}{4}$.

10. A

10. A gentleman's living is worth £.436-14-6 per annum: How much must he spend daily to lay up every four weeks £.5-18-4?

Answer 19s. 8d. $\frac{214}{365}$.

11. What quantity of water must I add to a pipe of mountain wine, value £.33 to reduce the first cost to 4s. 6d. the gallon?

Answer 20 $\frac{2}{3}$ gallons.

12. A merchant at London buys 64 tuns of French wine for £.460, the freight thereof cost £.220, loading and unloading £.10, custom and other charges £.23, and he would gain £.250 by the bargain: A gentleman comes and demands the price of 24 tuns, the question is what he must give?

Answer £.361-2-6.

CLASS III.

1. If I leave Exeter at 10 o'Clock on Tuesday morning for London, and ride at the rate of 2 miles an hour without intermission; and you set out of London for Exeter at 6 the same evening, and ride 3 miles an hour constantly; the question is, whereabouts on the road you and I shall meet, if the distance of the two cities be 130 miles?

Answer 61 $\frac{3}{4}$ miles from Exeter.

2. Shipped for Barbadoes 500 pair of stockings at 3s. 6d. per pair, and 1650 yards of baize at 15d. per yard; And I have received in return 348 gallons of rum at 6s. 8d. per gallon, and 750lb. of indigo at 16d. per lb. what remains due upon my adventure? Answer £.24-12-6.

3. A merchant from London carried money to Turkey and there gained by trade £.45-10 per cent, he spent of his gains 1465 dollars at 4s. 9d. per dollar, and yet had £.1782 clear gains left. I desire you'll tell me how much money he departed from London with?

Answer £.4681-3-7 $\frac{1}{2}$ $\frac{172}{10000}$.

4. If I buy 100 yards of ribbon at 3 yards for a shilling, and 100 ditto at 2 yards for a shilling, and sell it again for 2 shillings the 5 yards, the question is whether I get or lose, and how much?

Answer lose 3s. 4d.

5. The iron work that goes round the cupola of St. Paul's church, being in number 130 pieces, each weighing 8C. 0 $\frac{2}{5}$ s. 15lb. was the work of 7 men for 26 weeks, and each man's wages 20s. per week. Now supposing the iron to be worth 6d. the pound, when it is fixed: I demand what the whole comes to, and likewise what the undertaker gains by the work, allowing that the iron and painting stands him in 5d. per pound?

Answer, comes to £.2960-15 Gains £.311-9-2.

6. A may-pole 50 feet 11 inches long, at a certain time of day, will cast a shadow 98 feet 6 inches long: I would hereby find the breadth of a river, that runs within 20 feet 6 inches of the foot of a steeple 300 feet 8 inches high, which will at the same time throw the extremity of its shadow 30 feet 9 inches beyond the stream?

Answer 530 feet 5 inches nearly.

RECIPROCAL PROPORTION,

O R

The Rule of Three Inverse.

Definition.

"*T*HIS is when there are three terms given to find a fourth in the same proportion to the second, that the first is to the third." But this must be explained.—In the direct rule it was supposed that *more* required *more*, and *less*, *less* (i. e. *more* money requires *more* goods, and *less* money *less* goods, &c.) and the examples were accordingly all of that nature. But there are some relations of things (few indeed in comparison of the rest in ordinary affairs) where the contrary is true, or where *more* requires *less*, and *less*, *more*; as for instance: In the performing of any piece of work, the *more* hands there are employed, the *less* time it will take, and the *less* hands the *more* time; also the *greater* distance you are from any object, the *less* it will appear, and the *less* distance, the *larger*, &c. This being the case, in all questions in the rule of proportion we see 'tis necessary to examine them very carefully, in order to find of what nature they are, i. e. whether *direct* or in *inverse*; and if they be of the latter kind (our present subject) they are answered by the following

* *Rule.*

"*S*tate and reduce the terms as in the direct rule, and then the product of the first and second, divided by the third, quotes the answer."

An example or two will be sufficient illustration.

1. If in 10 days 6 men can do a piece of work, in how many days will 12 men do it?

If 6 men require 10 days what will 12 men require?

$$\begin{array}{r} 6 \\ - \\ 12 \overline{) 60} \\ - \end{array}$$

5 days, the answer.

2. If

* In the first example following, the product of the first and second number (i. e. 6 times 10 or 60) is evidently the days it would take one man in doing; therefore, if this be the case, 12 men will do it in one 12th. of the time (i. e. 5 days).—The exact agreement of this reasoning (which is applicable to any other instance) with the directions of the rule, proves it to be right.

2. If for * 24 shillings I have 1200*lb.* carried 36 miles, how many pounds can I have carried 24 miles for the same money?

If 36 miles carry 1200*lb.* what will 24 carry?

$$\begin{array}{r}
 36 \\
 \hline
 24) 43200 \text{ (1800 Answer.} \\
 \hline
 24 \\
 \hline
 192 \\
 192 \\
 \hline
 00
 \end{array}$$

3. If I lend my friend £.200 for 12 months, how long ought he to lend me £.150 to requite my kindness?

If £.200 be lent 12 months what time must £.150 be lent?

$$\begin{array}{r}
 200 \\
 \hline
 150) 2400 \text{ (16 months the answer.} \\
 \hline
 150 \\
 \hline
 900 \\
 900 \\
 \hline
 \hline
 \end{array}$$

Scholium.

Several authors after this article proceed immediately to the solution of questions in what they call the *double rule of three*, or (as some absurdly name their kinds) the *rule of three, of five, seven, nine, &c. numbers*. But as the questions there solved are often more tedious than instructing to beginners, we have thought it as well to refer their management to another place; nevertheless, if the student understands the nature of the two foregoing rules thoroughly, and would wish to have all that are a-kin together, he may take in here the general rule for all questions of the above-mentioned nature, called *Plural Proportion*, which he will find at the end of the work.

Practical

* In some questions both in this and the direct rule, there is now and then a superfluous term, as 24*s.* in this example, and they are pretty readily discovered, by considering, whether, if they were altered, the question would be affected by it; if not, they have no business in the solution.

Practical Questions.

CLASS I.

1. If 150 pioneers cast a trench in 24 hours, how many must be set on to perform it in 6 hours? Answer 600.

2. Suppose 358 pieces of coin at 14s. 7½d. each exchanged for 436 other pieces, at what rate was one of these other pieces valued? Answer 12 shillings.

3. If when wheat is 3s. 6d. a bushel, the three penny loaf weighs 4lb. 2oz. what ought it to weigh when the wheat is 5 shillings a bushel? Answer 2lb. 11oz.

4. Suppose 800 foldiers were placed in a garison, and their provisions were computed sufficient for 2 months, how many foldiers must depart, that the provisions may serve them 5 months? Answer 320 men.

5. If a man performs a journey in 9 days when the day is 11 hours long; in how many days will he perform the same when the day is 15 hours long? Answer 6⅔ days.

6. If for £.5-5 I have 14C.wt. carried 136 miles how many miles may I have 24C.wt. carried for the same money? Answer 79½.

CLASS II.

1. There is a cistern having a cock, which will empty it in 12 hours: How many cocks of the same capacity must there be to empty it in a quarter of an hour? Answer 48.

2 How many pounds of coffee at 5s. 9d. per pound is equal in value to 426lb. of tea at 13s 4d. per lb? Answer 987⅔ lb.

3. Borrowed of my friend £.64 for 8 months, and he hath occasion another time to borrow of me for 12 months, how much must I lend him to requite his former kindness to me? Answer £.42-13-4.

4. If when the days are 13¾ hours long, a traveller performs his journey in 35½ days, in how many days will he perform the same when the days are 11¼ hours long? Answer 43d. 9½h.

CLASS III.

1. A owes B £.480 to be paid in 5½ months, but at 4 months end he pays him £.280, upon condition that he keeps the remaining £.200 so much beyond the time, as is equal to the favour of paying £.280, before the time. Required how long that is? Answer 2⅙ months.

2. Two girls, Sally, and Jenny, after a couple of hours employ at their spinning wheels happened to have a dispute which had done most? Now unluckily their reels were not both of a length, Sally's being

Shilling, or pound, as it happens to fall amongst them in value; and then is the quantity itself the answer at the supposed price in imaginary pieces of coin, each equal to this sum. Next, turn this answer into the higher denomination of (pence) shillings, or pounds, it happens to approach to by dividing by the number of pieces, the answer is made up of, which make a (penny) shilling, or pound. Lastly, from this answer the true one may easily be deduced, by finding the times, parts of a time, or parts of these parts of a time, that it falls short of the real one, the sum of which turned into £s. (when necessary) is the answer required.

OR, the price may be supposed some even piece of money bigger than the real one, in the same manner (which is sometimes useful) and the sum of such parts, and parts of parts taken of it, as will when added make up the given price, is the answer."

The following examples will illustrate the whole, which we shall divide according to custom into different cases, and accompany each with a particular rule, as drawn from the general one; tho' not for any absolute necessity there is for this distinction, but to give all possible satisfaction in so useful a praxis. Farther, as there may be a great variety of orders of taking the parts, &c. we have used such a method of registering the steps of every example, as cannot fail of making that, and every other particular relating thereto extremely evident and intelligible.

* CASE

Required to find the value of 3273 ells at 10d. $\frac{3}{4}$ per ell?

We will first suppose them sold for 6d. per ell, then is the answer 3273 sixpences, which divided by 2 gives 1636s. 6d. Next we argue thus to get the real answer, that whatever it comes to at 6d. per ell, it will come to half as much at 3 pence; and whatever at 3d. it will come to $\frac{1}{3}$ of that at 1d. and whatever at 1d. half of it at $\frac{1}{2}$ d. and whatever at $\frac{1}{2}$ d. half of that at $\frac{1}{4}$ d. Now as 6d. 3d. 1d. $\frac{1}{2}$ d. and $\frac{1}{4}$ d. make when added 10d. $\frac{3}{4}$, so will the sum of the above parts when taken make the whole answer, in shillings, &c. which are then easily brought into £s.

2) 3273 Sixpences suppose.

2) 1636	6	Answer in shillings at 6 per ell.
3) 818	3	Do. - - - - - at 3
2) 272	9	Do. - - - - - at 1
2) 136	4 $\frac{1}{2}$	Do. - - - - - at $\frac{1}{2}$
68	2 $\frac{1}{4}$	Do. - - - - - at $\frac{1}{4}$

2) 0 293 $\frac{1}{2}$ 0 $\frac{3}{4}$ Do. - - - - - at 10 $\frac{3}{4}$ Full price.

£. 146 12 0 $\frac{3}{4}$ True answer.

OR, if the quantity had been supposed so many shillings; by taking the parts as directed in the second branch of the method, the answer would not only have been the same, but also every figure in the process, supposing the parts taken all alike.—Yet as this second method has a liberty in taking some parts, the other has not (because of the one shilling line) they may in several instances differ very much in appearance and expedition. See the 3d. example under case the 2d.

* CASE I.

"When the price is less than a penny."

Rule.

"In the case of the farthing, this can be only dividing by 4, 12, and 20; in the half-penny 2, 12, and 20, and in the $\frac{3}{4}$ d. suppose the quantity so many half-pence, which bring into pence, and to them add their half for the other farthing, and the sum divided by 12 and 20 is the answer."

Examples.

Required the value of the following quantities, at the prices annexed respectively?

$$4) 1763 \text{ at } \frac{1}{4}d.$$

$$12) 440 \quad \frac{3}{4}$$

$$2|0) 3|6 \quad 8$$

$$\underline{\underline{\text{£.1} \quad 16 \quad 8\frac{3}{4} \text{ Answer.}}}$$

$$2) 3761 \text{ at } \frac{1}{2}d.$$

$$12) 1880 \quad \frac{1}{2}$$

$$2|0) 15|6 \quad 8$$

$$\underline{\underline{\text{£.7} \quad 16 \quad 8\frac{1}{2} \text{ Answer.}}}$$

$$2045 \text{ at } \frac{3}{4}d.$$

$$2) 2045 \text{ half-pence suppose.}$$

$$2) 1022 \quad \frac{1}{2}$$

$$511 \quad \frac{3}{4}$$

$$12) 1533 \quad \frac{3}{4}$$

$$2|0) 12|7 \quad 9$$

$$\underline{\underline{\text{£.6} \quad 7 \quad 9\frac{3}{4} \text{ Answer.}}}$$

CASE 2.

"When the price is under a shilling."

Rule.

"Suppose the quantity sold for so many shillings, and then take such parts of the answer, and parts of these parts, as make up the given price: Divide the sum by 20 and the quotient is the amount required."

O 2

Examples.

* This is a very trifling case, but as we intend to rise gradually to a pound, &c. we give it a place for the sake of order.---Practical examples are quite needless.

Examples.

					If 1 stone cost $2\frac{3}{4}d$. what will 2641 cost?
2d	is of	1s	the	$\frac{1}{8}$	2641 shillings suppose.
$\frac{1}{2}$	is of	2d	the	$\frac{1}{4}$	460 6
$\frac{1}{4}$	is of	$\frac{1}{2}$	the	$\frac{1}{2}$	115 $1\frac{1}{2}$
$2\frac{3}{4}$	Full	price.			57 $6\frac{3}{4}$
			2 0	63 3	$2\frac{1}{4}$
					<u>£.31-13-$2\frac{1}{2}$ Answer.</u>
					If 1 ounce cost $5\frac{1}{2}d$. what will 3062 cost?
4d	is of	1s	the	$\frac{1}{3}$	3062 shillings suppose.
1d	is of	4d	the	$\frac{1}{4}$	1020 8
$\frac{1}{2}$	is of	1d	the	$\frac{1}{2}$	255 2
$5\frac{1}{2}$	Full	price			127 7
			2 0	140 3	5
					<u>£.70-3-5 Answer.</u>
					If 1lb. cost $10\frac{3}{4}d$. what will 3273 cost?
6d	is of	1s	the	$\frac{1}{2}$	3273 shillings suppose.
4d	is of	1s	the	$\frac{1}{3}$	1636 6
$\frac{3}{4}$	is of	6d	the	$\frac{1}{8}$	1091 0
$10\frac{3}{4}$	Full	price			204 $6\frac{3}{4}$
			2 0	293 2	$0\frac{3}{4}$
					<u>£.146-12-$0\frac{3}{4}$ Answer.</u>

Practical Questions.

CLASS I.

		£.	s.	d.
1.	1764 ounces at $1\frac{1}{2}d.$ per ounce - - - -	come to	11	— 6.
2.	325 pounds at $2\frac{1}{4}d.$ per pound - - - -	come to	3	— $11\frac{1}{4}.$
3.	827 yards at $4\frac{1}{2}d.$ per yard - - - -	come to	15	10 $1\frac{1}{2}.$
4.	2700 ounces at $7\frac{1}{4}d.$ per ounce - - - -	come to	81	11 3.
5.	732 pounds at $8\frac{1}{2}d.$ per pound - - - -	come to	25	18 6.
6.	538 yards at $11d.$ per yard - - - -	come to	24	13 2.

CLASS II.

1.	372 ounces at $1\frac{3}{4}d.$ per ounce - - - -	come to	2	14 3.
2.	632 pounds at $3\frac{1}{2}d.$ per pound - - - -	come to	9	4 4.
3.	86 yards at $5\frac{1}{4}d.$ per yard - - - -	come to	1	17 $7\frac{1}{2}.$
4.	163 ounces at $6\frac{1}{2}d.$ per ounce - - - -	come to	4	8 $3\frac{1}{2}.$
5.	98 pounds at $9\frac{1}{4}d.$ per pound - - - -	come to	3	15 $6\frac{1}{2}.$
6.	1720 yards at $11\frac{1}{2}d.$ per yard - - - -	come to	82	8 4.

CLASS III.

1.	327 ounces at $2\frac{1}{2}d.$ per ounce - - - -	come to	3	8 $1\frac{1}{2}.$
2.	67 pounds at $4\frac{1}{4}d.$ per pound - - - -	come to	1	3 $8\frac{3}{4}.$
3.	168 yards at $5\frac{3}{4}d.$ per yard - - - -	come to	4	— 6.
4.	87 ounces at $7\frac{1}{8}d.$ per ounce - - - -	come to	2	11 $7\frac{3}{4}.$
5.	670 pounds at $9\frac{1}{4}d.$ per pound - - - -	come to	25	10 $5\frac{1}{2}.$
6.	163 yards at $10\frac{1}{2}d.$ per yard - - - -	come to	7	2 $7\frac{1}{2}.$

CASE 3.

“When the price is above a shilling and less than two.”

Rule.

“Suppose the quantity sold for so many shillings again, and for the excess the given price is above a shilling take parts as before, then these lines all added together, and divided by 20, make the answer.”

Examples.

Examples.

					If 1 yard cost 1s. $3\frac{1}{2}d$. what will 269 cost?
$3d$	is of	1s	the	$\frac{1}{24}$	269 shillings suppose.
$\frac{1}{2}$	is of	$3d$	the	$\frac{1}{8}$	67 3
					11 $2\frac{1}{2}$
				2 0	34 7 $5\frac{1}{2}$
					<u>£.17-7-$5\frac{1}{2}$</u> Answer.
					If 1 stone of goods cost 1s. $6\frac{1}{4}d$. what will
					374 cost?
$4d$	is of	1s	the	$\frac{1}{24}$	374 shillings suppose.
$2d$	is of	$4d$	the	$\frac{1}{2}$	124 8
$\frac{1}{4}$	is of	$2d$	the	$\frac{1}{8}$	62 4
					7 $9\frac{1}{2}$
				2 0	56 8 $9\frac{1}{2}$
					<u>£.28-8-$9\frac{1}{2}$</u> Answer.
					If 1 pound cost 1s. $10\frac{1}{2}d$. what will 86 cost?
$6d$	is of	1s	the	$\frac{1}{24}$	86 shillings suppose.
$4d$	is of	1s	the	$\frac{1}{3}$	43 8
$\frac{1}{2}$	is of	$4d$	the	$\frac{1}{8}$	28 8
					3 7
				2 0	16 1 3
					<u>£.8-1-3</u> Answer.

Practical Questions.

CLASS I.

	£.	s.	d.
1. 279 pounds at 1s. $1\frac{1}{2}$ d. per pound - - - come to	15	13	$10\frac{1}{2}$.
2. 860 ells at 1s. $5\frac{1}{4}$ d. per ell - - - - - come to	61	16	3.
3. 98 ounces at 1s. $7\frac{1}{2}$ d. per ounce - - - come to	8	1	$3\frac{1}{2}$.
4. 136 yards at 1s. $9\frac{1}{2}$ d. per yard - - - - - come to	12	3	8.

CLASS II.

1. 647 pounds at 1s. $2\frac{1}{2}$ d. per pound - - - come to	38	8	$3\frac{3}{4}$.
2. 987 ells at 1s. $4\frac{3}{4}$ d. per ell - - - - - come to	68	17	$8\frac{1}{4}$.
3. 69 ounces at 1s. $8\frac{1}{2}$ d. per ounce - - - - - come to	5	17	$10\frac{1}{2}$.
4. 570 yards at 1s. $10\frac{1}{4}$ d. per yard - - - - - come to	52	16	$10\frac{1}{2}$.

CLASS III.

1. 279 pounds at 1s. $3\frac{1}{4}$ d. per pound - - - come to	17	11	$7\frac{1}{2}$.
2. 74 ells at 1s. $6\frac{1}{4}$ d. per ell - - - - - come to	5	12	$2\frac{1}{4}$.
3. 190 ounces at 1s. $9\frac{1}{3}$ d. per ounce - - - - - come to	16	17	$9\frac{1}{4}$.
4. 84 yards at 1s. $11\frac{1}{2}$ d. per yard - - - - - come to	8	4	6.

CASE 4.

"When the given price is any even number of shillings under 20."

* Rule.

"Multiply the quantity by half the number of shillings, doubling the first figure of the product for shillings, and the rest esteeming pounds."

Examples.

2643 lb. at 2s. per lb.	872 oz. at 8s. per oz.	276 ft. at 16 per ft.
1	4	8
<u>£. 264-6</u>	<u>£. 348-16</u>	<u>£. 220-16</u>

CASE

* The reason is obvious; for the quantity is still supposed shillings, which if multiplied by the number of shillings and divided by 20 would according to the general method be the answer; but as dividing by 20 halves every figure in the product, except the last, it must amount to the same thing to multiply, &c. as directed.---Farther, it may be observed, that when the price is just 2s. the answer is had by doubling the first figure for shillings and calling the rest pounds, which
easy

CASE 5.

"When the given price is any odd number of shillings under 20."

* Rule.

"Find the answer for the greatest even number of shillings, as in the last case; then taking the given quantity as shillings, add the pounds they contain to the above result for the answer required."

OR, the quantity multiplied by the shillings and divided by 20 gives the answer."

Examples.

$$\begin{array}{r}
 210) 2816 \text{ at } 7s. \text{ per lb.} \\
 \underline{3} \\
 85 \quad 16 \\
 14 \quad 6 \\
 \hline
 \text{£. } 100 \quad 2
 \end{array}$$

$$\begin{array}{r}
 210) 3712 \text{ ells at } 11s. \text{ per ell,} \\
 \underline{5} \\
 186 \quad 0 \\
 18 \quad 12 \\
 \hline
 \text{£. } 204 \quad 12
 \end{array}$$

$$\begin{array}{r}
 210) 2614 \text{ yards at } 19s. \text{ per yard.} \\
 \underline{9} \\
 237 \quad 12 \\
 13 \quad 4 \\
 \hline
 \text{£. } 250 \quad 16
 \end{array}$$

CASE

easy procedure has induced some authors (particularly Saxon) to suppose, in a great variety of cases, the price to be 2s. (as we have hitherto 1s.) and then from the answer thence resulting they deduced in like manner, as in the foregoing cases, the true one required.---The two following examples will make the method easy.

1. Given 167 lb at 10d.	2. Given 278 oz. at 7d.
8d. is of 2s. the $\frac{1}{2}$) 16 14 answer at 2s.	6d. is of 2s. the $\frac{1}{4}$) 27 16 answer at 2s.
$ \begin{array}{r} 2d. \text{ is of } 8d. \text{ the } \frac{1}{4}) \quad 5 \quad 11 \quad 4 \\ \underline{1} \quad 7 \quad 10 \\ \text{£. } 6 \quad 19 \quad 2 \text{ True answer.} \end{array} $	$ \begin{array}{r} 1d. \text{ is of } 6d. \text{ the } \frac{1}{6}) \quad 6 \quad 19 \\ \underline{1} \quad 3 \quad 2 \\ \text{£. } 8 \quad 2 \quad 2 \text{ True ans.} \end{array} $

* The rule is too obvious to need explanation.

These two cases answer only particular instances, which might have been included in that which follows; and from this consideration, and their great ease, 'tis judged unnecessary to lengthen them with practical questions.

CASE 6.

"When the price is under a pound with pence and farthings annexed."

* Rule.

"Multiply the quantity (supposed shillings) by the shillings, and take parts for the pence, &c. the sum of which divided by 20 is the answer.

OR, Suppose the quantity so many pounds, and take such parts of it, as make up the given price; then is the sum of these the answer."

Examples by the first method.

If 1 stone cost 12s. 8½d. what will 1763 cost?				
6d	is of	1s	the	½
1763 shillings suppose.				
12				
2d	is of	6d	the	⅓
½	is of	2d	the	¼
21156				
881 6				
293 10				
73 5½				
2 0 2240 4 9½				
£.1120-4-9½ Answer.				
If 1 yard cost 17s. 9½d. what will 273 cost?				
6d	is of	1s	the	½
273 shillings suppose.				
17				
1911				
273				
3d	is of	6d	the	⅓
½	is of	3d	the	⅕
4641				
136 6				
68 3				
11 4½				
2 0 485 7 1½				
£.242-17-1½ Answer.				

P

Examples.

* Instances in this case, where the shillings and pence are both of the same number,

Examples by the second method.

10s	is of	1l.	the	$\frac{1}{2}$	If 1 C. wt. cost 17s. 9 $\frac{1}{2}$ d. what will 273 pounds suppose. [273 cost?		
5s	is of	10s	the	$\frac{1}{2}$	136	10	
2s 6d	is of	5s	the	$\frac{1}{2}$	68	5	
3d	is of	2s 6d	the	$\frac{1}{10}$	34	2	6
$\frac{1}{2}$	is of	3d	the	$\frac{1}{6}$	3	8	3
					11	4 $\frac{1}{2}$	
					£. 242	17	1 $\frac{1}{2}$ Answer.
10s	is of	1l.	the	$\frac{1}{2}$	If 1 stone cost 13s. 4 $\frac{1}{2}$ d. what will 825 pounds suppose. [825 cost?		
2s 6d	is of	10s	the	$\frac{1}{4}$	412	10	
6d	is of	2s 6d	the	$\frac{1}{4}$	103	2	6
3d	is of	6d	the	$\frac{1}{2}$	20	12	6
1 $\frac{1}{2}$ d	is of	3d	the	$\frac{1}{2}$	10	6	3
					5	3	1 $\frac{1}{2}$
					£. 551	14	4 $\frac{1}{2}$ Answer.

Practical Questions.

CLASS I.

1. 4270 stone at 3s. 4d. per stone - - - comes to 711 13 4
 2. 270 ells at 6s. 8d. per ell - - - come to 90 - -
 3. 27 C.

number, are readilier done, by multiplying by the shillings, and to the product adding $\frac{1}{2}$ th. of itself (as the latter are just $\frac{1}{2}$ th. in value of the former).

See the adjoining example.

146 ells at 9s. 9d.

$$\begin{array}{r}
 9 \\
 12) 1314 \\
 \underline{109} \quad 6 \\
 210) 14213 \quad 6 \\
 \underline{\hspace{1cm}} \\
 \text{£. 71} \quad 3 \quad 6 \text{ Answer.}
 \end{array}$$

3. 27 C. weight at 7s. 6d. per C. - - - comes to $\begin{matrix} \text{£.} & \text{s.} & \text{d.} \\ 10 & 2 & 6. \end{matrix}$
 4. 96 yards at 8s. 4d. per yard - - - - come to $\begin{matrix} 40 & - & - \end{matrix}$.

CLASS II.

1. 376 stone at 9s. $10\frac{1}{2}$ d. per stone - - comes to $\begin{matrix} 185 & 13 & - \end{matrix}$.
 2. 68 ells at 11s. $4\frac{3}{4}$ d. per ell - - - - come to $\begin{matrix} 38 & 14 & 11. \end{matrix}$
 3. 130 C. weight at 15s. 6d. per C. - - comes to $\begin{matrix} 100 & 15 & - \end{matrix}$.
 4. 57 yards at 19s. $3\frac{1}{2}$ d. per yard - - - come to $\begin{matrix} 54 & 19 & 7\frac{1}{2}. \end{matrix}$

CLASS III.

1. 642 stone at 10s. $8\frac{1}{4}$ d. per stone - - comes to $\begin{matrix} 343 & 1 & 4\frac{1}{2}. \end{matrix}$
 2. 379 ells at 13s. $6\frac{1}{4}$ d. per ell - - - - come to $\begin{matrix} 256 & 4 & 4\frac{3}{4}. \end{matrix}$
 3. 628 C. weight at 16s. $5\frac{1}{2}$ d. per C. - comes to $\begin{matrix} 516 & 15 & 10. \end{matrix}$
 4. 54 yards at 18s. $7\frac{3}{4}$ d. per yard - - - come to $\begin{matrix} 50 & 6 & 10\frac{1}{2}. \end{matrix}$

CASE 7.

"When the price is any number of pounds with shillings, &c. annexed.

Rule.

"Multiply the quantity supposed pounds by the pounds in the price, and to the product add such parts of it, as make up the whole price, as usual; then is the sum of these the answer."

Examples.

10s	is of	1l	the	$\frac{1}{2}$	* If 1 yard cost £.1-17-6 what will [581 cost? 581 pounds suppose. 290 10 145 5 72 12 6 £.1089 7 6 Answer.
5s	is of	10s	the	$\frac{1}{2}$	
2s 6d	is of	5s	the	$\frac{1}{2}$	

P 2

If

* As 17s. 6d. in this instance is just half a crown short of a pound this example may

If 1 stone cost £.5-9-10 what will 84 cost?				
5s	is of	1l	the	$\frac{1}{4}$
84 pounds suppose.				
$\begin{array}{r} 84 \\ 5 \end{array}$				
4s	is of	1l	the	$\frac{1}{3}$
8d	is of	4s	the	$\frac{1}{6}$
2d	is of	8d	the	$\frac{1}{4}$
$\begin{array}{r} 420 \\ 21 \\ 16 \quad 16 \\ 2 \quad 16 \\ 14 \end{array}$				
£.461 6 Answer.				
If 1 stone cost £.23-17-9 what will 93 cost?				
10s	is of	1l	the	$\frac{1}{2}$
93 pounds suppose.				
$\begin{array}{r} 93 \\ 23 \end{array}$				
$\begin{array}{r} 279 \\ 186 \end{array}$				
5s	is of	10s	the	$\frac{1}{2}$
2s 6d	is of	5s	the	$\frac{1}{2}$
3d	is of	2s 6d	the	$\frac{1}{10}$
$\begin{array}{r} 2139 \\ 46 \quad 10 \\ 23 \quad 5 \\ 11 \quad 12 \quad 6 \\ 1 \quad 3 \quad 3 \end{array}$				
£.2221 10 9 Answer.				

Practical

may be better done, as follows: The hint contained in which procedure may be worth the reader's remembering.

2s. 6d. is of 1l. the $\frac{1}{5}$) 581 pounds suppose.

	1162	- - -	Value at 2l. the yard.
Sub.	72	12 6	Value at 2s. 6d. the yard.
	£.1089	7 6	Answer as before.

Practical Questions.

CLASS I.

	£.	s.	d.
1. 276 C. weight at £. 1-11-8 per C. - - comes to	437	—	—.
2. 389 yards at £. 3-10-6 per yard - - - come to	1371	4	6.
3. 85 stone at £. 6-5-3 per stone - - - comes to	532	6	3.
4. 98 hhds. at £. 10-10-6 per hhd. - - - come to	1031	9	—.

CLASS II.

1. 587 C. weight at £. 12-9-8 per C. - - comes to	7327	14	4.
2. 27 yards at £. 15-7-9 per yard - - - come to	415	9	3.
3. 627 stone at £. 5-9-11 per stone - - comes to	3445	17	9.
4. 56 hhds. at £. 7-7-10 per hhd. - - - come to	413	18	8.

CLASS III.

1. 37 C. weight at £. 19-12-6 per C. wt. comes to	726	2	6.
2. 83 yards at £. 27-14-6 per yard - - - come to	2301	3	6.
3. 78 stone at £. 36-12-8 per stone - - - comes to	2857	8	—.
4. 98 hhds at £. 17-5-10 per hhd. - - - come to	1694	11	8.

CASE 8

“When the price and quantity given are both of several denominations.”

Rule.

“Find the price of the integral part of the quantity, in the manner already taught; then for the odd denominations, take the parts of the given price they are of an integer, &c. and the sum of these results is the answer.”

An

An Example.

If 1 C. wt. cost £.5-16-10 what will 372 C. 3 Qrs. 21 lb. cost?				
10s	is of	1l	the	$\frac{1}{2}$
372 pounds suppose.				
5				
1860				
5s	is of	10s	the	$\frac{1}{2}$
1s	is of	5s	the	$\frac{1}{2}$
10d	is of	5s	the	$\frac{1}{6}$
93				
18 12				
15 10				
£.2173 2 Value of 372 C. weight.				
£.5 16 10				
2 Qrs	is of	1 C	the	$\frac{1}{2}$
1 Qr	is of	2 Qrs	the	$\frac{1}{2}$
14 lb	is of	1 Qr	the	$\frac{1}{2}$
7 lb	is of	14 lb	the	$\frac{1}{2}$
2 18 5				
1 9 $2\frac{1}{2}$				
14 $7\frac{1}{4}$				
7 $3\frac{1}{2}$				
5 9 $6\frac{1}{4}$ Value of the odds.				
2173 2 — Value of 372 C. wt.				
£.2178 11 $6\frac{1}{4}$ Their sum, the answer.				

Practical Questions.

CLASS I.

- | | £. | s. | d. |
|--|-----|----|--------------------|
| 1. 473 $\frac{1}{2}$ yards at 5s. 7d. per yard - - - come to | 132 | 3 | 8 $\frac{1}{2}$. |
| 2. 246 $\frac{1}{4}$ C. weight at 11s. 6d. per C. - - - comes to | 141 | 11 | 10 $\frac{1}{2}$. |
| 3. 98 $\frac{3}{4}$ stone at 8s. 4d. per stone - - - comes to | 41 | 2 | 11. |
| 4. 216 $\frac{1}{2}$ ells at 12s. 9d. per ell - - - come to | 137 | 16 | 6 $\frac{1}{2}$. |

CLASS

CLASS II.

	£.	s.	d.
1. $93\frac{1}{8}$ pounds at 7s. 6d. per pound - - come to	34	18	$5\frac{1}{4}$.
2. $134\frac{3}{4}$ ounces at £. 4-16-3 per ounce come to	648	9	$8\frac{1}{4}$.
3. 360C. 3Qrs. 16lb at £. 6-18-6 per C. come to	2499	3	$7\frac{3}{4}$.
4. 14T. 16C. 3Qrs. 21lb. at £. 12-16 per tun - come to	190	—	9.

CLASS III.

1. 17C. 1Qr. 17lb. at £. 1-4-9 per C. come to	21	10	8.
2. 9T. 15C. 3Qrs. 27lb. at £. 14 per tun come to	137	3	$10\frac{1}{2}$.
3. 218C. 3Qrs. 14lb. at £. 5-15- $7\frac{3}{4}$ per C. come to	1265	11	$11\frac{1}{4}$.
4. 596C. 3Qrs. 19 $\frac{1}{2}$ lb. at £. 48-14- $7\frac{3}{4}$ per C. come to	29089	9	6.

CASE 9.

“When the scale of the denominations of any thing is similar (or can be made similar) to sterling money.”

* Rule.

“If similar, set the whole down, as pounds, shillings, and pence, which is the answer of the whole at a pound; then deauce the true answer from it, as in the foregoing cases: But if it is not similar, suppose the integral part pounds, and then write the value of all the other denominations, on that supposition (when easily taken) in their proper places; and then from this sum may the answer be truly acquired in the same manner.”

Examples.

* This is a very useful compendium in *Exchange*, and the valuing of gold and silver plate, which is always done in Troy ounces, penny-weights, and grains,—The reason cannot be hard to see on a perusal of the examples.

Examples.

If 1 <i>l.</i> sterling be worth 32 <i>s.</i> 6 <i>d.</i> Flemish what is £.24-16-8 worth?				
10 <i>s.</i> 2 <i>s.</i> 6 <i>d.</i>	is of is of	1 <i>l.</i> 10 <i>s.</i>	the the	$\frac{1}{2}$ $\frac{1}{4}$
$\begin{array}{r} \text{£.} 24 \quad 16 \quad 8 \quad \text{Answer at 1l. suppose.} \\ 12 \quad 8 \quad 4 \\ 3 \quad 2 \quad 1 \\ \hline \text{£.} 40 \quad 7 \quad 1 \quad \text{Answer.} \end{array}$				
If 1 ton cost £.3-17-6 what will 7 <i>T.</i> 13 <i>C.</i> 3 <i>Qrs.</i> cost?				
10 <i>s.</i>	is of	1 <i>l.</i>	the	$\frac{1}{2}$
$\begin{array}{r} \text{£.} 7 \quad 13 \quad 9 \quad \text{amount at 1l. suppose.} \\ 3 \\ \hline 23 \quad 1 \quad 3 \\ 3 \quad 16 \quad 10\frac{1}{2} \\ 1 \quad 18 \quad 5\frac{1}{4} \\ 19 \quad 2\frac{1}{2} \\ \hline \text{£.} 29 \quad 15 \quad 9\frac{1}{4} \quad \text{Answer.} \end{array}$				
5 <i>s.</i> 2 <i>s.</i> 6 <i>d.</i>	is of is of	10 <i>s.</i> 5 <i>s.</i>	the the	$\frac{1}{2}$ $\frac{1}{2}$
If 1 ounce cost 5 <i>s.</i> 6 <i>d.</i> what will 367 <i>oz.</i> 18 <i>pwt.</i> 22 <i>grs.</i> cost?				
5 <i>s.</i>	is of	1 <i>l.</i>	the	$\frac{1}{4}$
6 <i>d.</i>	is of	5 <i>s.</i>	the	$\frac{1}{10}$
$\begin{array}{r} \text{£.} 367 \quad 18 \quad 11 \quad \text{amount at 1l. suppose.} \\ 91 \quad 19 \quad 8\frac{1}{4} \\ 9 \quad 3 \quad 11\frac{1}{2} \\ \hline \text{£.} 101 \quad 3 \quad 8\frac{1}{4} \quad \text{Answer.} \end{array}$				

Practical Questions.

CLASS I.

1. £.275-13-10 $\frac{1}{2}$ sterling at 34*s.* Ex. come to £.469-16-6 $\frac{1}{4}$ Flemish.
2. 593*oz.* 17*pwt.* 11*grs.* at 5*s.* 6 $\frac{3}{4}$ *d.* per ounce come to £.165-3-4 $\frac{1}{4}$.

CLASS

CLASS II.

	£.	s.	d.
1. 84oz. 15pwt. 13grs. at £.4-1 per ounce come to	343	6	11 $\frac{1}{4}$.
2. 16T. 3C. 1Qr. at 16s. per ton - - - come to	13	8	2 $\frac{1}{4}$.

CLASS III

1. 327oz. 17pwt 20grs. at £.3-17-6 per ounce			
come to	1270	11	7 $\frac{1}{4}$.
2. 3oz. 7pwt. 12grs. at £.7-19-8 per ounce			
come to	26	18	10 $\frac{1}{2}$.

Scholium.

These are the chief compendiums in this article, and though some * authors encrease the number to near a hundred, or more, yet if the general reasoning, we have been endeavouring to illustrate, be understood, any other of service will readily occur in the place it is wanted. And methinks that author pays his reader but a poor compliment (and also in effect his own instructions) who imagines, after he has led him thus far in numbers, he cannot of himself find out, that multiplying by 10 and dividing by 6 is the same thing as multiplying by 5 and dividing by 3, and other discoveries of the same magnitude, which often in this place they are carefully let into the secret of. But in the main, the practice of giving every little compendium, besides being tiresome, has other bad effects, as it prevents the student the encouraging pleasure of hitting upon them now and then himself. And farther, if we suppose him of such a capacity, as not to be able to do it, it is highly probable the same head will never carry, to any good purpose, such a large collection of rules huddled in implicitly.

These questions may be proved, if any think it worth while, by solving them different ways, as to the taking of parts, &c. or in general, by the golden rule.

Q

* T A R F

* Hutton especially gives 149 compendiums.

* TARE, TRET, &c.

Definitions.

"UNDER this title are delivered practical rules for deducting certain allowances, which go by these names, made by merchants in selling goods by weight.

Gross is the weight of the commodity, and that wherein it is contained, as box, bag, chest, barrel, &c.

Tare is an allowance made for the weight of these coverings, and is sometimes reckoned at so much a piece, and at others so much per C. weight, on the whole.

Tret is an allowance of 4lb for every 104 (or 1lb for 26) on goods liable to waste.

Cloff is an allowance to the citizens of London, for the turn of the scales, and is 2lb. for every 3 C. weight.

Suttle is a general name for what remains after some allowances are made, but not all.

Neat is what remains after all allowances are made."

'Tis usual first to allow the tare, and out of the remainder (or first futtle) to allow the tret, and out of the remainder (or second futtle) to allow the cloff.

It would be obvious to almost every arithmetician thus far advanced, in what manner to make these allowances, without any formal directions; nevertheless for method's sake, we will give a practical rule for each case, hoping they will need no other demonstration than a bare reference to their intent.

C A S E 1.

"When the tare is at so much per bag, barrel, &c."

Rule.

"Multiply the tare of one by the number, and subtract the product from the whole gross; the remainder is the neat (supposing no other allowance to be made)."

Example.

* These allowances beyond seas are called the *courtesies* of London, being used in no other place.

Example.

In 7 frails of raifins each weighing 5 C. 2 $\mathcal{Q}$ rs. 5 lb. grofs, tare 23 lb. per frail, how much neat?

lb.	C. $\mathcal{Q}$ rs. lb.
23	5 2 5
7	7
112) 161 (1 C.	38 3 7 Grofs.
112	1 1 21 Tare.
28) 49 (1 $\mathcal{Q}$ r.	Answer 37 1 14 Neat.
28	
21 lb.	

C A S E 2.

"When the tare is at so much per cent."

Rule.

"Take the parts of the grofs, the tare is of an C. weight as in practice, and deduct it from the grofs; but if the tare cannot be taken conveniently in this manner, the whole must be reduced into pounds, and the tare found by the rule of three."

Examples.

1. In 674 C. 2 $\mathcal{Q}$ rs. 16 lb. grofs; tare 14 lb. per C. how much neat?

14 lb. is of 112 the $\frac{1}{8}$	C. $\mathcal{Q}$ rs. lb.
674	2 16 Grofs.
84	1 9 Tare.
590	1 7 Neat.

2. In 346 C. 3 $\mathcal{Q}$ rs. 12 lb. grofs; tare 12 lb. per C. how many C. neat?

Q 2

346 C.

346C. 3Qrs 12lb.

$$\begin{array}{r} 4 \\ \hline 1387 \\ 28 \end{array}$$

If 112lb. give 12lb what will 38848 give?

$$\begin{array}{r} 11098 \\ 2775 \end{array}$$

$$\begin{array}{r} 38848 \text{ lb. gros.} \\ 4162 \text{ lb. tare.} \end{array}$$

$$\begin{array}{r} 12 \\ \hline 112) 466176 (4162 \text{ lb.} \\ 448 \\ \hline 181 \\ \text{£c.} \end{array}$$

112) 34686 (309C. 2Qrs. 22lb. Answer.

C A S E 3.

*"When tret is allowed with the tare."**Rule.**"Divide the futtle by 26, and the quotient is the tret, which subtracted from the futtle leaves the neat."**Example.*

In 72C. 3Qrs. 12lb. gros, tare 16lb. per C. tret 4lb. per 104, how much neat?

$$\begin{array}{r} \text{C. Qrs. lb.} \\ 16 \text{ lb. is of } 112 \text{ lb. the } \frac{1}{7}) \begin{array}{r} 72 \quad 3 \quad 12 \\ 10 \quad 1 \quad 18 \end{array} \begin{array}{l} \text{Gros.} \\ \text{Tare.} \end{array} \end{array}$$

$$\begin{array}{r} 26) \begin{array}{r} 62 \quad 1 \quad 22 \\ 2 \quad 1 \quad 17 \end{array} \begin{array}{l} \text{Suttle.} \\ \text{Tret.} \end{array} \end{array}$$

$$\begin{array}{r} \text{Answer} \quad 60 \quad - \quad 6 \quad \text{Neat.} \end{array}$$

C A S E 4.

*"When the cloff is allowed."**Rule.**"Multiply the C. weights futtle by 2, divide the product by 3, and the quotient is the pounds cloff, which taken from the futtle gives the neat."**Example.*

CLASS III.

1. What is the neat weight of 49 hogheads of tobacco, quantity gross 197C. 3Qrs. 16lb. tare 80lb. per hoghead, tret 4lb. per 104, and cloff 2lb. per 3C. weight? Answer 155C. 2Qrs. 22lb.
2. In 6 hogheads of sugar containing 56C. 2Qrs. 20lb. gross, tare 30lb. per hhd, 4lb. per 104 tret, and cloff 2lb. upon every 3C. how much neat? Answer 52C. 3Qrs. 6lb.

P R O F I T and L O S S.

Definition.

"THIS article contains the solution of those questions in merchandise relative to gain and loss in selling goods; and illustrates by examples the method of raising or falling their first price, so as to gain or lose so much per cent (or on selling to the value of £.100)."

Rule.

"On a bare sight of the questions it will appear they require nothing, but an easy application of the golden rule, practice, &c.

Examples.

1. * Bought tea for 9s. 9d. the pound, and sold it again at 11s. 10½d: what is gained per cent?

s.	d.
11	10½
9	6
2	4½

If

* Some authors have brought the consideration of time into these questions, and enquire, supposing in the above example, that the feller was not to be paid till 4 months, after what rate per cent he cleared per annum: This they determine thus; If 4 months give £.25 what will 12 give? i. e. £. 75, which on the supposition he can turn the £. 100 over to the same purpose every 4 months, is right. But how trifling and unsatisfactory such knowledge as this must be to the trader will appear on supposing he received ready money, or was only half an hour over the bargain, then would he gain by the same reasoning after the rate of £.443000 per cent per annum, which, as large as it seems, no wise man would hug himself upon, while there is a possibility of his not being employed to this purpose above three hours in the year.

If 9s. 6d. gain 2s. 4½d. what will £. 100 gain?

$$\begin{array}{r} 12 \\ \hline 114 \end{array}$$

$$\begin{array}{r} 12 \\ \hline 28 \\ 4 \\ \hline 114 \end{array}$$

$$\begin{array}{r} 20 \\ \hline 2000 \\ 12 \end{array}$$

4) 24000 Answer in farth. as both
mult. and div. are alike,

$$12) 6000$$

$$2|0) 50|0$$

£. 25 Answer,

2. How must I sell tea per lb. that cost me 13s 5d. to gain after the rate of £. 25 per cent?

If £. 100 become £. 125, what will 13s. 5d. become?

$$\begin{array}{r} 20 \\ \hline 2000 \\ 12 \\ \hline 24000 \end{array}$$

$$\begin{array}{r} 20 \\ \hline 2500 \end{array}$$

$$\begin{array}{r} 12 \\ \hline 161 \\ 2500 \\ \hline 80500 \\ 322 \end{array}$$

$$24|000) 402|500 (16s. 9\frac{1}{4}d.$$

$$\begin{array}{r} 24 \\ \hline 162 \\ 144 \\ \hline 18500 \\ 12 \end{array}$$

$$24) 222|000 (9d.$$

$$\begin{array}{r} 216 \\ \hline 6 \\ 4 \end{array}$$

$$24) 24 (\frac{1}{4}$$

But

4. Bought cloth at 7s. 6d. per yard, which proving not so good as expected, I am resolved to lose £. 17-10 per cent by it, how must I sell it per yard?

£.	£.	s.	d.	
10 is of 100 the $\frac{1}{10}$		7	6	
5 is of 10 the $\frac{1}{2}$			9	
2-10 is of 5 the $\frac{1}{2}$			4 $\frac{1}{2}$	
			2 $\frac{1}{4}$	
		1	3 $\frac{3}{4}$	Subtract.
		6	2 $\frac{1}{4}$	Answer.

5. * A Manchester chapman, going to a fair, sold fustians for 11s. 6d. the end, wherein was gained £. 15 per cent; and seeing no other chapman had so good, raiseth them at the latter end of the fair to 12s. I demana what he gained per cent by this last sale?

If 11s. 6d. raise (100l. to) 115, what will 12s. Do.?

2	24	2
23	460	24
	230	

23) 2760 (120l. Hence he gained 20l. per cent.

23	
46	
46	

6. Bought hofe in London at 4s. 3d. the pair, and sold them afterwards in Dublin at 6s. the pair; now taking the charges at an average to be 2d. the pair, and considering that I must lose 12 per cent by remitting my money home again, what do I gain per cent by this article of trade?

R

6s.

* This question is wrong answered in Hill's arithmetic.

s.			
6		If 4s. 5d. gain 1s. 7d. what will 100l. gain?	
<u>4</u>	<u>3</u>	12	12
2		—	—
<u>4</u>	<u>5</u> cost.	53	19
1	7 gained per pair.	—	24000
		76000	12
		38	24060

$$53) 456000 (12) 8603 \frac{3}{4}$$

$$2|0) 71|6 \quad 11$$

£. 35-16-11 $\frac{3}{4}$ gain per cent there.

If 100l. lose 12l. what will £. 135-16-11 $\frac{3}{4}$ lose?

£.	£.	£.	s.	d.
10 is of 100 the $\frac{1}{10}$	135	16	11 $\frac{3}{4}$	
2 is of 10 the $\frac{1}{5}$	13	11	8 $\frac{1}{4}$	
	2	14	4	
	16	6	— $\frac{1}{4}$	
	119	10	11 $\frac{1}{2}$	

Hence he gained here £. 19-10-11 $\frac{1}{2}$ per cent, the answer.

Practical Questions.

CLASS I.

1. If a yard of cloth is bought for 11s. and sold for 12s. 6d. what is the gain per cent? Answer £. 13-12-8 $\frac{3}{4}$.
2. If I buy goods for £. 2-2 per C. weight, and sell the same again at 5 $\frac{1}{4}$ d. per lb. I demand what is gained per cent? Answer £. 16-13-4.
3. Indigo sold at 3s. 2 $\frac{1}{2}$ d. per lb. there is gained £. 19-18-9 upon the 100l. of money; I demand what the C. weight cost first penny? Answer £. 14-19-7.
4. A Manchester-man buys 20 tun of cheese, with which he went into Ireland; it cost him £. 400, the freight and custom came to £. 50, his own expences and charges came to £. 16-13-4; how must he sell it per lb. to gain £. 20 per cent by it? Answer 3 pence.

CLASS

CLASS II.

1. A merchant selling corn at 8s. the bushel, gained £. 10 per cent, but afterwards being by a falling market forced to sell it for 7s; what did he gain or lose per cent by this sale?

Answer lost £. 3-15.

2. In selling cloth at 3s. 8d. per yard there is gained £. 12-10 upon the 100l. of money; now I desire to know, how I may sell the same per yard to gain £. 18-15 per cent? Answer 3s. 10½d nearly.

3. A merchant bought 20 equal pieces of broad cloth for £. 200, and finds to have gained £. 40 per cent, by selling it at 17s. 6d. per ell English, I demand how many yards were in each piece?

Answer 20 yards.

CLASS III.

1. If I buy 28 pieces of stuff at 4l. per piece, and sell 10 of the pieces at 6l, and 8 at 5l. at what rate must I sell the rest to gain 10 per cent by the whole?

Answer £. 2-6-4¾.

2. Bought comfits to the value of £. 41-3-3 for 3s. 1d. per lb. It happened that so many of them were damaged in carriage, that by selling what remained good at 4s. 6d. the pound, my returns were no more than £. 34-4. Pray how much of these goods were spoiled, and what did this part stand me in?

Answer 115lb. and £. 17-14-7.

* Commission, Brokerage, Insurance, &c.

Definitions.

“**U**NDER this title are collected questions relative to the allowance of so much per cent (or £. 100) made by merchants, &c. on certain occasions.

Commission is an allowance of so much per cent to a factor or correspondent abroad for buying and selling goods for his account.

Brokerage is an allowance of so much per cent to a person called a broker, for assisting merchants or factors in procuring or disposing of goods.

Insurance is a premium of so much per cent (sometimes per annum) given to certain persons, and offices, for a security of making good the loss of ships, houses, merchandizes, &c. which may happen from storms, fire, &c.”

Rule.

* Many other cases might have been added, as duties of all kinds, primage, storwage, &c. but they are so very near a-kin, that to instance in one is to illustrate them all.

Rule.

"This (like profit and loss) is nothing but an easy application of the rule of three."

Examples.

1. What comes the commiffion of £.500-13-6 to at $3\frac{1}{2}\%$ per cent?

If 100*l.* give $3\frac{1}{2}\%$. what will £.500-13-6 give?

$$\begin{array}{r}
 \frac{1}{2}) 500 \quad 13 \quad 6 \\
 \hline
 1502 \quad \text{—} \quad 6 \\
 250 \quad 6 \quad 9 \\
 \hline
 1|00) 17 \quad 52 \quad 7 \quad 3 \\
 \quad \quad 20 \quad \text{—} \\
 \quad \quad \text{—} \\
 \quad \quad s. 10 \quad 47 \quad \text{—} \\
 \quad \quad \quad 12 \quad \text{—} \\
 \quad \quad \quad \text{—} \\
 \quad \quad d. 5 \quad 67 \quad \text{—} \\
 \quad \quad \quad 4 \quad \text{—} \\
 \quad \quad \quad \text{—} \\
 \quad \quad \frac{1}{2} \quad 68
 \end{array}$$

Answer £.17-10-5 $\frac{1}{2}$ $\frac{68}{100}$.

In these cases, as the middle number is but small, it will always be the best to multiply it and the third together, as in the example.

2. When a broker sells goods to the amount of £.7105-5-10, what does his allowance come to at 5*s.* 6*d.* per cent?

If 100*l.* give 1*l.* what will £.7105-5-10 give?

$$\begin{array}{r}
 5s. \quad \frac{1}{4} \quad \left| \begin{array}{l} \text{£.} \quad s. \quad d. \\ 71 \quad 1 \quad \text{—} \frac{1}{2} \end{array} \right. \quad \begin{array}{l} 05-5-10 \\ 20 \\ \text{—} \\ 1 \quad 05 \\ 12 \\ \text{—} \\ 0 \quad 70 \\ 4 \\ \text{—} \\ \frac{1}{2} \quad 80 \end{array} \\
 6d. \quad \frac{1}{10} \quad \left| \begin{array}{l} 17 \quad 15 \quad 3 \\ 1 \quad 15 \quad 6\frac{1}{4} \end{array} \right. \quad \frac{1}{2} \quad 80 \\
 \hline
 19 \quad 10 \quad 9\frac{1}{4} \quad \text{Answer.}
 \end{array}$$

As these allowances are generally under 1*l.* it will be the readiest method, to find them always for 1 pound, and then take the required parts of that answer, by practice, as in the above example.

3. What

3. What comes the insurance of £.560-10 to at $10\frac{3}{4}$ per cent?

If 100 $\frac{1}{2}$ give $10\frac{3}{4}$ %, what will £.560-10 give?

$$\begin{array}{r}
 \frac{1}{2}) \begin{array}{r} 560 \\ 10 \end{array} \\
 \hline
 \begin{array}{r} 5605 \\ 280 \\ 140 \end{array} \quad \begin{array}{r} 5 \\ 2 \\ 6 \end{array} \\
 \hline
 \begin{array}{r} 60 \mid 25 \quad 7 \quad 6 \\ 20 \\ \hline 5 \mid 07 \\ 12 \\ \hline 0 \mid 90 \\ 3 \\ \hline \frac{3}{4} \mid 60 \end{array}
 \end{array}$$

Answer £.60-5-0 $\frac{3}{4}$.

Practical Questions.

CLASS I.

1. What is the commission of £.210-10-11, at $17\frac{1}{2}$ per cent?
Answer £.36-16-10 $\frac{3}{4}$.
2. What is the brokerage of £.796-14-7, at 6s. per cent?
Answer £.2-7-9 $\frac{1}{2}$.
3. What is the insurance of £.410-7-7 at $3\frac{2}{3}$ per cent?
Answer £.13-19-0 $\frac{1}{2}$.
4. What is the commission of £.648-17-2, at $2\frac{1}{2}$ per cent?
Answer £.16-4-5.

CLASS II.

1. What is the brokerage of £.210-10-8, at 15s. per cent?
Answer £.1-11-6 $\frac{3}{4}$.
2. What is the commission of £.365-12-7 at $4\frac{1}{3}$ per cent?
Answer £.15-7-1 $\frac{1}{2}$.
3. What is the insurance of £.1200 at $7\frac{1}{8}$ per cent?
Answer £.91-10.

CLASS III.

1. What may a broker demand for brokage, when he sells goods to the value of £.500-10-7, and I allow him 7s. 6d. per cent?
Answer £.1-17-6 $\frac{1}{4}$.
2. If I allow my factor $3\frac{3}{4}$ per cent, for commission, what may he demand on the laying out £.876-5-10 $\frac{1}{4}$?
Answer £.32-17-2 $\frac{1}{4}$.

PURCHASING

PURCHASING of STOCK

IN THE Public Funds.

Definition.

“**STOCK** is now become a general name for the capitals of our trading companies, and the money borrowed by the Government, at so much per cent, to defray the expences of the nation on various occasions.” And as the persons, who originally raise these capitals and supplies, have a power of transferring them to whom they think proper, the buying and selling certain sums of money, at * so much per cent, in these funds (which from the various circumstances of trade and nations alter intrinsically every day) is now become a general practice.

Rule.

“This is effected by the rule of three, in a manner similar to its application under the two last heads.”

Examples.

1. † What is the purchase of £.2054-16 South-sea-stock, at $110\frac{1}{4}$ per cent?

If 100 $\frac{1}{4}$ advance $10\frac{1}{4}$ what will £.2054-16 advance?

					$\frac{1}{4}$) 2054	16
						10
					20548	—
					513	14
				210	61	14
					20	—
				12	34	
					12	
£.	s.					
2054	16					
210	12	4				
2265	8	4	Answer.	4	08	

2. What

* The ruling prices are in the papers every day, and when above £.100, the excess is called so much *premium*, and when below, what they are short is called *discount*, as also when the same, they are said to be at *par*.

† This question and the next contain all the variety of simple purchasing,

2. What is the purchase of £.156-15, 3 per cent annuities, at $74\frac{1}{2}$ per cent?

If 100l. become $74\frac{1}{2}$ l. what will £.156-15 become?

$$\begin{array}{r}
 \frac{1}{2}) 156 \quad 15 \times 4 \\
 \quad \quad 10 \\
 \hline
 \quad 1567 \quad 10 \\
 \quad \quad \quad 7 \\
 \hline
 \quad 10972 \quad 10 \\
 \quad \quad 627 \quad - \\
 \quad \quad \quad 78 \quad 7 \quad 6 \\
 \hline
 116 \quad 77 \quad 17 \quad 6 \\
 \quad \quad 20 \\
 \quad \quad - \\
 15 \quad 57 \\
 \quad \quad 12 \\
 \quad \quad - \\
 6 \quad 90 \\
 \quad \quad 4 \\
 \quad \quad - \\
 3 \quad 60
 \end{array}$$

Answer £.116-15-6 $\frac{3}{4}$.

3. If I buy *Old south-sea annuities*, which bear interest at 3 per cent for $78\frac{1}{4}$ per 100l. how much interest do I make of my money?

If £.78-5 give 3l. what will 100l. give?

$$\begin{array}{r}
 \quad 20 \\
 \hline
 1565 \\
 \\
 \text{Answer } \text{£.} 3-16-8. \\
 \\
 \quad 20 \\
 \hline
 2000 \\
 \quad 3 \\
 \hline
 1565) 6000 \text{ (3l.} \\
 \quad 4695 \\
 \hline
 \quad 1305 \\
 \quad \quad 20 \\
 \hline
 1565) 26100 \text{ (16s. 8d.} \\
 \quad 1565 \\
 \hline
 \quad 10450 \\
 \quad \quad 50c.
 \end{array}$$

4. A

4. A person made 5 per cent of his money in buying stock, that carried $4\frac{1}{8}$ per cent interest, at what rate was the stock bought?

If 5% require 100%, what will $\pounds.4-7-6$ require?

$$\begin{array}{r} 20 \\ \hline 100 \\ 12 \\ \hline 1200 \end{array}$$

$$\begin{array}{r} 20 \\ \hline 87 \\ 12 \\ \hline 12|00) 1050|00 \\ \hline \pounds.87\frac{1}{2} \text{ Answer.} \end{array}$$

5. Suppose $3\frac{1}{2}$ per cent stock cost $\pounds.75\frac{1}{4}$, what must be given for $4\frac{1}{2}$ per cent, to make it an equal purchase?

If $3\frac{1}{2}$ give $\pounds.75-5$, what will $4\frac{1}{2}$ give?

$$\begin{array}{r} 2 \qquad 20 \qquad 2 \\ \hline 7 \qquad 1505 \qquad 9 \\ \hline 9 \\ 7) 13545 \\ \hline 2|0) 193|5 \\ \hline \pounds.96-15 \text{ Answer.} \end{array}$$

6. Whether is it better to purchase 3 per cent annuities at $85\frac{1}{2}$ per cent, or Bank-stock at $4\frac{1}{2}$, for $130\frac{1}{2}$ per cent?

If

If £85½ give 3 per cent what will £.130½ give?

$$\begin{array}{r}
 \begin{array}{r} 2 \\ \hline 171 \end{array} \\
 \begin{array}{r} 2 \\ \hline 261 \\ 3 \\ \hline 171 \overline{) 783} \text{ (4l.} \\ 684 \\ \hline 99 \\ 20 \\ \hline 171 \overline{) 1980} \text{ (11s.} \\ 171 \\ \hline 279 \\ 171 \\ \hline 99 \\ 12 \\ \hline 171 \overline{) 1088} \text{ (6d.}
 \end{array}$$

Hence as the 3 per cent annuities bring in (by the above stating) after the rate of £.4-11-6 for £.130-10, and the other but £.4-10, they are the better purchase.

Practical Questions.

CLASS I.

1. What is the purchase of £.575-10 *Bank-stock* at $131\frac{3}{4}$ per cent?
Answer £.758-4-5.
2. What is the purchase of £.577-19 *Bank-annuities* at $96\frac{3}{4}$ per cent?
Answer £.559-3-3¼.
3. What is the purchase of £.758-17-10 *India-stock* at $124\frac{5}{8}$ per cent?
Answer £.945-15-4¼.
4. What is the purchase of £.254-17 *Bank-annuities* at $97\frac{1}{4}$ per cent?
Answer £.274-16-9¾.

CLASS II.

1. What is the purchase of £.2750-17 *South-sea old annuities* at $102\frac{5}{8}$ per cent?
Answer £.2823-1-2½.
2. What is the purchase of 29l. 3 per cent annuities at 74 per cent?
Answer £.21-9-2¼.
3. Suppose I buy 4 per cents at 93l. how much per cent do I make of my money?
Answer £.4-6-0¼.

CLASS III.

1. If $4\frac{1}{2}$ per cent stock give $121\frac{1}{2}l$, what should $5\frac{1}{2}$ give to make an equal purchase? Answer $\pounds. 148\frac{1}{2}$.
2. Bought 650*l*. Bank annuities at $90\frac{3}{8}$ per cent, and paid brokerage $\frac{1}{8}$ per cent, what did the whole amount to? Answer $\pounds. 588-5$.

B A R T E R.

Definition.

“*UNDER this head are solved those questions which relate to the exchanging of goods among merchants, on certain conditions, so that neither party may receive damage.*”

Rule.

“*From a view of the examples, they will readily appear to require only the rule of three, &c.*”

Examples.

1. How many dozen of candles at 5*s*. 2*d*. per dozen must be given for 3*C*. 2*Q**rs*. 16*lb*. of tallow at 37*s*. 4*d*. per *C*. weight?

If 112*lb*. cost 37*s*. 4*d*. what will 3*C*. 2*Q**rs*. 16*lb*. cost?

$$\begin{array}{r} 12 \\ \hline 448 \end{array}$$

$$\begin{array}{r} 4 \\ \hline 14 \\ 28 \\ \hline 118 \\ 29 \\ \hline 408 \\ 448 \\ \hline 3584 \\ 1792 \end{array}$$

$$\begin{array}{r} 112) 182784 (12) 1632 \\ \underline{112} \\ 70784 \\ \underline{67200} \\ 3584 \\ \underline{3584} \\ 0 \end{array}$$

&c.

$$2|0) 13|6$$

$\pounds. 6-16$ value of the tallow.

If

If 5s. 2d. by 1 dozen what will £. 6-16 buy?

$$\begin{array}{r}
 20 \\
 \hline
 136 \\
 12 \\
 \hline
 62) 1632 \text{ (26 doz. 3lb. Answer.} \\
 124 \\
 \hline
 392 \\
 372 \\
 \hline
 20 \\
 12 \\
 \hline
 62) 240 \text{ (3lb.}
 \end{array}$$

2. A and B barter, A hath 41 C. of hops at 30s. per C. for which B gives him 20l. in money, and the rest in prunes at 5d. per pound: How many prunes did B give A besides the 20 pounds?

$$\begin{array}{r}
 10s. \text{ is } \frac{1}{2}) 41 \text{ C. at } £. 1-10. \\
 20 \quad 10 \\
 \hline
 61 \quad 10 \text{ the value of the hops.} \\
 20 \quad - \\
 \hline
 £. 41 \quad 10 \text{ to be laid out in prunes.}
 \end{array}$$

Hence—If 5d. buy 1lb. what will £. 41-10 buy?

$$\begin{array}{r}
 20 \\
 \hline
 832 \\
 12 \\
 \hline
 5) 9960 \\
 28) 1992 \text{ lb. (4) } 71 \\
 196 \\
 \hline
 32 \\
 28 \\
 \hline
 4
 \end{array}$$

17C. 3Qrs. 4lb. Anf.

3. A and B barter, A hath 123 pieces of cloth, which he values at £. 2-2-6 per piece ready money, and at £. 2-7-3 in barter, for which B would give him Spanish tobacco, which he rates at £. 7-14-9 per C. ready money, now I demand how B's tobacco ought to be valued in barter to equal the other, and also how many C. weight must be given for the cloth?

2s. 6d. $\frac{1}{8}$) 123 at £. 2-2-6.

$$\begin{array}{r}
 2 \\
 \hline
 246 \\
 15 \quad 7 \quad 6 \\
 \hline
 261 \quad 7 \quad 6 \text{ value of the cloth.}
 \end{array}$$

If £. 7-14-9 buy 1C. what will £. 261-7-6 buy?

$$\begin{array}{r}
 20 \\
 \hline
 154 \\
 12 \\
 \hline
 1857
 \end{array}$$

$$\begin{array}{r}
 20 \\
 \hline
 5227 \\
 12 \\
 \hline
 5571
 \end{array}$$

1857) 62730 (C. 33-3-3 $\frac{724}{1857}$ tobacco given.

$$\begin{array}{r}
 5571 \\
 \hline
 \text{£}c.
 \end{array}$$

If £. 2-2-6 become £. 2-7-3 what will £. 7-14-9 become?

$$\begin{array}{r}
 20 \\
 \hline
 42 \\
 12 \\
 \hline
 510
 \end{array}$$

$$\begin{array}{r}
 20 \\
 \hline
 47 \\
 12 \\
 \hline
 567
 \end{array}$$

$$\begin{array}{r}
 20 \\
 \hline
 154 \\
 12 \\
 \hline
 1857 \\
 567 \\
 \hline
 510) 1052919 (12) 2064
 \end{array}$$

510) 1052919 (12) 2064

20) 17|2

B's barter price per C. £. 8-12-0 $\frac{272}{318}$.

Scholium.

Scholium.

Authors in general have many such questions as the above in this rule, and seem, by their practice, to think it necessary to find the barter price of B's tobacco, before the quantity required can be determined, but as they will then have raised both their prices proportionably, 'tis plain the quantity found from the barter price will be the same, as would result from the ready money price. This being the case, 'tis a needless operation here, and indeed every where else, as might be easily shewn. For in real exchange of goods, when both sides have named their ready money prices, and are really sensible, that to adjust others in the same proportion, has no effect in giving either party an advantage, what can be more absurd than to trouble themselves (or their arithmetical friends) with these unmeaning operations? From this obvious consideration, were it not better in this rule to omit all questions in which this same barter price is concerned? For besides their instruction being little, and their use none, they have a very discouraging effect upon beginners, who, not being able to see the good intent of such a particular choice of circumstance in the question, nor after that perhaps the reason of the solution, conclude that there must be some weakness in their reasoning and mystery they cannot penetrate. And indeed this may well be the case, as in short, by hunting after this phantom of their own starting, authors have made Barter one of the most difficult rules in arithmetic, and not only puzzled their readers, but also * themselves. However not to appear too singular in our opinion, we will comply a little with other writers, and add a few more questions of this stamp, for the farther consideration of the reader.

4. A has cloth worth 15*s.* a yard, and B's cloth is worth 12*s.* per yard, how high must B put his cloth to gain 10 per cent by the barter?

If 100*l.* become 110*l.* what will 12*s.* become?

$$\begin{array}{r} 20 \\ \hline 2000 \end{array} \quad \begin{array}{r} 20 \\ \hline 2200 \\ 12 \end{array}$$

$$\begin{array}{r} 2|000) 26|400 \\ \hline 13*s.* \end{array}$$

Answer 13*s.* 2½*d.*

$$\begin{array}{r} 400 \\ 12 \\ \hline 4|800 \\ \hline 2*d.* \end{array} \quad \begin{array}{r} 800 \\ 4 \\ \hline 3|200 \\ \hline \frac{1}{4} \end{array}$$

5. A

* See preface to *Wells's* arithmetic.

5. A has 100 pieces of filks, which are worth but 3*l.* a piece ready money, yet he barter them with B at 4*l.* per piece, and at that rate takes their value of B in wool at $\text{£}.$ 7-10 per C. which is worth but 6*l.* in ready money; the question is to know what quantity of wool pays for the filks, and which of the two A or B is the gainer ?

100 picces at 3*l.* per piece come to 300*l.*
 100 Do. - at 4*l.* Do. - - come to 400*l.*

If $7\frac{1}{2}$ *l.* buy 1 C. what will 400*l.* buy?

$$\begin{array}{r} 2 \\ \hline 15 \end{array}$$

$$\begin{array}{r} 2 \\ \hline 15 \overline{) 800} \quad (53\frac{1}{3} \text{ C. wool which pays for the filk.} \\ 75 \\ \hline 50 \\ 45 \\ \hline 5 \end{array}$$

And, If 1 C. be worth 6*l.* what is $53\frac{1}{3}$ C. worth?

$$\begin{array}{r} 3 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 3 \\ \hline 160 \\ 6 \\ \hline 3 \overline{) 960} \\ \hline 320 \text{ l.} \end{array}$$

Hence 320*l.* is what A receives for his filks, and as they cost but 300*l.* he gains 20*l.* by the barter.

6. A and B barter thus; A has 100 yards of cloth worth 12*s.* per yard ready money, but in barter he will have 13*s.* 6*d.* and will have also $\frac{1}{4}$ of the barter value in ready money; B hath sugar at 8*d.* a *lb.* how much sugar ought B to deliver, and how is it to be raised to equal the barter?

$$\begin{array}{r} 13 \text{ s. } 6 \text{ d.} \\ 12 \end{array}$$

4) 16200 pence the 100 yards of cloth come to at the barter price.

Hence 4050 is the ready money to be given by B to A.

* Now

* Now as 4050 pence is paid ready money we must see what cloth it will bring in at the ready money price of 12s. per yard, as that is bought and not bartered; Hence

If 12s. buy 1yd. what will 4050 pence buy?

12

144) 4050 ($28\frac{1}{8}$ yards. This taken from the whole 100 yards leaves $71\frac{7}{8}$ yards, for which A is to have sugar. Therefore

If 1yd. give 144d. what will $71\frac{7}{8}$ give?

8

8

8

575

144

8) 82800

8) 10350 Answer in pence.

Consequently $1293\frac{1}{4}$ lb. is the sugar A is to give B.

As

* This natural and obvious method of solving questions of the nature of the above is recommended by Mr. *Malcolm*, in preference to another, which had been used by former authors on the like occasion. But Mr. *Wells*, in the preface to his arithmetic, gives his opinion in the favour of the old rule, and intimates its being more agreeable to reason, and that Mr. *Malcolm's* is not general, inasmuch as it cannot be applied when the prices only are given. To the latter part of this charge I agree with Mr. *Wells*, if he means it cannot be applied to the discovery of a barter price for the monied man, so compounded out of the two considerations of his giving part money and part goods, as to let the other person get no advantage of him. But as this (besides being a question of very small moment) seems a different enquiry, and consequently requires a different process in the solution, than what Mr. *Malcolm's* rule was intended for, I cannot see any fault there is in its not including it. As to the reasonableness of the two methods (where both are applicable) I beg leave to observe, that if there be any reason in going upon the express supposition of the question, Mr. *Malcolm's* has the preference. For what is paid for in ready money should certainly be had at a ready money price, or else the distinction of terms is lost in the very article, where every thing is supposed to depend upon them. But the old rule supposes all the goods, as well those for which ready money is received as the other, to be delivered at the barter price, and hence is obliged to make the party thus imposed upon raise his barter price in such a manner, as to take off the effect of the other's exaction, and from this I appeal to any reasonable person if Mr. *Malcolm* was not pretty right in observing of the old rule, that "This method is a ridiculous going round about to no purpose, and committing two errors, or doing two pieces of injustice, that one may correct the other, when there is a more simple and natural way of doing." But not to lay too great a stress upon a thing of so little consequence, we shall subjoin both rules, (equally true upon the suppositions abovementioned) and then leave the reader to the choice of that, which he can the easier comprehend, and reconcile to his notion of this kind of questions.—The old rule is "Subtract the money received both from the ready money value, and the barter value of his goods, who received the said money, and then proportion the other man's price to these remainders." Mr. *Malcolm's* rule is "Make A deliver goods to B at the ready money price for the sum received, and then find how much of B's goods (raised in the same proportion as A's) are equivalent to the remainder of A's goods."

As to the other part of the question the answer is thus found.

If 12s. become 13s. 6d. what will 8 pence become?

$$\begin{array}{r} 8 \\ \hline 12) 108 \text{ —} \\ \hline \end{array}$$

9 pence, the proportionable barter price for the sugar.

7. A has currants worth 4 pence per pound, but in barter charges them 6 pence, and also requires $\frac{1}{2}$ of that in ready money. B has candles at 6s. 8d. the dozen, and he in barter charges them but 7s. should these persons deal together for the value of 20l. how much must A get of B?

If 6d. become 4d. what will 20l. become?

2d. (left) is of 6d the $\frac{1}{3}$ 20

$$\begin{array}{r} 6 \ 13 \ 4 \\ \hline \text{£. } 13 \ 6 \ 8 \text{ what A's currants cost him.} \\ \hline \end{array}$$

Again.

If 7s. become 6s. 8d. what will 10l.

$$\begin{array}{r} 20 \\ \hline 200 \\ 80 \\ \hline 7) 16000 \\ \hline 12) 2285\frac{5}{7} \\ \hline 210) 1910 \ 5. \\ \hline \end{array}$$

9 10 $5\frac{5}{7}$ What B's candles cost him.

Mence ----- 19 10 $5\frac{5}{7}$ is what in effect he gave
A for ----- 13 6 8 therefore their -----
difference ----- 6 3 $9\frac{3}{7}$ is what A got the better of
B or the answer.

8. A

8. A has 40*lb.* of cloves at 6*s.* per *lb.* which he charges at 7*s.* 6*d.* in barter with B, who has velvet at 20*s.* per yard ready money. Neither of them know any thing of accounts, but however they agree at last, that A, in consideration of having $\frac{1}{3}$ of his barter price in cash, shall discount 10 per cent of it, and that B's velvet shall be reckoned at 22*s.* per yard. The question then is what velvet must be delivered by B, and what he will in reality lose by the agreement?

If 100*l.* become 90*l.* what will 7*s.* 6*d.* become?

$$\begin{array}{r} \text{10*l.* (left) is of 100*l.* the } \frac{1}{10} \text{) } \begin{array}{r} \text{s. } d. \\ 7 \quad 6 \\ \hline 9 \end{array} \end{array}$$

Hence - - - 6 9 is A's barter price when 10 per cent is deducted.

$$\begin{array}{r} \text{Again—If 1*lb.* of cloves give } \begin{array}{r} \text{s. } d. \\ 6 \quad 9 \\ \hline 4 \end{array} \text{ what will 40*lb.* give?} \\ \hline \begin{array}{r} 1 \quad 7 \quad - \\ \hline 10 \end{array} \end{array}$$

Therefore 13 10 — is the value of the cloves at the said barter price.

$$\begin{array}{r} \text{5*s.* is of 20*s.* the } \frac{1}{4} \text{) } 40 \\ \hline \text{2*s.* 6*d.* is of 5*s.* the } \frac{1}{2} \text{) } 10 \\ \hline 5 \end{array}$$

Farther. 40*lb.* of cloves at 7*s.* 6*d.* per *lb.* come to 15*l.* the $\frac{1}{3}$ of which is 5*l.*

$$\begin{array}{r} 13 \quad 10 \\ \hline 5 \quad - \end{array}$$

Hence £. 8 10 is what B is to deliver A in velvet.

Therefore—If 22*s.* buy 1 yard what will £. 8-10 buy?

quantity of velvet B delivers.

$$\begin{array}{r} 22 \overline{) 170} \text{ (7 } \frac{8}{11} \text{ yards — the } \\ \text{£c.} \end{array}$$

T

Again

Again—If 22s. become 20s. what will 170s. become?

$$\begin{array}{r} 20 \\ \hline 22) 3400 (210 \quad 15 \frac{1}{2} \\ \underline{440} \\ 800 \\ \underline{880} \\ 200 \\ \underline{220} \\ 0 \end{array}$$

money value of B's velvet.

£. 7-14-6 $\frac{1}{2}$ The ready

Also 40lb. of cloves at 6s. per lb. comes to 12l. the real value of A's cloves. But 5l. in cash and £. 7-14-6 $\frac{1}{2}$ in goods, amount to £. 12-14-6 $\frac{1}{2}$. Therefore it appears that B loses 14s. 6 $\frac{1}{2}$ d. by the agreement.

Practical Questions.

CLASS I.

1. A would exchange 1C. 22rs. 26 $\frac{1}{2}$ lb. of pepper, worth 2s. 1d. a pound, with B, for cotton worth 10d. per lb. How much cotton must B give A for his pepper? Answer 4C. 12r. 17 $\frac{1}{2}$.

2. Two men barter, A and B. A hath 249 gallons of brandy at 5s. 4d. ready money, for which B would give him checks which he rates at 27s. 5d. per piece, I desire you'll tell me how many pieces B must give A for his brandy? Answer 48 $\frac{1}{2}$ $\frac{1}{2}$.

3. 20 bags of hops each 3C. 22rs. bartered for 336C. of Brazil wood at 18s. per C. I demand what the hops were sold at per C? Answer £. 4-6-4 $\frac{3}{4}$.

4. A hath 608 yards of cloth worth 14s. per yard for which B gives him £. 125-12, in ready money, and 85C. 22rs. 24lb of bees-wax, the question is, what did B reckon his bees-wax at per C. weight? Answer £. 3-10.

5. A hath linen-cloth worth 20d. an ell ready money; but in barter he will have 2s. B has broad-cloth worth 14s. 6d. per yard ready money, at what price ought the broad-cloth to be rated in barter? Answer 17s. 4 $\frac{3}{4}$ d.

6. A hath 100 yards of kersey at 3s. per yard ready money, which he barter with B at 3s. 6d. taking small hair-buttons at 15 pence per grofs, which are but worth 12 pence: How many grofs of buttons will pay for the kersey, and whether does A or B get the better bargain, and how much? Answer, A's goods are worth 15l. and B's 14l. Therefore B has the advantage by one pound.

CLASS

CLASS II.

1. A and B barter; A hath 320 dozen of candles, at 4s. 6d. per dozen, for which B gives him 30*l.* in money, and the rest in cotton, at 8 pence per pound; I desire to know how much cotton B gave A besides the money?

Answer 5*C.* 3*Qrs.* 16*lb.*

2. How many reams of paper at 33½*d.* per ream must B give A for 37 pieces of Irish cloth, which is worth 32*s.* 4*d.* per piece?

Answer 428 $\frac{3}{67}$.

3. C has candles at 6*s.* per dozen ready money, but in barter he will have 6*s.* 6*d.* D has cotton at 9*d.* per pound ready money; I demand what price the cotton must be at in the barter? Also how much cotton must be given for 100 dozen of candles?

Answer, the cotton is 9½*d.* per *lb.* in barter, and the quantity will be 7*C.* 0*Qrs.* 16*lb.*

4. A has 210 yards of cloth at 4*s.* per yard, which he exchanges with B for large spoons at 5*s.* and tea spoons at 2*s.* each; being willing for every 2 large spoons to have 5 tea spoons. How many spoons of each sort must he have for his cloth?

Answer 84 large and 210 small.

CLASS III.

1. Two merchants A and B, barter with one another thus, A has 43 yards of broad cloth, worth 9*s.* 2*d.* per yard, but in barter he will have 11*s.* a yard. B has shalloon, worth 2*s.* per yard, which he charges at 2*s.* 6*d.* How much shalloon must A receive for his cloth? and what does he gain or lose by the bargain?

Answer, A receives 189 $\frac{1}{3}$ yards, and loses 15*s.* 9½*d.* by the bargain.

2. A with an intention to clear 30 guineas, in a bargain with B, rates hops at 16*d.* per *lb.* that stood him in 10*d.* B apprized of that, sets down malt, which cost him 20*s.* a quarter, at an adequate price: How much malt did they contract for? Answer 420 bushels.

FELLOWSHIP.

Definition.

“THIS is in general a rule, by which merchants trading in company with a joint stock, adjust in a reasonable manner every person's share of the gain or loss. It divides itself into two kinds, usually called Simple Fellowship, and Fellowship with Time, of which in their order.”

I. Simple Fellowship.

Definition.

"This is when different stocks are employed for any certain equal time."

* Rule.

"By the Rule of Three divide the gain or loss in proportion to their respective stocks, saying,

*As the whole stock,
Is to the whole gain or loss;
So is each man's particular stock,
To his particular share of the gain or loss."*

Examples.

1: Three men company, A put in £.505-5, B put in £.709-10, C put in 897^l. and at making up their accounts, they found they had gained £.400-8-4; now I demand each man's part of the gain?

£.	s.
505	5
709	10
897	—
2111	15

If £.2111-15 gain £.400-8-4 what will £.505-5 gain?

20	20
42235	8008
	12
	96100
	10105

42235) 971090500 (12) 22992 $\frac{23380}{42235}$.

£c.

2|0) 191|6

£.95-16-0 $\frac{23380}{42235}$ A's share.

If

* This can need no explanation.

If 42235*s.* gain 96100*d.* what will £.709-10 gain?

$$\begin{array}{r}
 \begin{array}{r}
 14190 \\
 \hline
 42235 \overline{) 1363659000} \quad (32287 \\
 \text{£}c.
 \end{array}
 \qquad
 \begin{array}{r}
 20 \overline{) 14190} \\
 \hline
 14190
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 12) 32287 \quad \frac{17555}{42235} \\
 \hline
 2 \overline{) 0} 269 \overline{) 0} \\
 \hline
 \text{£. } 134-10-7 \quad \frac{17555}{42235} \text{ B's share.}
 \end{array}$$

If 42235*s.* gain 96100*d.* what will 897*l.* gain?

$$\begin{array}{r}
 \begin{array}{r}
 17940 \\
 \hline
 42235 \overline{) 172403400} \quad (40820 \\
 \text{£}c.
 \end{array}
 \qquad
 \begin{array}{r}
 20 \overline{) 17940} \\
 \hline
 17940
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 12) 40820 \quad \frac{1300}{42235} \\
 \hline
 2 \overline{) 0} 340 \overline{) 1} \quad 8 \\
 \hline
 \text{£. } 170-1-8 \quad \frac{1300}{42235} \text{ C's share.}
 \end{array}$$

The last stating in these questions will mostly be needless (if the arithmetician can depend on his work) for the sum of all the gains, except one, taken from the whole, must leave that one. But if he thinks proper to carry the work through, he will have an opportunity of proving it, for all the particular gains, when added, must exactly make up the whole gain.—Thus in the above example.

	£.	s.	d.		
A's share is	95	16	—	$\frac{23380}{42235}$	23380
B's Do. - -	134	10	7	$\frac{17555}{42235}$	17555
C's Do. - -	170	1	8	$\frac{1300}{42235}$	1300
Whole gain	400	8	4	Proof.	42235

Rems. when added
make the divisor;
hence 1*d.* is carried
for them.

As it will always be the case, that the sum of the remainders will make some number of times the divisor (if all the middle terms be reduced alike) they must be carried for accordingly.

2. Three

2. Three persons make a joint stock of which A put in £. 184-10, B put in £. 96-15, and C, £. 76-5, they trade and gain £. 220-12; what must each person have of this gain?

£.	s.
184	10
96	15
76	5
357	10

If £. 357-10 gain £. 220-12, what will £. 184-10 gain?

20	20	20
7150	4412	3690
	3690	

7150) 16280280 (2276s. $\frac{688}{713}$ OR £. 113 16 $\frac{688}{713}$ A's share
£c.

If 7150s. gain 4412s. what will £. 96-15 gain?

1935	20
7150) 8537220 (1194s. $\frac{122}{713}$	1935
£c.	

OR £. 59 14 $\frac{122}{713}$ B's share

If 7150 gain 4412s. what will £. 76-5 gain?

1525	20
7150) 6728300 (941s. $\frac{15}{713}$	1525
£c.	

OR £. 47 1 $\frac{15}{713}$ C's share

£. 220 12 — Proof.

3. Four merchants in partnership A, B, C, and D put in 180l. 240l. 350l. and 430l. respectively for 5 years certain, and at the end of that time they find they have gained 3600l. what is each person's share of the gain?

£.
180
240
350
430
—
1200
—

If 1200*l.* gain 3600*l.* what will 280*l.* gain?—As soon as this is stated, 'tis easy to see the two first terms may be reduced, as mentioned in the contractions of the rule of three, one being just 3 times as big as the other. Hence the question will be readily answered thus.

If 1*l.* gain 3*l.* what will 180*l.* gain?

$$\begin{array}{r} 3 \\ \hline 540 \text{ A's gain.} \\ \hline \end{array}$$

If 1*l.* gain 3*l.* what will 240*l.* gain?

$$\begin{array}{r} 3 \\ \hline 720 \text{ B's gain.} \\ \hline \end{array}$$

If 1*l.* gain 3*l.* what will 350*l.* gain?

$$\begin{array}{r} 3 \\ \hline 1050 \text{ C's gain.} \\ \hline \end{array}$$

£.
540
720
1050
1290
—
3600 Proof.
—

If 1*l.* gain 3*l.* what will 430*l.* gain?

$$\begin{array}{r} 3 \\ \hline 1290 \text{ D's gain.} \\ \hline \end{array}$$

In cases where the second term may be divided exactly by the first, without a remainder, as the above, the quotient is called a *common multiplier*, and the work evidently then becomes exceeding easy. Now by *decimals* this multiplier may always be found (if thought proper) which renders them very useful in all questions of the like nature.

4. Two men company, A put in £.302-7, and they gained £.150-15 of which B's part came to £.87-11-4, I demand from this how much B put into company?

A T R E A T I S E

£.	s.	d.	
150	15	—	Whole gain.
87	11	4	B's gain
63	3	8	A's gain.

If £. 63-3-8 require £. 302-7 stock, what will £. 87-11-4 gain?

$$\begin{array}{r}
 20 \\
 1263 \\
 12 \\
 \hline
 15164
 \end{array}
)
 \begin{array}{r}
 20 \\
 6047 \\
 21016 \\
 \hline
 127083752
 \end{array}
 (2|0) 838|0 \frac{9432}{15164}$$

£. 419-0 $\frac{9432}{15164}$ Answer.

5. Three persons enter joint trade, to which A contributes 210/. B 312/. they clear 140/. whereof £. 37-10 belongs of right to C. That person's stock and the several gains of the other two are required?

£.	s.		£.
140	—	whole gain	210
37	10	C's gain	312
102	10	The sum of A and B's gain.	522 Do. stocks.

If 522/. gain £. 102-10 what will 210/. gain?

$$\begin{array}{r}
 20 \\
 2050 \\
 210 \\
 \hline
 522) 430500 (2|0) 82|4 \ 9 \ \frac{106}{322}
 \end{array}$$

£. 41-4-9 $\frac{106}{322}$ A's gain.

£.	s.	d.	
102	10		
41	4	9 $\frac{106}{322}$	
61	5	2 $\frac{416}{322}$	B's gain.

If

If £. 102-10 gain, require 522l. stock, what will £. 37-10 require?

$$\begin{array}{r}
 20 \\
 \hline
 2050 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 750 \\
 \hline
 2050 \ 391500 \ (\text{£. } 190-19-6 \frac{600}{2030} \text{ C's stock.} \\
 \text{£c.}
 \end{array}$$

6. Three partners A, B, and C, gain in a bargain £. 12-10, whereof it is agreed, that for every 2s. A has, B is to take 3s. and C 5s. required each person's share?

s. If 10s. give £. 12-10 what will 2s. give?

$$\begin{array}{r}
 2 \\
 3 \\
 5 \\
 \hline
 10 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 10) \ 12 \ 10 \\
 \hline
 1 \ 5 \text{ Common 2d. term.} \\
 2 \\
 \hline
 \text{£. } 2 \ 10 \text{ A's. share.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{£. } s. \\
 1 \ 5 \\
 3 \\
 \hline
 3 \ 15 \text{ B's share.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{£. } s. \\
 1 \ 5 \\
 5 \\
 \hline
 6 \ 5 \text{ C's share.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{£. } s. \\
 \text{A } 2 \ 10 \\
 \text{B } 3 \ 15 \\
 \text{C } 6 \ 5 \\
 \hline
 12 \ 10 \text{ Proof.} \\
 \hline
 \end{array}$$

Besides questions of the nature of the above, this obvious method of dividing money in certain proportions, may include some others, like the following one, which, tho' not altogether agreeable to the title, will fall in here better than in any other place.

7. Three men hired a shepherd to keep their sheep, and were to give him £. 7-15; the first committed to his care 540, the second 710, and the third 876, now I demand what each man ought to pay?

$$\begin{array}{r}
 540 \\
 710 \\
 876 \\
 \hline
 2126 \\
 \hline
 \text{U}
 \end{array}$$

If

If 2126 require £.7-15 what will 540 require?	Ans. 1	19 $\frac{785}{2126}$.
If 2126 ————— 7-15 ————— 710 —————	Ans. 2	11 $\frac{1624}{2126}$.
If 2126 ————— 7-15 ————— 876 —————	Ans. 3	3 $\frac{1842}{2126}$.
Proof 7 15		

Practical Questions.

CLASS I.

1. A, B, and C, trading to Guinea, with 480*l.* 680*l.* and 840*l.* in three years time gained 1010*l.* How much is each man's share of the gain? Answer, A, £.242-8, B, £.343-8, and C, £.424-4.

2. A's stock is £.16-4-6, B's is £.20-8-9, C's £.36-5-9; they lose by trade £.18-4-9, required each man's loss?

Answer, A's £.4-1-1 $\frac{1}{2}$, B's £.5-2-2 $\frac{1}{4}$, and C's £.9-1-5 $\frac{1}{2}$.

3. Three merchants A, B, and C, trading together, gained £.516-10-10. A put in £.816-12-6, B put in £.913-10-10, and C put in £.407-9-8. I demand what was each man's gain?

Answer, A £.197-6-7- $\frac{33504}{313036}$, B £.220-14-11 $\frac{1}{2}$ - $\frac{126952}{313036}$, C £.98-9-3 $\frac{1}{4}$ - $\frac{352580}{313036}$.

4. Three merchants A, B, and C, join their monies to make a stock of 12400*l.* with which they traded, and gained 2480*l.* of which A gets 686*l.* B 870*l.* and C 924*l.* what was each merchant's stock at first? Answer, A's 3430*l.* B's 4350*l.* and C's 4620*l.*

CLASS II.

1. In an article of trade, A gains 14*s.* 6*d.* and his adventure was 35*s.* more than B's, whose share of profit is but 8*s.* 6*d.* What are the particulars of their stock? Answer, A's £.4-4-7, and B's £.2-9-7.

2. Four failors, A, B, C, and D, join together, and buy 150 gallons of brandy for £.22-10, of which A paid £.7-10, B £.6-15, C £.5-5, and D 3*l.* How many gallons then ought each man to have to his share? Answer, A 50, B 45, C 35, and D 20 gallons.

3. Two men company and put in together £.703-13, with which they gained a certain sum, whereof for every 36*l.* A had, B had £.44-10; now I demand how much each man put in apart?

Answer, A £.314-13-6- $\frac{320}{1610}$, B £.388-19-5 $\frac{3}{4}$ - $\frac{410}{1610}$.

CLASS III.

1. A and B venturing equal sums of money, clear by joint trade 154*l.* By agreement, A was to have 8 per cent, because he spent his time in the execution of the project, and B was to have only 5: The question is what was allotted A for his trouble?

Answer £ 35-10-9 $\frac{1}{4}$ nearly.

2. Three

2. Three men A, B, and C, trade together a little while, and gain £.48-13-4. A put in at first 220*l.* and B and C together, put in 330*l.* now B's share of the gain was £.18-5; what then did A and C gain, and B and C put in respectively?

Answer, A's gain was £.19-9-4, and C's gain £.10-19. B's stock £.206-5, and C's stock £.123-15.

II. Fellowship with Time.

Definition.

"This is when different (or equal) stocks are employed different times."

* Rule.

"Multiply each man's stock into the time of its continuance, and then proportion the gain or loss according to these products, by saying,

*As the total sum of the products,
Is to the whole gain or loss;
So is each man's particular product,
To his particular share of the gain or loss."*

Examples.

1. A and B make a joint stock. A put in 250*l.* for 3 months, and B, 325*l.* for 4 months; they gain £.102-10, what each man's share of the gain?

250	325	750
<u>3</u>	<u>4</u>	<u>1300</u>
750 A's stock and time.	1300 B's Do.	2050 Sum.

U 2

As

* The reason of this will soon be made pretty clear. For as money is a thing, that is supposed to be turned to a daily account, as to profit, whether in trade or put to interest, it is plain on this supposition, that any sum, as 250*l.* in the first example, employed 3 months is the same thing, as 3 times that sum, or 750*l.* engaged for 1 month.—Again, 325*l.* for 4 months is the same thing as 4 times as much, or 1300*l.* for 1 month. Now having found out a method of bringing the time of continuance to an equality, as it were (i. e. 1 month) the gain must consequently (as in the last article) be divided in proportion to the stock, that is, in proportion to the above products.—The reason of the rule therefore is evident in this instance, and must consequently hold in all others, that are similar.

A T R E A T I S E

As 2050 is to £. 102-10, so is 750 to A's gain

$$\begin{array}{r} 20 \\ \hline 2050 \quad 2|0) \quad 75|0 \\ \hline \quad \quad \quad \text{£. 37-10 A's gain.} \\ \hline \end{array}$$

As 2050 is to 2050 so is 1300 to B's gain.

$$\begin{array}{r} 2|0) \quad 130|0 \\ \hline \quad \quad 65 \text{ l. B's gain.} \\ \hline \end{array} \quad \begin{array}{r} 37 \quad 10 \\ 65 \quad - \\ \hline \text{£. 102} \quad 10 \text{ Proof.} \\ \hline \end{array}$$

2. Three persons make a stock, the first puts in 96 l. for 8 months; the second 58 l. for 14 months; the third 40 l. for 22 months: They gain 240 l. what is each man's share of the gain?

$\begin{array}{r} 96 \\ 8 \\ \hline 768 \\ \hline \end{array}$	$\begin{array}{r} 58 \\ 14 \\ \hline 812 \\ \hline \end{array}$	$\begin{array}{r} 40 \\ 22 \\ \hline 880 \\ \hline \end{array}$	$\begin{array}{r} 768 \\ 812 \\ 880 \\ \hline 2460 \\ \hline \end{array}$
--	---	---	---

As 2460 is to 240 l. so is 768 to the gain of the first.

$$\begin{array}{r} 240 \\ \hline 2460) \quad 184320 \quad (\text{£. 74-18-6} \frac{108}{240} \\ \hline \text{£.} \end{array}$$

As 2460 is to 240 l. so is 812 to the second Do.

$$\begin{array}{r} 240 \\ \hline 2460) \quad 194880 \quad (\text{£. 79-4-4} \frac{158}{240} \\ \hline \text{£.} \end{array}$$

As 2460 is to 240 l. so is 880 to the third Do.

$$\begin{array}{r} 240 \\ \hline 2460) \quad 211200 \quad (\text{£. 85-17-0} \frac{216}{240} \\ \hline \text{£.} \end{array}$$

	£.	s.	d.	
1.	74	18	6	$\frac{108}{240}$
2.	79	4	4	$\frac{108}{240}$
3.	85	17	0	$\frac{216}{240}$
	240	0	0	Proof.

3. A and B were in company thus; A had 50% in stock for 10 months, and B had his stock in for 8 months, their gain was in proportion as 2 to 3, what was B's stock?

50%
10

500 A's stock and time. Then, As 2 is to 3 so is 500 to B's stock and time.

$$\begin{array}{r} 3 \\ 2) 1500 \\ 8) 750 \\ 93\frac{3}{4} \text{ B's stock.} \end{array}$$

4. A gains 20% in 6 months, B 18% in 5 months, and C 28% in 9 months, whose stock is 72%. what are the stocks of A and B?

$$\begin{array}{r} 72 \\ 9 \\ \hline 648 \text{ C's stock multiplied into his time.} \end{array}$$

Then: As 28% is to 648 so is 20% to A's stock and time:

$$\begin{array}{r} 20 \\ \hline 28) 12960 \text{ (462}\frac{6}{7} \text{)} \\ \text{£c.} \end{array}$$

$$\begin{array}{r} 6 \quad 462\frac{6}{7} \\ 7 \quad 7 \\ \hline \end{array}$$

Hence 42) 3240 (77 $\frac{1}{7}$ %; A's stock.

Again.

Again. As 28 $\frac{1}{2}$ is to 648 so is 18 $\frac{1}{2}$ to B's Do.

$$\begin{array}{r} 28) 11664 (416\frac{2}{3} \\ \underline{11200} \\ 464 \\ \underline{4500} \\ 144 \end{array}$$

$$\begin{array}{r} 5 \quad 416\frac{2}{3} \\ 7 \quad \quad 7 \\ \hline \end{array}$$

Hence 35) 2916 (83 $\frac{1}{3}$ B's stock.

5. Three merchants went partners 18 months; A put into stock, at first 200 ℓ . and at 8 months end he put in 100 ℓ . more, and two months after that, he put in 50 ℓ . more; B put in at first 550 ℓ . at 4 months end he took out 140 ℓ . and at 3 months after he took out 110 ℓ . more; C put in at first 600 ℓ . at 2 months end he took out 250 ℓ . and at 12 months after he put in 300 ℓ . At the expiration of the time they find they had gained 526 ℓ . what was each man's just share?

	200	300	
	100	50	
			1600
200	300	350	600
8	2	8	2800
1600	600	2800	5000 A's stock and time.

	550	410	
	140	110	
			2200
550	410	300	1230
4	3	11	3300
2200	1230	3300	6730 B's stock and time.

	600	350	
	250	300	
			1200
600	350	650	4200
2	12	4	2600
1200	4200	2600	8000 C's stock and time.

$$\begin{array}{r} 5000 \\ 6730 \\ 8000 \\ \hline 19730 \text{ Sum.} \end{array}$$

As 19730 is to 526l. so is 5000 to A's gain.

$$\begin{array}{r} 5000 \\ \hline 19730 \end{array}) 2630000 \text{ (£. 133-5-11 } \frac{1757}{1973} \text{.}$$

As 19730 is to 526l. so is 6730 to B's gain.

$$\begin{array}{r} 6730 \\ \hline 19730 \end{array}) 3539980 \text{ (£. 179-8-5 } \frac{167}{1973} \text{.}$$

As 19730 is to 526l. so is 8000 to C's gain.

$$\begin{array}{r} 8000 \\ \hline 19730 \end{array}) 4208000 \text{ (£. 213-5-7 } \frac{49}{1973} \text{.}$$

	£.	s.	d.	
A's	133	5	11	$\frac{1757}{1973}$
B's	179	8	5	$\frac{167}{1973}$
C's	213	5	7	$\frac{49}{1973}$
	526	—	—	Proof.

A single instance of another kind evidently answered in a similar manner.

6. A, B, and C, hold a piece of ground in common, for which they are to pay £.36-10-6. A put in 23 oxen 27 days, B 21, 35 days, C 16, 23 days, what ought each man to pay of the said rent?

23	21	16	621
27	35	23	735
621	735	368	1724

As

	£.	s.	d.		£.	s.	d.
As 1724 is to 36	10	6	fo	is 621 to A's share i. e.	13	3	$1\frac{1}{2}$ 624
As 1724 is to 36	10	6	fo	is 735 to B's share i. e.	15	11	$5\frac{1}{2}$ 724
As 1724 is to 36	10	6	fo	is 368 to C's share i. e.	7	15	$11\frac{1}{2}$ 1724

Proof 36 10 6

Scholium.

After all, this article is only speculation, as in real trade difference of time I believe can seldom happen. However as it is a possible case, and the reasoning upon it just, and of a kind that is very frequent, and useful in many other things, 'tis not amiss to consider it.

Several authors under one of these heads insert questions in *Factorship*, as they call it, or what relates to adjusting the profits of a partnership, in which one acts the whole, and values his service at so much. But as they vary in their reasoning upon it, and have gone no farther towards an equitable solution, than what would readily occur to the persons in their engagements, 'tis perhaps as well to let it rest there.

Practical Questions.

CLASS I.

1. Three persons A, B, and C, composed a small joint stock of 90*l.* thus: A had in it 20*l.* for 3 months; B 30*l.* for 5 months; and C 40*l.* for 7 months; their gain in all was £. 57-3-4, what was each man's share of it? Answer, A's 7*l.* B's £. 17-10, and C's £. 32-13-4.

2. A put into company 640*l.* for 5 months, B 400*l.* for 6 months, and C 590*l.* for 8 months. They gained 500*l.* what is each person's share of it? Answer, A's £. 155-0-9 $\frac{1}{4}$ $\frac{216}{1032}$, B's £. 116-5-6 $\frac{3}{4}$ $\frac{986}{1032}$, and C's £. 228-13-7 $\frac{1}{2}$ $\frac{912}{1032}$.

3. Three shepherds take a piece of ground for £. 200-10, in which A put 860 sheep for 7 months, B put 750 for 6 months, and C, 500 for 4 months: What ought each man to pay of the above rent? Answer, A 96*l.* $\frac{509}{1232}$, B 72*l.* $\frac{81}{1232}$, C 32*l.* $\frac{36}{1232}$.

4. Three merchants join in trade; A put in 400*l.* for 9 months; B, 680*l.* for 5 months; C, 120*l.* for 12 months, but by misfortune lose to the value of 500*l.* what is each man's share of it?

Answer, A's £. 213-5-4 $\frac{3}{4}$ $\frac{284}{844}$, B's £. 201-8-5 $\frac{784}{844}$, and C's £. 85-6-1 $\frac{3}{4}$ $\frac{620}{844}$.

CLASS II.

1. A, B, and C, jointly raise a stock of 4600*l.* A's part thereof continued in 8 months, B's 6 months, and C's 4 months; their total gain was 800*l.* of which A's dividend was 400*l.* B's £. 266-13-4, C's £. 133-6-8, what did each person deposit at the first?

Answer, A put in 1800*l.* B, 1600*l.* and C, 1200*l.*

2. A

2. A puts in 30*l.* for 30 months (each 30 days) and 10 days; B puts in 40*l.* for 20 months and 15 days; C puts in 60*l.* for 16 months and 5 days; they gain £.67-10, and each man's part of it is required?

Answer, A's £.22-15, B's £.20-10, and C's £.24-5.

3. Three men company, A put in £.519-10 for $8\frac{1}{2}$ months, B put in £.706-15, for $10\frac{1}{4}$ months, C put in £.876-5, for $12\frac{3}{4}$ months; they gained £.303-3; what was each man's part of the said gain? Answer, A £.58-12 $\frac{1075340}{1826570}$, B £.96-3 $\frac{1226595}{1826570}$, and C £.148-6 $\frac{1351205}{1826570}$.

CLASS III.

1. In a joint trade B putteth in 200*l.* more than A; B continueth his stock but 5 months, A $7\frac{1}{2}$; they gain alike; what money had each of them in stock? Answer, A, 400*l.* B, 600*l.*

2. A puts in 168*l.* for 5 months; B puts in a sum of money for 8 months, and C 400*l.* for a certain time; they gain 90*l.* whereof A must have 18*l.* B 12*l.* and C 60*l.* how much was the stock of B? and how long was C's stock in company?

Answer, B's stock 70*l.* and C's time 7 months.

INTEREST.

Definitions.

I NTEREST is the gratuity allowed the lender by the borrower for the use of any sum of money at so much per cent per annum, and for such a determinate space of time.

Principal is the money lent, for which interest is to be received.

* Rate is the sum agreed on per cent per annum between the parties.

Amount, is the principal and interest added together."

Interest resolves itself into two kinds, on different considerations, called *Simple* and *Compound*.

I. Simple Interest.

Definition.

"This is what arises from the continuance of a sum of money lent for a certain time (greater or less) in the proportion of so much per cent per annum."

X

Rule.

* By act of Parliament this rate is not to exceed 5*l.* per cent per annum.

* Rule.

Find the interest of the principal for one year by the rule of three. Then from the result, by the method of practice, may be found the interest, for any time greater or less, if it be given (as is most common) in years and † calendar months."

Examples.

1. What is the interest of £. 249-16-8 for $7\frac{1}{2}$ months at 5 per cent per annum?

			If 100l. give 5l. what will £. 249 16 8 † give?		
			5		
6m. is of 1y. the $\frac{1}{2}$	£. s. d.	12 9 10	One year's int.	12	49 3 4
					20
1 $\frac{1}{2}$ is of 6m. the $\frac{1}{4}$		6 4 11		9	83
					12
					—
			£. 7 16 1 $\frac{3}{4}$	Answer.	10 00

2. What is the interest of £. 530-16-8 for $8\frac{1}{4}$ months $4\frac{3}{4}$ per cent per annum?

* $\frac{1}{2}$) 530

* This rule can need no explanation, besides the work itself.

† The months, whereof 12 make a year.

‡ In real business pence (and farthings) are neglected in interest, discount, &c. as of small moment, and often the shillings also, or else taken in at 10 shillings, just as they are of worth. And in the main, the odd pence and farthings in the data of many questions in this work are a little improper upon such like considerations, except we urge, that in teaching these sciences, examples may be proposed difficult and tedious (in any shape) purely to inure the student to calculation, and to make the others of a less perplexed nature, which he will chiefly have to do with afterwards, appear the more easy to him &c.

m.		£.	s.	d.	
6	$\frac{1}{2}$	25	4	$3\frac{1}{2}$	Interest for one year.
2	$\frac{1}{3}$	12	12	$1\frac{3}{4}$	
$\frac{1}{4}$	$\frac{1}{8}$	4	4	$\frac{1}{2}$	
			10	6	
		£. 17	6	$8\frac{1}{4}$	Answer.

	£.	s.	d.
* $\frac{1}{2}$)	530	16	8
			4
	2123	6	8
$\frac{1}{2}$)	265	8	4
	132	14	2
25	21	9	2
	20		
4	29		
	12		
3	50		
	4		
$\frac{1}{2}$	00		

3. What is the interest of £.64-17-8 for 3 years $4\frac{1}{2}$ months at 4 per cent per annum?

m.		£.	s.	d.	
4	$\frac{1}{3}$	2	11	$10\frac{3}{4}$	Interest for one year.
				3	
$\frac{1}{2}$	$\frac{1}{8}$	7	15	$8\frac{1}{4}$	
			17	$3\frac{1}{2}$	
			2	$1\frac{3}{4}$	
		£. 8	15	$1\frac{1}{2}$	Answer.

	£.	s.	d.
	64	17	8
			4
2	59	10	8
	20		
11	90		
	12		
10	88		
	4		
$\frac{3}{4}$	52		

4. What is the interest of £.140-15 for 5 years $6\frac{3}{4}$ months at $4\frac{3}{4}$ per cent per annum?

X 2

$\frac{1}{4}$) 140

* The method is so easy, that the statings need not be wrote.

m.		£.	s.	d.	
6	$\frac{1}{2}$	6	13	$8\frac{1}{2}$	Interest for one year.
				5	
		33	8	$6\frac{1}{2}$	
$\frac{3}{4}$	$\frac{1}{8}$	3	6	$10\frac{1}{4}$	
			8	$4\frac{1}{4}$	
		£.37	3	9	Answer.

$\frac{1}{2}$)	140	15	4
	563	—	
$\frac{1}{2}$)	70	7	6
	35	3	9
6	68	11	3
	20		
13	71		
	12		
8	55		
	4		
$\frac{1}{2}$	20		

In some cases it is customary to consider the time elapsed different ways; as in computing the interest on the public-bonds of the *South-sea*, and *India Companies*, &c. the time is generally taken in months and days, and on *Exchequer bills*, in quarters of a year and days. Now as the months and quarters differ some little matter in length, the same actual time elapsed may by this means be esteemed differently, and consequently effect the result proportionably. Therefore in compliance with this, and to extend what is said above to these cases, we must next shew,

“The method of computing interest for days.”

* Rule.

“Multiply the principal by the rate, and that product by the number of days; the last product divide by 36500, and the quotient is the answer.”

Examples.

* The questions resolved by this rule are also very naturally answered by two statings in the rule of three, and on comparing the methods the above will readily appear to be drawn from them. Thus in the first example that follows, the first stating would be,

If 100*l.* give 5*l.* what will £.419-5-6 give?

The answer to which we will suppose A. This done the second stating would be;

If 365 days give A what will 54 days give?

Now A, which is to be multiplied into 54 is (from the first) the product of the principal and rate divided by 100. But suppose the operation of dividing by 100 was omitted, and that A stood for the product of the principal and rate only, then would A when multiplied into 54 (in the course of solving [the stating] give

Examples.

1. Required the interest of £.419-5-6 for 54 days at 5 per cent per annum?

$$\begin{array}{r}
 \text{£. } 419 \quad 5 \quad 6 \\
 \underline{20} \\
 8385 \\
 \underline{12} \\
 100626 \\
 \underline{5} \\
 503130 \\
 \underline{54} \\
 365|00 \quad 271690|20 \quad (12) \quad 744 \\
 \text{£s.} \quad \quad \quad 2|0 \quad 6|2 \\
 \underline{\text{£. } 3-2} \quad \text{Answer.}
 \end{array}$$

2. What is the interest due upon an India-bond of 500*l.* value, at $3\frac{1}{2}$ per cent per annum, from the 30th. of September 1763 to the 18th. of June 1764? (i. e. 8 months and 18 days).

<i>m.</i>		<i>£.</i>	<i>s.</i>		$\frac{1}{2}$ 500 <i>l.</i>
6	$\frac{1}{2}$	17	10	Interest for one year.	<u>3</u>
2	$\frac{1}{3}$	8	15		1500
		2	18	4	<u>250</u>
		11	13	4 for 8 months.	17 50
			17	3 for the 18 days.	20
		<u>£. 12</u>	<u>10</u>	<u>7</u> Answer.	10 00

For

a product 100 times as much as it should be. But this excess may be rectified by making the first number (365) which is to divide that product 100 times as large as it is, or 36500. Now on examining the operations resulting from this method of arguing they will be found to be exactly what the rule directs.—Hence its truth is demonstrated.

Practical Questions.

CLASS I.

1. What is the interest of £.284-7-6 for 3 years and 7 months at £.4-10 per cent per annum? Answer £.45-17-1½.
2. What is the interest of £.742-11-5 for 2 years 9½ months at 4 per cent per annum? Answer £.82-5-11¾.
3. What is the interest of £.519-10-4 for 4 years 5¾ months at 4½ per cent per annum? Answer £.104-14-3¼.
4. What is the interest of £.1046-11 for 3 years 8¼ months at 4¾ per cent per annum? Answer £.183-6-1½.
5. What is the interest of £.175-17 for 2 years and ¾, at 4½ per cent per annum? Answer £.21-15-2¼.
6. What is the interest of 246*l.* for 164 days at 4 per cent per annum? Answer £.4-8-5.

CLASS II.

1. What is the interest of £.397-5-4 for 3 years and 75 days at 5 per cent per annum? Answer £.63-13-4.
2. What is the interest of £.660-15-8½ for 5 years, 9 months, and 14 days at 3¾ per cent per annum? Answer £.143-8-7¼.
3. What is the interest of £.47-10 for 4¾ years and 52 days at 4½ per cent per annum? Answer £.10-9-1¾.
4. What is the interest of £.946-14-10 from July 21st. 1758 to April 7th. 1764 at 3½ per cent per annum? (i. e. 5 years, 260 days). Answer £.189-5-7¼.

CLASS III.

1. What interest is due on a bond of £.634-16-4, at 4½ per cent, from June 15th. 1762, to September 10th. 1764? (i. e. 2 years and 87 days). Answer £.63-18-10.
2. A gentleman left his niece by will £.558-15, to be paid her, when she came to age with interest, at 4 per cent. Now she came to age in 5 years 9 months and 21 days; what has she to receive in all? Answer £.688-10-11.

II. * *Compound Interest.**Definition.*

"This is when after interest becomes due, and continues in the borrower's hand it is considered as added to the old principal, and making a new one for the next year, and so on, creating a new principal from year to year, for the succeeding one, by adding to the old one the interest of that, which went immediately before."

* *Rule.*

* Tho' it be made unlawful to take compound interest (probably as Mr. Malcolm observes for the encouragement of trade) yet as it is a very equitable consideration, and usually allowed in the theories relating to the purchasing annuities, pensions, &c. and taking leases in reversion, this hint thrown in here may not be amiss.—See Scholium at the latter end of Interest by Decimals in the second part.

* Rule.

"Find the first years interest, as per the last, which add to the principal, then find in the same manner the interest of that sum, which add as before.—Proceed thus for the number of years required, and then the sum of all the interests, or the first principal, taken from the last amount, is the answer."

Example.

What is the compound interest of £. 320-10 for $4\frac{1}{2}$ years at 5 per cent per annum?

	£.	s.	d.		OR	£.	s.	d.
† $\frac{1}{20}$)	320	10	—	1st. years principal.				
	16	—	6	1st. years interest. - - - - -	16	—	6	
$\frac{1}{20}$)	336	10	6	2d. years principal.				
	16	16	$6\frac{1}{4}$	2d. years interest. - - - - -	16	16	$6\frac{1}{4}$	
$\frac{1}{20}$)	353	7	$—\frac{1}{4}$	3d. years principal.				
	17	13	4	3d. years interest. - - - - -	17	13	4	
$\frac{1}{20}$)	371	—	$4\frac{1}{4}$	4th. years principal.				
	18	11	—	4th. years interest - - - - -	18	11	—	
$\frac{1}{20}$)	389	11	$4\frac{1}{4}$	5th. years principal.				
$\frac{1}{2}$)	19	9	$6\frac{3}{4}$	5th. years interest.				
	9	14	$9\frac{1}{4}$	$\frac{1}{2}$ of 5th. years interest - - - - -	9	14	$9\frac{1}{4}$	
	389	11	$4\frac{1}{4}$	Principal for 5th. year.				
	399	6	$1\frac{1}{2}$	Whole amount.				
	320	10	—	First principal.				
	£. 78	16	$1\frac{1}{2}$	Answer. - - - - -	£. 78	16	$1\frac{1}{2}$	

Scholium.

By having any three of the four different terms, *principal*, *rate*, *time*, and *interest* given, the other may be discovered, and in the first of these heads is not hard to do by the rule of three. But as the case

* Every step of the operation explains itself.

† In the instances of 5 per cent, the interest is very easily found by taking $\frac{1}{20}$ of the principal as *5l.* is the $\frac{1}{20}$ of 100*l.*—The remains at the end of these operations are never regarded.

case of *principal, rate, and time* given to find the interest (already fully illustrated) is the most common and useful question in usury, we think it better to refer the rest to the second part of this work, where the subject is resumed, and treated in a better method by the help of *Decimals*.

Practical Questions.

CLASS I.

1. What is the compound interest of 550*l.* for $3\frac{1}{2}$ years at 5 per cent per annum? Answer £.102-12-2 $\frac{1}{2}$.
2. What is the compound interest of 480*l.* for 6 years at 5 per cent per annum? Answer £.163-4-10 $\frac{1}{2}$.

CLASS II.

- What is the amount at compound interest of £.15-10 for 9 years at $3\frac{1}{2}$ per cent per annum? Answer £.21-1-4 $\frac{1}{4}$.

CLASS III.

- What is the compound interest of £.57-10-6 for 5 years 7 months and 15 days at 5 per cent per annum? Answer £.18-3-8 $\frac{1}{2}$.

REBATE or DISCOUNT.

Definition.

"*THIS rule discovers what present money will in equity discharge a debt due hereafter, or what allowance (called Rebate or Discount) the creditor should make the debtor for the favour.*"

* Rule.

"By the rule of three say, As 100*l.* with its interest for the time at the rate agreed on, is to 100*l.* so is the sum to be paid hereafter, to what will pay it at present."—Hence the rebate or allowance is the difference of these two.

Y

Example.

* The equity of the rule is its reason: For a person paying money before it is due gives the advantage he might have made of it (at interest &c.) into his creditor's hands; hence the present money must be such a sum, as put to interest for the given time will amount to the whole debt, and this is evidently effected in the stating.

Some indeed find the *interest* of the whole sum for the time, and call that the *rebate*, which is always too much: But if a few agree upon it to spare the labour of an exacter calculation, 'tis to be hoped, the more knowing will not think the additional labour there is in the above rule too great a sacrifice to the justice of it.

As £.100-6-9 is to 100*l.* so is £.549-10-6 to the present worth

$$\begin{array}{r} 20 \\ \hline 2006 \\ 12 \\ \hline 24081 \end{array}$$

$$\begin{array}{r} 20 \\ \hline 10990 \\ 12 \\ \hline 24081 \end{array} \quad 131886 \text{ (£.547-13-6 } \\ \text{£}c.$$

£.	s.	d.	
549	10	6	Sum due.
547	13	6	Present worth,
1	17	—	Discount.

Practical Questions.

CLASS I.

1. What is the present worth of 150*l.* payable in three months, at 5 per cent discount? Answer £.148-2-11½.
2. What is the present worth of 75*l.* due 15 months hence at 5 per cent? Answer £.70-11-9½.
3. What is the discount of £.275-10, for 7 months at 5 per cent? Answer £.7-16-1¾.
4. What is the discount of £.898-9-7 payable at 1½ years, at 4½ per cent? Answer £.56-16-3.

CLASS II.

1. What is the discount of £.143-17-6 for 2 years and 23 days at 5 per cent? Answer £.13-9-0¾.
2. What is the present worth of £.133-10 due in 14 days at 5 per cent? Answer £.133-4-10¾.
3. What is the present worth of £.309-12-6¾ payable in 8 years and 73 days at 5¾ per cent? Answer £.210-8-4.

CLASS III.

1. What is the present worth of a bill of £.6790-1-8¼ due in 7¼ years and 50 days at 4¾ per cent? Answer £.4992-5-9.
2. What is the interest and discount of £.799-7-7¾ at 4½ per cent? Answer, Interest £.28-15-6¾, and Discount £.27-15-6¾.

E Q U A T I O N of Payments.

Definition.

“WHEN several sums are due at different times from one person to another, this rule discovers a mean (or equated) time of paying the whole without injuring either party.”

* Rule.

“Multiply each particular payment by the time it is to be paid in, then divide the sum of these products by the whole debt, and the quotient is the equated time.”

Examples.

1. A owes B 50*l.* to be paid in 6 months, 60*l.* to be paid in 7 months, and 80*l.* to be paid in 10 months: Required the equated time to pay the whole?

6 times

* This rule is founded upon a supposition, that the equated time is right “when the interest of the sum of the debts from the time of the question is equal to the sum of the interests of the several debts from the time of the question to their several terms of payment.” Or, which amounts to the same, “when the sum of the interests of the several debts, that are payable before the equated time from their terms to that term, is equal to the sum of the interests of the debts payable after the equated time, from that time to their terms.” This has appeared a reasonable state of the case to many, but is not altogether so, as is well shewn by Mr. *Malcolm* in his *Arithmetic*, page 616, where he has investigated another rule, that is unexceptionable. But as the operations there required are very tedious, and in their result differ but a very trifle from the answer given by this old one, it will always be made use of as the most eligible on these occasions.

This being premised we will next shew its truth on the above supposition.—In the example that follows, the use of 50*l.* for 6 months is the same to the debtor as 6 times 50, or 300*l.* would be to him for 1 month. Likewise the use of 60*l.* for 7 months may be looked on, to him, the same as 7 times 60, or 420*l.* for 1 month. And, in the same manner 80*l.* for 10 months, the same as 10 times 80, or 800*l.* for 1 month. Hence the use of the sum of these i. e. 1520*l.* in his hands for 1 month is shewn to be equal to that of the above separate sums, their respective times. But the money he owes is in all only 190*l.* therefore by the rule of three inverse we must find a time, when the use of 190*l.* will be to him equal to 1520*l.* for 1 month, and the question is answered.—Now for this the stating is,

If 1520*l.* require 1 month what will 190*l.* require?

and the solution (8 months) is got by dividing 1520 by 190.—Hence as every step of the above reasoning agrees with what is directed in the rule, it is proved to be right.

6 times 50 make 300
7 times 60 make 420
10 times 80 make 800

190) 1520 (8 months the equated time.
1520

2. A owes B 150*l.* to be paid in 40 days, 96*l.* to be paid in 53 days, and 310*l.* to be paid in 76 days; required the equated time?

40 times 150 make 6000
53 times 96 make 5088
76 times 310 make 23560

556) 34648 (62 days the answer.
6*c.*

3. A owes B £. 52-7-6, to be paid in $4\frac{1}{2}$ months, £. 80-10, to be paid in $3\frac{1}{2}$ months, and £. 76-2-5 to be paid in 5 months; required the equated time?

	£.	s.	d.	make	£.	s.	d.
9 times	52	7	6	471	7	6	
7 times	80	10	—	563	10	—	
10 times	76	2	6	761	5	—	

209 — —
20

4180
12

50160)

1796 2 6
20

35922
12

431070 $\frac{1}{2}$ m. d.
(8-8 OR 4m. 8d. taking
30 days to the month.

Practical Questions.

CLASS I.

1. A owes B 205*l.* to be paid in 2 years, and 200*l.* to be paid in 1 year; required the equated time?

Answer, 1 year, 6 months and $\frac{30}{405}$ of a month.

2. B owes C 800*l.* whereof 200*l.* is to be paid in 3 months, 100*l.* at 4 months, 300*l.* at 5 months, and 200*l.* at 6 months; required the equated time?

Answer, 4 months 18 days.

3. C

3. C owes D £.12-6-8 to be paid in 10 months, £.36-11-4 to be paid in 15 months, and £.72-6-8 to be paid in 18 months; required the equated time?
Answer, 16 months 8 days.

CLASS II.

1. A owes B £.6-4-8 to be paid in 18 days, £.12-6-10 to be paid in 32 days, £.7-6-2 to be paid in 40 days, and £.8-4-9 to be paid in 50 days, required the equated time? Answer $35\frac{1}{2}$ days.

2. X owes Y 16*l.* to be paid in $3\frac{1}{2}$ months, 20*l.* to be paid in $\frac{3}{4}$ of a month, 12*l.* to be paid in $6\frac{1}{4}$ months, and 50*l.* to be paid in $2\frac{3}{4}$ months; required the equated time? Answer $2\frac{1}{4}$ ($\frac{5}{8}$) months.

CLASS III.

A owes B 240*l.* to be paid in 6 months, but in $1\frac{1}{2}$ months pays him 60*l.* and in $4\frac{1}{2}$ months after that 80*l.* more, required how much longer than 6 months B should in equity defer the rest?

Answer $3\frac{9}{10}$ months.

E X C H A N G E.

Definitions.

“*UNDER this head those questions are solved, which relate to the receiving money in one country for the same value paid in another.*”

* *Par of exchange (as 'tis called) is the intrinsic value of foreign money compared with sterling.*

† *Course of exchange is an occasional fluctuating value, used instead of the intrinsic one (or par) owing to its being plentiful, scarce, and other circumstances of trade and nations.”*

Rule.

“*On examination this will be found to be done by an obvious application of the rule of three, and the 9th. case of practice.*”

This

* This is generally some piece of coin expressed in pence of ours, except in part of the Low Countries, with whom the exchange is in pounds sterling.

† That country 'tis obvious, in whose money the course of exchange is reckoned, has always a greater advantage the lower it runs.

This subject of exchange is very * extensive, but as the method of solution is pretty general, it will be sufficient to illustrate it in the chief places, with which England exchanges, as follows, under each of which are given the common tables of currencies, the par of exchange, limits of the course, and other necessary requisites and observations.

I. + LONDON *with* AMSTERDAM *and* all HOLLAND.

"At Amsterdam and Rotterdam (the chief places of trade and exchange in Holland) they keep their accounts generally in guilders, stivers, and penningens, and exchange with London in pounds † Flemish (which are subdivided in the same manner as pounds sterling)."

The following "*table*" contains the chief money of exchange in these parts.

8 penningens - - - - -	} make one	groot, penny (or denier de
2 groots (or 16 penningens)		stiver.
6 stivers (or 12 pence) - -		schilling.
20 stivers - - - - -		guilder (or florin).
2½ guilders - - - - -		rix-dollar.
6 guilders (or 20 schillings)		pound Flemish.

“ In Holland there are two sorts of money called bank and current. The bank is paid into the Bank (at Amsterdam) but the current passes among the inhabitants, and is of a worse kind, tho’ still divided in the same manner. Hence as the exchange with England is negotiated in bank money, and perhaps received, or equated with some other part in current, it will often happen in the course of business, that these must be changed one into another. Now the difference of these is called the agio, and is reckoned at so much per cent: Hence the two following rules for this purpose abovementioned easily occur.

1. To turn current money into bank (or banco, as 'tis mostly called).

Rule.—As 100 (any thing) with its agio added to it, is to 100, so is any given sum current, to its value in banco.

- ## 2. To

* So extensive, that it would of itself require a volume to take in all the particulars that belong to it. But those who are desirous of being farther acquainted with these calculations, the nature of transacting *bilis*, adjusting *arbitrations*, and other things of the same kind, may meet with as much satisfaction in *Postlethway's Dictionary of trade and commerce*, the *British Negotiator*, or the *Negotiator's Magazine*, as can be had from books alone.

† Or England in general.

† The pound Flemish, like the pound sterling, is imaginary, as are most moneys of exchange and account, equated in these negotiations.

2. To turn banco money into current.

Rule.—*As 100 is to the agio, so is the banco, to a certain sum, which added to it gives the current.*

Par is 37 $\frac{1}{2}$ schillings Flemish for 1 pound sterling, course of exchange from 33s. 4d. to 37s. 6d. and agio from 3 to 5 per cent.

Examples.

1. In £.754-10 sterling how many pounds Flemish, exchange at 33s. 6d?

This may be answered by the rule of three, or rather by the 9th. case of practice.

10s. is of 1l. the	$\frac{1}{2}$	£.	754	10	at 33s. 6d. per £.
2s. 6d. is of 10s. the	$\frac{1}{4}$		377	5	
1s. is of 10s. the	$\frac{1}{10}$		94	6	3
			37	14	6
			£.1263 15 9 Answer.		

2. In £.1263-15-9 Flemish how many pounds sterling, exchange as above?

If 33s. 6d. become 1l. what will £.1263-15-9 become?

12	20
402	25275
	12

402) 303309 (£.754-10 Answer.
 6c.

3. In £.527-15 sterling how many guilders, exchange at 35s?

$$\begin{array}{r|l}
 \begin{array}{l} 10s. \\ 5s. \end{array} & \begin{array}{l} 1 \\ 2 \\ 1 \\ 2 \end{array} \\
 \hline
 & \begin{array}{l} \text{£.} \quad s. \quad d. \\ 527 \quad 15 \\ 263 \quad 17 \quad 6 \\ 131 \quad 18 \quad 9 \end{array}
 \end{array}$$

$\text{£.}923 \quad 11 \quad 3$ Flemish.
 $\quad \quad \quad 6$ guilders in a pound Flemish.

$5538 \quad ft. \quad pe.$
 $\quad \quad 3 \quad 6 \quad \text{in } 11 \text{ shilling.}$
 $\quad \quad \quad 1 \quad 8 \quad \text{in } 3 \text{ pence.}$

Guilders $5541 \quad 7 \quad 8$ Answer.

OR, the latter part thus:

$$\begin{array}{r}
 \text{£.}923-11-3 \\
 \underline{20}
 \end{array}$$

$$\begin{array}{r}
 18471 \\
 \underline{12}
 \end{array}$$

$$40) 2216515$$

Guilders $5541 \quad 15$ pence or $7ft. 8pe.$ as before.

4. In $5541gu. 7ft. 8pe.$ how many pounds sterling, exchange as above?

$$\begin{array}{r}
 gu. \quad ft. \quad pe. \\
 6) 5541 \quad 7 \quad 8
 \end{array}$$

$\text{£.}923 \quad 3$ guilders remain or 120 pence, this with 15 pence in $7ft. 8pe.$ is 135 pence, or 11s. 3d. hence the guilders &c. are $\text{£.}923-11-3$ Flemish.

OR thus for the £s. Flemish.

$5541gu. 7ft. 8pe.$
 Mult. by 40 and take in $15d.$ for the $7ft. 8pe.$

$$12) 221655 \text{ pence.}$$

$$20) 18471 \quad 3$$

$\text{£.}923-11-3$ Flemish as before..

Z

(Then)

(Then) If 35*s.* become 1*l.* sterling what will £.923-11-8 become?

This solved gives £.527-15 the answer.

5. In £.50-15-4 sterling how many guilders current, exchange 34*s.* 6*d.* and agio $3\frac{1}{3}$?

		£.	s.	d.	
10 <i>s.</i>	$\frac{1}{2}$	50	15	4	
4 <i>s.</i>	$\frac{1}{5}$	25	7	8	
6 <i>d.</i>	$\frac{1}{8}$	10	3	$\frac{3}{4}$	
		1	5	$4\frac{1}{4}$	
		87	11	$5\frac{1}{4}$	Flemish.
		6			

Guilders 525 9 taking in 11*s.* $5\frac{1}{4}$ *d.* as 3
guilders 9 fivres.

(Next) If 100 guilders gain $3\frac{1}{3}$ guilders what will $\begin{smallmatrix} gu. & ft. \\ 525 & 9 \end{smallmatrix}$ gain?
 $3\frac{1}{3}$

gu.	ft.	pe.	
525	9	—	banco.
17	10	4	agio.
Answer.	542	19	4 current.

1576	7
175	3
17	51 10
	20
	—
10	30
	16
	—
4	80

6. In 542*gu.* 19*ft.* 4*pe.* current how many pounds sterling, exchange, and agio as in the last?

If 103*gu.* 6*ft.* 10*pe.* (i.e. $103\frac{1}{3}$ *gu.*) become 100*gu.* what will 542*gu.* 19*ft.* 4*pe.* become?

103 <i>gu.</i> 6 <i>ft.</i> 10 <i>pe.</i>
20
2066
16 penningens in a stiver.
33066

542 <i>gu.</i> 19 <i>ft.</i> 4 <i>pe.</i>
20
10859
16
33066) 17374800 (525 <i>gu.</i> 9 <i>ft.</i> banco.
£ <i>s.</i>

6) 525*gu.*

gu. ft.
6) 525 9

£.87 3 guilders remain, or 120 pence, which with 18*d.* the pence in 9 stivers, is 138 pence or 11*s.* 6*d.* therefore we have £.87-11-6 Flemish in all.

(Then) If 34*s.* 6*d.* become 1 pound sterling what will £.87-11-6 become?

This solved gives £.50-15-4 the answer.

Practical Questions.

CLASS I.

1. If I pay in London 492*l.* sterling, what may I draw my bill for to Amsterdam, exchange at 34*s.* 5*d.*? Answer £.846-13 Flemish.
2. If I pay at Amsterdam £.2057-1-8 Flemish, what sterling money shall I receive in London, exchange at 33*s.* 4*d.*? Answer £.1234-5 sterling.
3. Change 749*gu.* 15*ft.* current money into banco, agio $4\frac{3}{8}$ per cent? Answer 718*gu.* 6*ft.* 7*pe.*
4. London being to remit to Amsterdam £.413-18-11, 'tis required to know how much current money that will come to, the exchange being 35*s.* 2*d.* and agio $4\frac{1}{2}$? Answer 4563*gu.* 12*ft.* 14*pe.*

CLASS II.

1. Remitted from Lancaster to Amsterdam a bill of exchange of £.285-10 sterling, exchange at 33*s.* 9*d.* how many guilders must the bill be drawn for? Answer 2890*gu.* 13*ft.* 12*pe.*
2. A merchant at Rotterdam remits a bill of exchange of 7621*gu.* 7*ft.* to be paid in London; how much sterling money must the said bill be drawn for, exchange at 33*s.* 4*d.*? Answer £.762-2-8.
3. A merchant delivered at London 120*l.* sterling to receive 200*l.* Flemish in Amsterdam; what was the exchange? Answer £.1-13-4.

CLASS III.

1. London remits to Holland at 35*s.* 6*d.* exchange: In one month afterwards London draws upon Holland at 34*s.* 6*d.* The question is what does London make per cent by this negotiation? Answer £.2-17-11 $\frac{1}{2}$.
2. A merchant at Amsterdam owes another in London £.623-10-6 sterling, what will the bill amount to both in banco and current, exchange at 37*s.* and the agio at $3\frac{1}{2}$? Answer 6921*gu.* 2*ft.* 8*pe.* banco, and 7163*gu.* 7*ft.* 4*pe.* current.

II. L O N D O N with H A M B U R G H.

“ At Hamburg (the chief place of exchange in all Germany) they keep their accounts generally in marks, schillings lub, and pennings lub, but exchange with London in pounds, shillings, and pence Flemish.”

“ Table” of their chief moneys of exchange.

$\left\{ \begin{array}{l} 12 \text{ pennings (deniers or} \\ \text{pence) lub (also 2 groots} \\ \text{or pence Flemish) - - -} \\ 6 \text{ schill. lub (or 12 pen. Flem.)} \\ 16 \text{ schill. lub (or stivers Flem.)} \\ 2 \text{ marks - - - - -} \\ 3 \text{ marks (or } 1\frac{1}{2} \text{ drittle) - -} \\ 20 \text{ schill. Flemish, or } 7\frac{1}{2} \\ \text{marks or } 2\frac{1}{2} \text{ rix-dollars} \end{array} \right\}$	} make one	$\left\{ \begin{array}{l} \text{schilling lub (or stiver} \\ \text{Flemish).} \\ \text{schilling Flemish.} \\ \text{mark (or } 2s. 8d. \text{ Flemish).} \\ \text{drittle (or sletch dollar).} \\ \text{rix-dollar (or } 8s. \text{ Flemish).} \\ \text{pound Flemish.} \end{array} \right\}$

“ They have a bank here as at Amsterdam, and also bank and current monies.”

Par $35\frac{6}{13}s.$ course of exchange from $33s. 4d.$ to $38s.$ and agio from 30 to 40 per cent.

Examples.

1. What sterling money will a bill of 6271 Hamburg marks amount to, exchange at $34s. 6d$?

If $34s. 6d.$ give $1l.$ sterling what will 6271 marks give?

$$\begin{array}{r} 12 \\ \hline 414 \\ \hline \end{array}$$

32 pence in $2s. 8d.$ Flemish.

$$\begin{array}{r} 12542 \\ 18813 \\ \hline \end{array}$$

414) 200672 (£.484-14-3 Answer.
£c.

2. In £.362-10 sterling how many Hamburg marks, exchange at $35s$?

$$\begin{array}{r|l}
 \begin{array}{l} 10s. \\ 5s. \end{array} & \begin{array}{l} \frac{1}{2} \\ \frac{1}{2} \end{array} \\
 \hline
 \begin{array}{l} \pounds. \\ s. \end{array} & \begin{array}{l} 362 \\ 181 \\ 90 \end{array} \begin{array}{l} s. \\ 10 \\ 5 \\ 12 \end{array} \begin{array}{l} \text{at } 35 \\ \\ 6 \end{array} \\
 \hline
 \pounds.634 & 7 \ 6 \text{ Flemish.} \\
 20 &
 \end{array}$$

$$\begin{array}{r}
 12687 \\
 \text{Mult. by } 6 \text{ the schill. lub in a shill. Flemish,} \\
 \text{and take in 3 for 6d. Flemish.} \\
 16) 76125 \text{ schill. lub.} \\
 \hline
 4757m. 13sch. \text{ the answer.} \\
 \hline
 \end{array}$$

3. In 4757m. 13sch. how many pounds sterling, exchange as in the last?

$$\begin{array}{r}
 4757m. 13sch. \\
 16 \\
 \hline
 6) 76125 \text{ schill. lub.} \\
 \hline
 2|0) 1268|7 \quad 3 \text{ schill. remain or 6d. Flemish.} \\
 \hline
 \pounds.634-7-6 \text{ Flemish.} \\
 \hline
 \end{array}$$

(Then) If 35s. become 1l. what will $\pounds.634-7-6$ become?

This solved is $\pounds.362-10$, the answer.

4. In $\pounds.24-14-4\frac{1}{2}$ sterling how many marks current, exchange at 33s. 11d. and agio $19\frac{1}{2}$?

$$\pounds.24-14-4\frac{1}{2}$$

£. 24-14-4½ sterling will be found as before to be £. 41-18-4 Flemish.

$$\begin{array}{r} 20 \\ 838 \\ 6 \end{array}$$

If 100 *sch.* gain 19½ *sch.* what will 5030 *schillings* gain?

<i>sch.</i>	<i>ph.</i>	
5030	— banco.	45270
980	10 agio.	5030
		2515
16)6010	10 current.	
375 <i>m.</i> 10 <i>sch.</i> 10 <i>ph.</i> the answer.		980 85
		12
		10 20

5. In 375 *m.* 10 *sch.* 10 *ph.* current how many pounds sterling, exchange and agio as above?

If 119 *m.* 8 *sch.* become 100 *m.* what will 375 *m.* 10 *sch.* 10 *ph.* become?

$$\begin{array}{r} 16 \\ \hline 1912 \\ 12 \\ \hline 22944 \end{array}$$

$$\begin{array}{r} 16 \\ \hline 6010 \\ 12 \\ \hline \end{array}$$

22944) 7213000 (314 *m.* 6 *s.* banco nearly.
£c.

(Again) If 33 *s.* 11 *d.* become 1 *l.* what will 314 *m.* 6 *s.* become?

$$\begin{array}{r} 12 \\ \hline 407 \end{array}$$

$$\begin{array}{r} 32 \\ \hline 407) 10060 \text{ (pence Flemish or)} \\ \hline \end{array}$$

£. 24-14-4½ the answer.

Practical Questions.

CLASS I.

1. London remits to Hamburg £. 724-18-6 sterling; what must be received in Hamburg for this remittance, exchange at 33 *s.* 4 *d.*?

Answer 9061 *m.* 9 *s.*

2. In £. 42-12-7 sterling how many marks, exchange at 34 *s.* 3 *d.*?

Answer 547 *m.* 8 *s.*

3. Ham-

3. Hamburg draws upon London for 8234*m.* 10*s.* at 33*s.* 10*d.* exchange; I would know what must be paid for this draught in London?
Answer £.649-0-8 $\frac{1}{4}$.

CLASS II.

1. Suppose Lancaster draws upon Hamburg for 8872*m.* 1*s.* 10*ph.* How much sterling money must be paid for that bill in Lancaster, supposing the exchange at 35*s.* 2*d.* Answer £.672-15-4 nearly.

2. Hamburg is indebted to London for the neat proceed of a parcel of East-India goods 8732 marks current; I would know how much sterling the same sum will amount to, the exchange taken at 34*s.* 5*d.* and the agio 33 $\frac{1}{3}$? Answer £.507-8-6 $\frac{3}{4}$.

CLASS III.

Bought goods in Hamburg to the value of 36845 rix-dollars current, for how much sterling money ought I to sell them to gain 10 per cent, exchange being supposed 33*s.* 6*d.* and agio 20?
Answer £.8065-11-5.

III. LONDON with FRANCE.

"Throughout the dominion of France they keep their accounts in livres, sols, and deniers * Tournois, and exchange with London by the ecu or crown."

Table.

$\left\{ \begin{array}{l} 12 \text{ deniers} \\ 20 \text{ sols} \\ 3 \text{ livres} \\ 10 \text{ livres} \\ 24 \text{ livres} \end{array} \right.$	$\left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \text{make one}$	$\left\{ \begin{array}{l} \text{sol (or 2 dardenes or 4 lairds).} \\ \text{livre (or franc).} \\ \text{ecu (or crown).} \\ \text{pistole.} \\ \text{Louis d'ore.} \end{array} \right.$
--	--	--

Par 29 $\frac{1}{4}$ *d.* (per ecu) and the course from 30 to 34 pence.

Examples.

1. In £.243-18 sterling how many livres Tournois, exchange at 30 pence?

If

* A term equivalent to sterling with us.

If 30 pence give 3 livres what will £. 243-18 give?

$$\begin{array}{r}
 20 \\
 \hline
 4878 \\
 12 \\
 \hline
 58536 \\
 3 \\
 \hline
 3|0) 17560|8 \\
 \hline
 5853 \text{ liv. } 12 \text{ sols Answer.}
 \end{array}$$

2. In 5853 *liv.* 12s. Tournois how many pounds sterling, exchange as above?

If 3 *livres* give 30 pence what will 5853 *liv.* 12s. give?

$$\begin{array}{r}
 20 \\
 \hline
 60 \\
 2) 117072 \\
 \hline
 12) 58536 \\
 \hline
 2|0) 487|8
 \end{array}$$

OR better thus by practices

$$\begin{array}{r}
 2s. 6d. \text{ is } \frac{1}{8}) 5853 \quad 12 \\
 \hline
 3) 731 \quad 14 \\
 \hline
 \text{£. } 243 \quad 18 \text{ Answer.}
 \end{array}$$

£. 243-18 sterling the answer.

Practical Questions.

CLASS I.

1. In £. 164-14-6 sterling how many French crowns, exchange at $31\frac{1}{2}$? Answer 1255 *cr.* *oli.* 2s. 10d.
2. In 5427 *li.* 12s. 9d. Tournois how many pounds sterling, exchange at $32\frac{5}{8}$? Answer £. 245-18-9½.
3. Paris remits to London 4739 *liv.* 10s. 6d. and would know how much sterling money the said remittance will amount to in London at 33 pence exchange? Answer £. 217-4-6½.

CLASS II.

1. Lancaster remits to Lions £. 568-10 sterling, at $32\frac{1}{4}$ exchange: How much will that amount to in Lions? Answer 12692 *li.* 1s. 10d.

2. Sup-

2. Suppose Dunkirk remits to London 765 *liv.* 12*s.* how much sterling money will that make, taking the exchange at 22 *liv.* 10*s.* per £. sterling? Answer £. 340-4-11½.

CLASS III.

Bought a horse in Paris for 46 pistoles, how much sterling did he stand me in, exchange at 31½ *d*? Answer £. 20-2-6.

IV. LONDON with SPAIN.

"Nothing very general can be said of the exchange with Spain: However we shall instance in a few of their principal towns, as Madrid, Cadiz, Bilboa, Malaga, Seville, &c. where they keep their accounts in piastras, rials, and maravedies. They exchange with London by the piastra of old plate, which is the case also of all their places of trade in the Streights, Mediterranean, Africa, and the West-Indies."

Table.

$\left\{ \begin{array}{l} 4 \text{ maravedies vellon, or} \\ 2\frac{1}{8} \text{ maravedies plate} \\ 8\frac{1}{2} \text{ quartos, or 34 mara-} \\ \text{vedies vellon} \\ 16 \text{ quartos, or 34 mara-} \\ \text{vedies plate} \\ 8 \text{ rials of old, or 10 of} \\ \text{new plate} \\ 5 \text{ old or 4 new plate piastras} \end{array} \right\}$	} make one	$\left\{ \begin{array}{l} \text{quarto vellon.} \\ \text{rial vellon.} \\ \text{rial plate (or old exchange} \\ \text{plate).} \\ \text{piastra, piso, or piece of} \\ \text{eight.} \\ \text{pistole.} \end{array} \right\}$

"It appears from the above table, that the two sorts of rials (*i. e.* vellon and plate) are to one another in value as 32 to 17, and also that the old plate is 25 per cent better than the new, tho' the piastra of exchange is of equal value in all."

Par is 43 pence (per piastra), and the course from 35 to 45 pence.

Examples.

1. In £. 456-15-6 sterling how many piastras at 40 pence exchange?

A a

If

If 40 pence give 1 piaftre, what will £.456-15-6 give?

$$\begin{array}{r}
 20 \\
 \hline
 9135 \\
 12 \\
 \hline
 410 \overline{) 10962} 6 \\
 \hline
 \text{Piaftres } 2740 \quad 26 \text{ remains.} \\
 8 \\
 \hline
 2018 \\
 \hline
 \text{Rials } 5 \quad 8 \text{ remains.} \\
 34 \\
 \hline
 2712 \\
 \hline
 7 \text{ maravedies, nearly.}
 \end{array}$$

2. In 2740*p.* 5*r.* 7*m.* how many pounds sterling, exchange as above?

If 1 piaftre give 40 pence what will 2740*p.* 5*r.* 7*m.* give?

This solved is £.456-15-6 the answer.

Practical Questions.

CLASS I.

1. In 1748 rials how many pounds sterling, exchange at 43*d*?
Answer £.39-2-11½.
2. How many piaftres at 36*d.* exchange will answer a bill of £.594-6 sterling?
Answer 3962.
3. If I pay here 2500*l.* what Spanish money may I draw my bill for payable at Madrid, exchange being supposed at 38*d*?
Answer 15789*p.* 3*r.* 27*m.*

CLASS II.

1. Cadiz is indebted to Lancaster 1591*p.* 5*r.* 17*m.* exchange at 45*d.* The question is, how much sterling the said money will amount to?
Answer £.298-8-9½.
2. In 3264 rials vellon, how many rials plate? Answer 1734

CLASS III.

Spain remits to London 342*p.* 5*r.* at 40*d.* exchange, what will the remittance amount to, and how many rials vellon would be equivalent?
Answer £.57-2-1, and 5159*r.* 18*m.* vellon.
V. LON.

V. LONDON with PORTUGAL.

"At Lisbon and Oporto (the chief places of exchange in Portugal) they keep their accounts in millreas and reas, and exchange with London by the millrea."

Table.

20 reas	-----	} make one	vintin.	}
100 reas (or 5 vintins)	--		testoon.	
400 reas (or 4 testoons)	-		crufado.	
1000 reas (or 2½ crufados)	-		millrea.	
4800 reas (or 48 testoons)	-		moidore.	
6400 reas (or 64 testoons)	-		Joanese.	

Par 5s. 7½d. (per millrea), and the course of exchange from 5s. 3d. to 5s. 8d.

Examples.

1. In £.680-7-8¼ how many millreas, exchange at 5s. 6d?

If 5s. 6d. give 1 millrea, what will £.680-7-8¼ give?

This solved makes 2474m.r. 125r. the answer.

2. In 2474m.r. 125r. how many pounds sterling, exchange as above?

Calling millreas pounds, and reas farthings a piece dropping 1 for every 25 (as 960 farthings make a pound) we have the following practice question.

		£.	s.	d.	
5s.	¼	2474	2	6	at 5s. 6d.
6d.	⅙	618	10	7½	
		61	17	¾	
		£.680	7	8¼	the answer.

Practical Questions.

CLASS I.

1. If I have a bill for 1764m.r. 350r. exchange at 5s. 6d. how much sterling money must be received for the same?

Answer £.485-3-11.
2. If

A a 2.

2. If I pay a bill at London of £.1193-17-6 $\frac{1}{2}$, what must I draw for on my correspondent at Lisbon, exchange at 5*s.* 5 $\frac{5}{8}$ *d*?

Answer 4366*m.r.* 183*r.*

3. London remits to Portugal £.387-7-6 sterling, required what this remittance will come to at 5*s.* 3 $\frac{1}{2}$ *d.* exchange?

Answer 1475*m.r.* 714*r.*

CLASS II.

1. Lancaster remits to Lisbon £.436-16-10 $\frac{1}{2}$ sterling; exchange 5*s.* 3 $\frac{3}{8}$ *d.* How much Portugal money will it amount to?

Answer 1654*m.r.* 320*r.*

2. What sum must be received in London for 1728*m.r.* 144*r.* exchange being at 5*s.* 4 $\frac{1}{2}$ *d*?

Answer £.464-8-9 $\frac{1}{4}$

CLASS III.

In 214 cruzados 350 reas, how many pounds sterling, exchange at 5*s.* 6 $\frac{7}{8}$?

Answer £.59-14-6 $\frac{1}{2}$.

VI. L O N D O N with I T A L Y.

“ Nothing general can be said under this head, as the coins and current species differ in almost every city, &c. therefore an instance in a chief place or two must suffice.

In Venice, Genoa, and Leghorn they keep their accounts generally in ducats (*banco*), pezzos, and dollars, and exchange with London by these respectively, supposing each piece divided into 20 parts, called *foldi* (or *sols*) and these *foldi* again into 12 parts, called *denari*, in a manner similar to our pound sterling. Sometimes indeed accounts are kept, and the exchange negotiated in * Venice in ducats and *grossi*, whereof 24 make a ducat. Also in each of these places books are sometimes kept in *lire*, *foldi*, and *denari*, in which the *lire*, *foldi*, and *denari* bear the same proportion one to another, that pounds, shillings, and pence do with us. Now as by the following table we see that the ducat, pezzo, and dollar contain each 6 $\frac{1}{2}$, 5 $\frac{3}{4}$, and 6 *lire* respectively, 'tis obvious every subdivision in the scale of descent in the first method of keeping accounts is 6 $\frac{1}{2}$, 5 $\frac{3}{4}$, and 6 times the value of the corresponding one in the latter method, which particular ought to be carefully attended to.”

Table.

* When Venice does not exchange in this latter method, the *foldi*, and *denari* of the ducat are called *sols*, and *deniers d'or*, to distinguish them perhaps from the other sort in the table.

Table.

12 denari - - - - -	} make one	foldi.	}
20 foldi - - - - -		lire.	
5 $\frac{1}{6}$ foldi - - - - -		grossi.	
24 grossi, 6 $\frac{1}{2}$ lires, or 124 foldi.		Venice ducat.	
5 $\frac{3}{4}$ lires, or 115 foldi - - -		Genoa pezzo (or piaftre).	
6 lires, or 120 foldi - - - -		Leghorn dollar (or piaftre).	

"In Venice they have two banks, and three sorts of money. One bank deals in the best sort of money called banco, and the other in a middling sort called banco current. The remaining kind of money is termed picoli, and in this goods and merchandizes are generally computed. The agio of the bank current is fixed at 20 per cent in favour of the bank, and the agio of the picoli is about 39 or 40 per cent in favour of the bank, or 19 or 20 in favour of the bank current."

	d.		d.	d.
Par. { Venice ducat	52 $\frac{1}{4}$	}, Course of exchange from	{ 45 to 54	
{ Genoa piaftre	54		{ 47 to 58	
{ Leghorn dollar	54		{ 47 to 58	

Examples.

1. In 5214du. 12gr. banco how many pounds sterling, exchange at 54d?

Taking the grossi at 10d. per piece (on the supposition that 24 make a £.) we have

		£.	s.	
4s.	$\frac{1}{5}$	5214	10	at 4s. 6d.
6d.	$\frac{1}{8}$	1042	18	
		130	7 3	
		£. 1173	5 3	the answer.

2. In £. 1173-5-3 sterling how many Venice ducats, exchange as above?

If 54d. give 1 ducat what will £. 1173-5-3 give?

This solved makes 5214du. 12gr. the answer.

3. In

3. In 446 *pez.* 15 *so.* 10 *de.* of Genoa, how many pounds sterling, exchange at 56 *d*?

		<i>pez.</i>	<i>so.</i>	<i>de.</i>	
4 <i>s.</i>	$\frac{1}{3}$	446	15	10	at 4 <i>s.</i> 8 <i>d.</i>
6 <i>d.</i>	$\frac{1}{3}$	89	7	2	
2 <i>d.</i>	$\frac{1}{3}$	11	3	4 $\frac{1}{2}$	
		3	14	5 $\frac{1}{2}$	
		£. 104	5	$-\frac{1}{4}$	the answer.

4. In £. 104-5-0 $\frac{1}{4}$ sterling how many Genoa pezzos, exchange as in the last?

As 56 *d.* is to 1 pezzo so is £. 104-5-0 $\frac{1}{4}$ to 446 *pez.* 15 *so.* 10 *de.* the answer.

Practical Questions.

CLASS I.

1. How much sterling money may a person receive in London, if he pays in Genoa 976 pezzos, exchange at 53 *d*? Answer £. 215-10-8.

2. A merchant remitted £. 234-15-6 sterling to Leghorn, how many dollars will he receive there, supposing the exchange at 54 $\frac{1}{2}$ *d*? Answer 1033 *do.* 17 *so.* 5 *de.*

3. Genoa is indebted to London for 8728 *pe.* 16 *so.* 8 *d.* of their money, I demand how much sterling it will amount to, exchange being at 45 *d*? Answer 1636-13-1 $\frac{1}{2}$.

CLASS II.

1. Lancaster remits to Venice £. 293-9-8 sterling, exchange at 52 *d.* What must be paid for this remittance at Venice? Answer 1354 *du.* 12 *gr.*

2. If in London I pay 211-11-0 $\frac{1}{4}$ sterling, how many pezzos may I receive in Genoa, exchange at 53 $\frac{1}{2}$ *d*? Answer 951 *do.* 4 *so.* 11 *d.*

CLASS III.

In 3878 *du.* 15 *gr.* bank current, agio 20 per cent, how many pounds sterling, exchange at 54 *d*? Answer £. 727-4-10 $\frac{1}{4}$.

VII. LON.

VII. LONDON with DUBLIN and the WEST-INDIES.

"In those places accounts are kept in the same manner as at London, yet the value of their money differs very much; especially in the latter part, where their currency is mostly paper, they discount great sums per cent for sterling money. In the West-India islands it generally runs betwixt 25 and 50 per cent discount, and in Ireland from about 6 to 15 per cent." —The par betwixt England and Ireland is 100*l.* English, for 108½ Irish.

Examples.

1. A gentleman remits to Ireland £. 575-15 sterling, what will he receive there, the exchange being at 10 per cent?

$$\begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ \frac{1}{10}) \begin{array}{r} 575 \quad 15 \\ 57 \quad 11 \quad 6 \end{array} \\ \hline 633 \quad 6 \quad 6 \text{ the answer.} \end{array}$$

2. The West-Indies draw on London for £. 294-18-6 sterling, how much West-India money will the draught amount to, exchange at 47 per cent?

As 100*l.* is to 147*l.* so is £. 294-18-6 to £. 433-10-9½ the answer.

Practical Questions.

CLASS I.

1. If I remit £. 618-13-8 sterling from London to Dublin; what is the sum to be paid in Dublin, exchange at 8½ per cent?

Answer £. 669-14-5¼.

2. In 549*l.* Jamaica currency how many pounds sterling, discount 40 per cent?

Answer £. 392-2-10¼.

CLASS II.

What must be paid in Lancaster for a remittance of £. 633-6-6 Irish, exchange at 10 per cent?

Answer £. 575-15.

CLASS III.

An Officer in New-York has £. 150-7-6 sterling due to him, how much New-York currency must he receive, exchange at 71¾ per cent?

Answer £. 257-15-8½.

Scholium.

Scholium.

Thus much may serve for a tolerable specimen of exchange, in which subject real experience it seems must alone make an artist, as all the authors we have turned over for information, differ from one another in their accounts of things (and even often contradict themselves). This, with a perplexed indigested manner of delivering the materials, has rendered it a subject not likely to yield much satisfaction to those, who have no other means of becoming acquainted with it, than the assistance of books.

Before we put an end to this article we must deliver the rule called

*“Conjoined Proportion.”**Definition.*

“THIS is when the proportion betwixt one species and another (either of monies, weights, or measures) is given; betwixt this other and a third; betwixt this third and a fourth, and so on, to find

1st. How much of the first sort is equal to a given quantity of the last, and

2d. How much of the last is equal to a given quantity of the first.”

For the solution of both which cases we have the following general rule.

** General rule.*

“Set down the terms compared with some mark or distinction betwixt them (as this =) one under another in order as they occur, and the odd one at last under that perpendicular column where no other term of its name is found. Then multiply continually for a dividend that perpendicular column which contains the odd term, and the other for a divisor, and the quotient is the answer.”

Examples.

** These questions are all naturally solved by the rule of three (using as many statings save one, as there are different species compared), and this method only directs to multiply the same terms, &c. that would fall out in the course of these operations. This will be manifest by solving the example that follows, by different statings: Thus*

- 1. If 23lb. make 20lb. what will 155lb. make?—Suppose Alb.*
- 2. If 180lb. make Alb. what will 72lb. make?*

Now, from the first stating, 155 is to be multiplied by 20, and, neglecting the division by 23, its product, from the 2d. is multiplied again by 72: But as the division by 23 was omitted, these three will give a product 23 times too big to produce a right answer, when divided by 180 (the divisor in the second stating) therefore 180 must be made 23 times more than it is, to bring out a right result. Now this reasoning, agreeing with what the rule directs, proves its truth, nor can it fail in any other question of the like nature.

Example.

1. If 20*lb.* at London make 23*lb.* at Antwerp, and 155*lb.* at Antwerp make 180*lb.* at Leghorn, how many pounds at London are equal to 72*lb.* at Leghorn?

20	London 20 = 23 Antwerp.	23
155	Antwerp 155 = 180 Leghorn.	180
	Leghorn 72 =	
3100		4140
72		
	4140) 223200 (53 $\frac{378}{414}$ the answer.	
223200	<i>Ans.</i>	

2. If 40*lb.* at London make 36*lb.* at Amsterdam, and 90*lb.* at Amsterdam make 116*lb.* at Dantzick, how many pounds at Dantzick are equal to 122*lb.* at London?

36	London 40 = 36 Amsterdam.	90
116	Amsterdam 90 = 116 Dantzick.	40
	= 122 London.	
4176		3600
122		
	3600) 509472 (141 $\frac{1872}{3600}$.	
509472	<i>Ans.</i>	

Practical Questions.

CLASS I.

1. If 140 braces at Venice are equal to 156 braces at Leghorn, and 7 braces at Leghorn equal to 4 ells English, how many braces at Venice are equal to 16 ells English? *Answer* 25 $\frac{80}{814}$.

2. If 35 ells at Vienna make 24 aulns of Lyons; and 3 aulns of Lyons, 5 ells at Antwerp; and 4 ells at Antwerp 5 ells at Francfort: How many ells of Vienna are equal to 700 ells at Francfort? *Answer* 490.

3. If 6*lb.* of sugar be equal in value to 7*lb.* of raisins; 5*lb.* of raisins to 2*lb.* of almonds; 3*lb.* of almonds to 5*lb.* of currants; 2*lb.* of currants to 18*d.* How many pence are the value of 3*lb.* of sugar? *Answer* 21 pence.

CLASS II.

1. If 3 dozen pair of gloves be equal in value to 2 pieces of ribbon; 3 pieces of ribbon to 7 dozen of points; 6 dozen of points to 2 yards of Flanders lace; and 3 yards of Flanders lace to 81 shillings; how many dozen pair of gloves may be bought for 28 shillings? *Answer* 2 dozen.

B b

2. If

2. If 10*lb.* at London be equal to 9*lb.* at Amsterdam; 45*lb.* at Amsterdam to 49*lb.* at Bruges; and 98*lb.* at Bruges equal to 116*lb.* at Dantzick; how many *lb.* at Dantzick are equal to 112*lb.* at London?

Answer 129 $\frac{22}{100}$ *lb.*

CLASS III.

Suppose a bill for 500*l.* sterling negotiated as follows, *viz.* First from London to Amsterdam at 38*s.* Flemish for a *£.* sterling, then from Amsterdam to Francfort at 6*s.* Flemish for 66 cruitzers, and from Francfort to Paris at 54 cruitzers for a French crown: How many French crowns must be paid for the last negotiation?

Answer 3870 $\frac{10}{27}$ crowns.

The solution of these questions may often be greatly “*contracted*” thus,

* “If one and the same number be found in each of the perpendicular columns, it may be struck out of them both: Also, if any two terms in different columns can be divided by any other number, the quotients may be used in their stead. — ’Tis usual to make a dash through the terms as they are contracted (called cancelling, as below) in order to see the better how the work goes on, and to avoid mistakes, &c.”

Example.

$$\left. \begin{array}{l} 6 = 3 \\ 18 = 18 \\ 9 = 9 \\ 17 = \end{array} \right\} \text{Contrac. to} \left. \begin{array}{l} 6 = 3 \\ = 18 \\ 9 = \\ 17 = \end{array} \right\} \text{OR} \left. \begin{array}{l} 2 = 1 \\ = 7 \\ 1 = \\ 17 = \end{array} \right\} \text{OR} \left. \begin{array}{l} 1 = 1 \\ = 1 \\ 1 = \\ 17 = \end{array} \right\} \text{i. e. } 17 = 1.$$

Scholium.

By this last rule the business of arbitration of exchanges is effected; but as it would take up too much of our room to handle this particular to any advantage, we must again refer those, who may find it necessary to be farther acquainted with things of this nature, to the authors who have professedly treated upon them.—[See note page 167.]

ALLIGATION.

* The reason is obvious, as these operations will only lessen the divisor and dividend equally.

ALLIGATION.

Definition.

"UNDER this head those questions are solved, which relate to the mixing of several simples together."

It is generally divided into two kinds called *Alligation Medial*, and *Alligation Alternate*.

I. Alligation Medial.

Definition.

"This is when the prices and quantities of several simples are given to be mixed, to find the mean price of any part of the mixture."

* Rule.

"Say, by the rule of three, as the whole composition, is to its total value, so is any required part, to its mean value."

Examples.

1. A Grocer mixed 4 C. weight of sugar, at 56s. per C. 7 C. weight at 43s. per C. and 5 C. weight at 37s. per C. I demand the price of 2 C. weight of this mixture?

56s.	43s.	37s.	
4	7	5	4
224	301	185	5
301			
185			16 Total sum.
710 Total value.			

As 16C. is to 710s. so is 2C. to the price required.

$$\begin{array}{r}
 8) 710 \\
 \underline{00} \\
 2|0) 8|8 \quad 9 \\
 \underline{00} \\
 \text{£.4-8-9 Answer.} \\
 \underline{00} \\
 \text{Bb 2}
 \end{array}$$

2. 55½

* This application of the rule of three (like many others already gone past) explains itself.

2. $55\frac{1}{2}$ gallons of Canary, $48\frac{3}{4}$ of white, and $50\frac{1}{4}$ gallons of Rhenish wine are to be mixed together, I demand what one quart of this mixture is worth, supposing the Canary at $2s. 8\frac{1}{2}d.$ the quart, white at $1s. 7\frac{1}{2}d.$ and the Rhenish at $2s. 2d.$ per quart?

$2s. 8\frac{1}{2}d.$	$1s. 7\frac{1}{2}d.$	$2s. 2d.$	$55\frac{1}{2}$	$48\frac{3}{4}$	$50\frac{1}{4}$
<u>12</u>	<u>12</u>	<u>12</u>	<u>4</u>	<u>4</u>	<u>4</u>
32	19	26	222	195	201
4	4	4	130	78	104
<u>130</u>	<u>78</u>	<u>104</u>	5660	1560	804
			222	1365	2010
			<u>28860</u>	<u>15210</u>	<u>20904</u>
222					28860
195					15210
201					
<u>618</u>					<u>64974</u>

If 618qts. cost 64974far. what will 1 cost?

$$618) 64974 (4) 105 \frac{84}{618}$$

$$12) 26 \frac{1}{4}$$

$2s. 2\frac{1}{4}d. \frac{84}{618}$ the answer.

Practical Questions.

CLASS I.

1. A druggist has 27lb. of large cloves, at 6s. a pound, 15lb. of a middling sort at 2s. 6d. a pound, and 10lb. of a coarser sort at 2s. 2d. a pound: How may a mixture of these be sold by the pound?

Answer 4s. $3\frac{2}{3}\frac{1}{2}d.$

2. A tobacconist would mix 20lb. of tobacco at 9d. the pound with 60lb. at 12d. 40lb. at 18d. and 12lb. at 2s. the pound. The question is what a pound of this mixture is worth?

Answer 1s. $2\frac{1}{4}d. \frac{80}{132}$

3. If with 40 bushels of corn at 4s. per bushel, there are mixed 10 bushels, at 6s. the bushel, 30 bushels at 5s. the bushel, and 20 bushels at 3s. the bushel; what will 10 bushels of that mixture be worth?

Answer £. 2-3.

CLASS II.

1. Of three sorts of rich perfumes suppose one to be worth 2s. 6d. a dram; the second 3s. 9d. and the third at 6s. 3d. a dram. What will

will the value of a dram of the perfume be, compounded of 10 drams of the first sort, 8 drams of the second, and 12 of the third?

Answer 4s. 4d.

2. $37\frac{1}{2}$ C. weight of Sugar at £. 2-7-9 per C. $24\frac{1}{4}$ C. at £. 2-17-4, is to be mixed with $19\frac{3}{4}$ C. at £. 3-8 per C. Now I demand what one C. weight of this mixture will be worth? Answer £. 2-15-7 $\frac{1}{4}$.

CLASS III.

A wine merchant mixes 54 gallons of wine at 7s. a gallon; 35 gallons of another sort at 9s. the gallon; 26 gallons of cyder at 10d. per gallon, and 12 gallons of water together; 'tis required what one gallon of this mixture stood him in?

Answer 5s. 7 $\frac{5}{12}$ d.

II. Alligation Alternate.

Definition.

"This is the converse of the former, and discovers what quantities of several simples of such prices, must be mixed to make the composition of a given price."

* Rule.

"Having reduced (if need be) the given and required prices to one denomination; on the left hand side of a perpendicular line place the mean price, and on the right of it, one under another, the several prices given. Then with a crooked line join, or link, any two of the given prices together, that are one greater, and the other less than the mean, till they are all joined, or have got a yoke-fellow; remembering, that, if there be occasion, one term may be linked to as many as it will admit of according to the

* That this method of solution is true, will appear from a more attentive consideration of the question that follows, in which we may observe, that every pound at 5d. (taken into the mixture) will gain 2d. by the sale, and every pound at 11d. lose 4d. (i. e. the differences between the given prices respectively and the mean). Now as there is nothing either to be gained or lost by this new mean price on the supposition, and as taking pound for pound of each would manifestly lose 2d. pence every time, we must, to restore the balance, have more of that, which gains slowly, than of that which loses fast, and this disparity in the quantity must evidently be just the same, as the disparity of the gain and loss applied to different sides; that is, there must be twice the quantity used of that sort, which gains only half of what the other sort loses. Now this is what the method of changing the places of the differences (as directed) plainly effects: For 4 pound of that, which gains 2d. per lb. balances the loss of 2lb. of that which loses 4d. per lb. i. e. 8d. is thrown either way. This reasoning then is evidently conclusive in all cases where two simples are mixed; and since in cases, where there are more than two, a greater is always linked with a less, there must result the like balance from each couple, and consequently from the whole made up of these couples.—Hence the rule is true in general.

the above restriction, (as in the case, where there is but one greater or less than the mean). Next take the difference betwixt the mean, and all the given prices, and place these differences, not against the prices they come from, but against those that are at the other extremities of the crooked lines respectively, noting, that if there be one that is linked to two or more, it must have its difference taken as often, as it is linked, and placed against its yoke-fellow, as directed, and each of their differences brought to stand against it. Then are these differences (and if there be more than one against any term, the sum of them) the quantities required."

The perusal of the following examples will make the method plain.

Examples.

1. Required the quantities of two sorts of sugar, one at 5*d.* per *lb.* and the other at 11*d.* that will when mixed be worth 7*d.* a pound?

$$7d. \left\{ \begin{array}{c} 5 \\ 11 \end{array} \right\} \begin{array}{c} 4lb. \\ 2lb. \end{array} \quad \text{From this it appears there must be } 4lb. \text{ of the } 5 \text{ penny and } 2lb. \text{ of the } 11 \text{ penny sugar.}$$

2. Required what quantities of tobacco at 9*d.* 14*d.* and 15*d.* per *lb.* must be taken to make a mixture worth 12*d.* per *lb.*?

$$12d. \left\{ \begin{array}{c} 9 \\ 14 \\ 15 \end{array} \right\} \begin{array}{c} 3.2 \\ 3 \\ 2 \end{array} \parallel \begin{array}{c} 5lb. \text{ at } 9 \\ 3lb. \text{ at } 14 \\ 3lb. \text{ at } 15 \end{array} \quad \text{gives the answer.}$$

Scholium.

'Tis very obvious, that any set of numbers in the same proportion with these already found (i. e. others half as much as each, twice, thrice &c. as much as each) will answer the question: Hence on that score an innumerable number of answers may be obtained. But in cases where there are more than three terms, and two or more of these both bigger and less, than the mean term (as in the following) the manner of linking may be greatly varied, and every variation of this kind produces an answer different from the other, with regard to the value and order of the terms among themselves. Hence from each of these in the same manner may any number of other proportional answers be drawn, which adds greatly to the variety. And yet after all, this method does not give all the solutions these kinds of questions are capable of, nor perhaps those which might be the most convenient. But as the rule is perhaps of no great moment, we shall

shall not enlarge it with more artifices, which might be used to extend the number yet farther, &c. but refer the curious in these speculations to the * writers of algebra, by whom alone they can be handled to any purpose beyond what is already taught.

3. A merchant would mix wines at 14s. 19s. 15s. and 22s. per gallon, so as the mixture may be worth 18s. What quantity of each must be taken?

$$\begin{array}{c} 14 \\ 15 \\ 19 \\ 22 \end{array} \left\{ \begin{array}{c} 4 \\ 1 \\ 3 \\ 4 \end{array} \right\} \parallel \parallel$$

$$\text{OR } 18 \left\{ \begin{array}{c} 14 \\ 15 \\ 19 \\ 22 \end{array} \right\} \begin{array}{c} 1 \\ 4 \\ 4 \\ 3 \end{array} \parallel \parallel$$

$$\text{OR } 18 \left\{ \begin{array}{c} 14 \\ 15 \\ 19 \\ 22 \end{array} \right\} \begin{array}{c} 1.4 \\ 1 \\ 4.3 \\ 4 \end{array} \begin{array}{c} 5 \\ 1 \\ 7 \\ 4 \end{array} \parallel \parallel$$

$$\text{OR } 18 \left\{ \begin{array}{c} 14 \\ 15 \\ 19 \\ 22 \end{array} \right\} \begin{array}{c} 4 \\ 1.4 \\ 3 \\ 4.3 \end{array} \begin{array}{c} 4 \\ 5 \\ 3 \\ 7 \end{array} \parallel \parallel$$

$$\text{OR } 18 \left\{ \begin{array}{c} 14 \\ 15 \\ 19 \\ 22 \end{array} \right\} \begin{array}{c} 1.4 \\ 4 \\ 4 \\ 4.3 \end{array} \begin{array}{c} 5 \\ 4 \\ 4 \\ 7 \end{array} \parallel \parallel$$

$$\text{OR } 18 \left\{ \begin{array}{c} 14 \\ 15 \\ 19 \\ 22 \end{array} \right\} \begin{array}{c} 1 \\ 1.4 \\ 4.3 \\ 3 \end{array} \begin{array}{c} 1 \\ 5 \\ 7 \\ 3 \end{array} \parallel \parallel$$

$$\text{OR } 18 \left\{ \begin{array}{c} 14 \\ 15 \\ 19 \\ 22 \end{array} \right\} \begin{array}{c} 1.4 \\ 1.4 \\ 4.3 \\ 4.3 \end{array} \begin{array}{c} 5 \\ 5 \\ 7 \\ 7 \end{array} \parallel \parallel$$

The above seven answers are all that can possibly be got by variety of linking.

T o

* The knowing and ingenious Mr. B. Martin in particular, in a little work called *The Young Student's Memorial Book* &c. has laid down a few theorems, which carry the work to its last perfection.

Two more cases of this rule are generally mentioned by authors, which they call *Alternation Partial*, and *Alternation Total*, and as what they differ in from the above is only an obvious application of the rule of three, a bare instance in each might be sufficient; nevertheless to comply with custom we shall give the rules, &c. as usual.

Alternation Partial.

Definition.

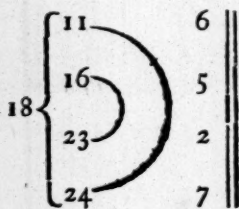
"This is, when in the mixture the quantity of one of the simples is limited."

** Rule.*

"Find an answer to the question by linking, as above: Then say (if the quantity, or difference falling against the given price be not accidentally right), as the quantity opposite the price of the simple limited, is to the quantity required, so are the rest of the differences severally, to the quantities to be taken on the consideration."

Examples.

1. I have a hoghead of wine worth 2s. a quart, which I would mix with three other sorts of 11d. 16d. and 23d. a quart respectively, what quantity of each must be mixed with the hoghead to make a quart worth 18 pence?



Then As 7 is to 63 so is 6 to 54 at 11 per gal.

As 7 is to 63 so is 5 to 45 at 16 Do.

As 7 is to 63 so is 2 to 18 at 33 Do.

Other answers might be had by different linking.

2. A grocer would mix teas of 12s. 6d. 11s. and 8s. per lb. with 25lb. at 15s. so as to make the mixture half a guinea per lb. required the quantity of each?

10½

* The rule is obvious as was said above, but sometimes a little labour may be spared in its application: For when the given quantity is only linked with one other, that other need only be altered in the same proportion as it, and the rest taken as they are, as is manifest.

$$\begin{array}{r|l}
 \begin{array}{l} 8 \\ 11 \\ 12\frac{1}{2} \\ 15 \end{array} & \begin{array}{l} 4\frac{1}{2} \\ 2 \\ \frac{1}{2} \\ 2\frac{1}{2} \end{array}
 \end{array}
 \left\| \begin{array}{l} \text{lb.} \quad \text{lb.} \quad \text{lb.} \quad \text{lb.} \\ \text{Then As } 2\frac{1}{2} \text{ is to } 25 \text{ so is } 4\frac{1}{2} \text{ to } 45 \text{ at } 8s. \text{ per lb.} \\ \text{As } 2\frac{1}{2} \text{ is to } 25 \text{ so is } 2 \text{ to } 20 \text{ at } 11s. \text{ do.} \\ \text{As } 2\frac{1}{2} \text{ is to } 25 \text{ so is } \frac{1}{2} \text{ to } 5 \text{ at } 12s. 6d. \text{ do.} \end{array}$$

These answers might still be varied by different linking.

Scholium.

It will readily appear, that if two or more quantities were limited, the question might be solved by this rule, by "first finding a mean price for the limited simples when mixed, and placing that in the question, instead of those it came from, and lastly altering all the other differences in the same proportion which that, falling against this mean price, bears to the sum of the quantities limited."—But this hint is only started for the reader to pursue or leave as he thinks proper.

Alternation Total.

Definition.

"This is when the total quantity of the mixture is limited."

Rule.

"Find an answer as before, by linking; then say, as the sum of the quantities (or differences) thus determined, is to the quantity given; so is each in particular, to the quantity to be taken of its price."

Examples.

1. How many gallons of water must be mixed with wine worth 3s. the gallon, so as to fill a vessel of 100 gallons, that a gallon may be afforded at 2s. 6d?

$$\begin{array}{r|l}
 \begin{array}{l} 36 \\ 30d. \left\{ \begin{array}{l} 36 \\ 0 \end{array} \right. \end{array} & \begin{array}{l} 30 \\ 6 \\ \hline 36 \end{array}
 \end{array}
 \left\| \begin{array}{l} \text{Then As } 36 \text{ is to } 100 \text{ so is } 30 \text{ to } 83\frac{1}{3} \text{ of wine.} \\ \text{As } 36 \text{ is to } 100 \text{ so is } 6 \text{ to } 16\frac{2}{3} \text{ of water.} \end{array}$$

C c.

2. A

2. A grocer has currants, at 4*d.* 6*d.* 9*d.* and 11*d.* per *lb.* and he would make a mixture of 240*lb.* so that it might be afforded at 8 pence: How much of each sort must he take?

	<i>lb.</i>	<i>lb.</i>	<i>lb.</i>	<i>lb.</i>	<i>d.</i>
8 {	4	3		Then As 10 is to 240	so is 3 to 72 at 4 per <i>lb.</i>
	6	1		As 10 is to 240	so is 1 to 24 at 6.
	9	2		As 10 is to 240	so is 2 to 48 at 9.
	11	4		As 10 is to 240	so is 4 to 96 at 11.
		10			

The questions under the last three rules may easily be proved, by having recourse to Alligation Medial, and actually trying if the solutions brought out will answer the given conditions.

Practical Questions.

CLASS I.

1. A grocer would mix three sorts of sugar together, *viz.* one sort at 10*d.* per *lb.* another sort at 7*d.* and another sort at 6*d.* How much of each sort must he take, that the whole may be worth 8*d.* the *lb.*?

Answer { 3*lb.* at 10*d.* per *lb.*
2*lb.* at 7*d.*
2*lb.* at 6*d.*

2. A druggist hath several sorts of tea, *viz.* one sort at 12*s.* per *lb.* another at 11*s.* a third at 9*s.* and a fourth at 8*s.* per *lb.* I demand how much of each sort he must mix together, that the whole quantity may be afforded at 10*d.* per *lb.*?

Answer 1 { *lb.* *d.*
2 at 12 per *lb.*
1 at 11
1 at 9
2 at 8

Answer 2 { *lb.* *d.*
3 at 12 per *lb.*
2 at 11
2 at 9
3 at 8

Answer

$$\text{Answer 3 } \left\{ \begin{array}{l} 1 \text{ at } 12 \text{ per lb.} \\ 2 \text{ at } 11 \\ 2 \text{ at } 9 \\ 1 \text{ at } 8 \end{array} \right.$$

$$\text{Answer 4 } \left\{ \begin{array}{l} 1 \text{ at } 12 \text{ per lb.} \\ 3 \text{ at } 11 \\ 3 \text{ at } 9 \\ 1 \text{ at } 8 \end{array} \right.$$

$$\text{Answer 5 } \left\{ \begin{array}{l} 3 \text{ at } 12 \text{ per lb.} \\ 1 \text{ at } 11 \\ 3 \text{ at } 9 \\ 2 \text{ at } 8 \end{array} \right.$$

$$\text{Answer 6 } \left\{ \begin{array}{l} 2 \text{ at } 12 \text{ per lb.} \\ 3 \text{ at } 11 \\ 1 \text{ at } 9 \\ 3 \text{ at } 8 \end{array} \right.$$

Answer 7 — 3lb of each sort.

3. A distiller would mix 40 gallons of French brandy, at 12s. per gallon, with English at 7s. and spirits at 4s. per gallon: What quantity of each sort must he take to afford it at 8s. per gallon?

Answer, 40 gallons French, 32 English, and 32 spirits.

4. A brewer has three sorts of ale, viz. at 10d. at 8d. and at 6d. per gallon; and he would have a composition of 30 gallons worth 7d. per gallon. I demand how much of each sort he must have?

Answer, 5 gallons at 10d. 5 at 8d. and 20 at 6d.

CLASS II.

1. How many gallons of Sherry at 4s. 6d. per gallon, Lisbon at 5s. and Sack at 7s. must be mixed together to make the compound worth 6s. 2½d? Answer, 34½ gal. of Sack, 9¾ of Lisbon, and 9¾ of Sherry.

2. How much barley at 5s. and rye at 4s. must be mixed with 10½ bushels of wheat at 8s. to make the compound worth 6s. per bushel?

Answer 7 of each sort.

3. A goldsmith would mix alloy with gold of 24 carraets (or degrees of fineness) another sort of 21 carraets, and another sort of 19 carraets, so that 190 ounces may bear 17 carraets fine. How much of each sort of gold must he take, and what quantity of alloy, esteeming it (as is reasonable in these cases) of 0 carraets fine?

$$\text{Answer } \left\{ \begin{array}{l} 50\frac{1}{2} \text{ oz. of each sort of gold,} \\ 38\frac{1}{2} \text{ oz. of alloy.} \end{array} \right.$$

CLASS III.

1. A vintner hath two vessels, one will hold 50 gallons, and the other 30, and would know how much water he must mix with the wine at 4s. per gallon to fill the bigger vessel, that every gallon drawn may be worth 3s. the gallon, and with wine at 2s. 6d. the gallon, to fill the lesser vessel, that a gallon may be worth 2s?

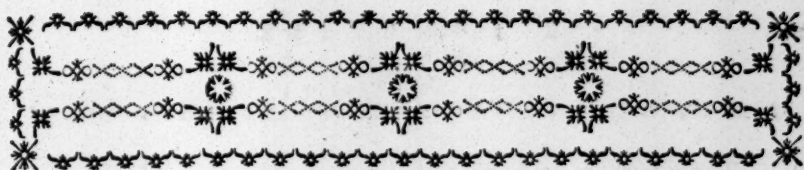
Answer, $37\frac{1}{2}$ gal. of wine to $12\frac{1}{2}$ of water for the first, and 24 gal. of wine to 6 of water for the second.

2. A druggist had 3 sorts of drugs; one was worth 4s. per lb. another 5s. and another 8s. and out of these he made two parcels, one was 21 lb. at 6s. per lb. and the other 35 lb. at 7s. per lb. How much of each sort did he take, for each parcel?

Answer, First. 6 lb. at 4s. 6 lb. at 5s. and 9 lb. at 8s.—Second 5 lb. at 4s. 5 lb. at 5s. and 25 lb. at 8s.

End of the first part.

A R A T I O N A L



A

RATIONAL and PRACTICAL

TREATISE

OF

ARITHMETIC.

PART II.

Introduction.

IN the former part of this work we have delivered the algorithm of abstract whole, and applicate numbers, and shewn their use in a variety of questions, digested under their proper heads, relating to business and the ordinary affairs of mankind. And as in these cases perfect exactness is not required, what is there delivered needs no farther improvement upon that account: But as in several enquiries of a mathematical and philosophical nature, where calculation is wanted, 'tis requisite to be exceeding nice, we must now endeavour to render the operations of numbers more perfect, and to provide means for their management, when any the smallest parts of one are concerned, and to form expressions in every application perfectly adequate to the enquiry.

To effect this, and lay before the reader some other curious, and useful operations in arithmetic, this second part is intended; and as a good deal of precision, and even elegance may be now thrown into our proceedings by the help of certain * characters or signs, we shall, as one preparative thereto, subjoin them with their explanation, observing that the knowledge of them can by no means be dispensed with in the perusal of the following articles.

The

* As no one can now be at a loss in the management of the simple operations, in general, the steps only of the work will hereafter be pointed out by these signs.

The characters with their explanation are as follow.

1. The character or sign $+$ (*more*) is the mark for addition, and placed betwixt two numbers signifies, that the latter is to be added to the former. Thus $8+10$ (read 8 more 10) is an expression for 8 added to 10.

2. The sign $-$ (*less*) is the mark for subtraction, and placed betwixt two numbers implies, that the latter is to be taken from the former. Thus $12-5$ (read 12 less 5) is an expression for the difference betwixt 12 and 5.

3. The sign $\times$ (*into*) is the mark of multiplication, and betwixt two numbers signifies, that they are to be multiplied together. Thus 9×7 (read 9 into 7) is an expression for 9 times 7 or 7 times 9.

4. The sign $\div$ (*by*) is the mark for division, and signifies, that the number going before the sign is to be divided by the number following it. Thus $15 \div 3$ (read 15 divided by 3) shews that 15 is to be divided by 3. But the most common way of expressing division is by setting down the dividend, or number to be divided, above a small line and the divisor under it. Thus $\overset{15}{\underset{3}{\div}}$ implies that 15 is to be divided by 3: Also if 120 is to be divided by 7, the expression is wrote thus $\overset{120}{\underset{7}{\div}}$.

5. the sign $=$ (*equal*) signifies, that the numbers or expressions on each side thereof are equal one to another. Thus $5+10=15$ signifies, that 5 added to 10 is equal to 15; and $12-7=5$ implies, that 7 taken from 12 leaves 5, or 12 made less by 7 is equal to 5; and $4 \times 7=28$ implies, that 4 multiplied by 7 gives a product equal to 28, and $\overset{14}{\underset{2}{\div}}=7$ signifies, that 14 divided by 2 gives a quotient equal to 7.

6. Statings in the rule of three are briefly expressed by certain points between the terms. Thus $5:10::6:12$ is read, as 5 is to 10, so is 6 to 12. Hence the two points ($:$) between the two first terms signify constantly *is to*, the four points ($::$) between the second and third terms signify *so is*, and the two points between the third and fourth imply *to*.

When several terms or numbers are connected together by lines drawn either above or below them; it implies, that the result of these terms or numbers, ordered as their signs denote, is to be taken as one term or number. Thus $5+4-3 \times 5$ shews, that what remains, when from the sum of 5 and 4, 3 is taken, (or 6,) is to be multiplied by 5; and $\frac{21-3+2}{4}$ implies, that when 3 is taken from the sum of 21 and 2, the remainder or 20 is to be divided by 4, &c.

The

The following are a set of promiscuous examples for the farther consideration of the learner.

$$6+7-5=8$$

$$18-8+7=17$$

$$\overline{4+3} \times 6 = 42$$

$$\overline{8-3} \times 3 = 15$$

$$\frac{6+8}{2} = 7$$

$$\frac{8-2+4}{2} = 5$$

$$\frac{\overline{3+7} \times 6}{4} = 15$$

$$\frac{\overline{12-4} \times \overline{2+3}}{17-9} = 5$$

$$6 + \frac{18-3}{5} = 9$$

$$8 + \overline{9-4} \times 6 = 38$$

$$3 : 7 :: 18 : \frac{7 \times 18}{3} = 42$$

$$5 : 4 :: 21 : \frac{4 \times 21}{5} = 16\frac{4}{5}$$

The above attained, another preparative to what follows is a few "definitions" with respect to a distinction, that is made of numbers into * *Prime* and *Composite*, &c.

1. *Prime numbers* are those, which can be divided by no number but themselves and unity, as 1, 2, 3, 5, 7, 11, 17, 19, 23, 29, &c.

2. *Composite numbers* are those, which may be divided by some other number besides themselves, and unity, as 4, 6, 8, 9, 10, 12, 14, 15, 16, &c.

Farther,

3. One number is said to be the *measure* of another, when it is contained in it a certain number of times exactly, without a remainder. This *measure* is also sometimes called an *aliquot* part of the other number, and this other number a *multiple* of it: And in any two numbers, of which this cannot be said, the less is called an *aliquant* part of the greater. Thus 4 measures 12, because it is contained in 12 exactly 3 times. 4 also is an aliquot part of 12, and 12 a multiple of 4; but 5, 7, 8, 9, 10, and 11, are all aliquant parts of 12.

4. Also

* These distinctions in numbers with others of the same kind, called *numbers odd and even*, *figurate numbers*, &c. and the several properties that arise from them, have furnished out a great deal of speculation in some antient arithmetical treatises, but to us, at present, this article contains all that are necessary.

4. Also a number is called the *common measure* of two or more numbers, when it will divide each of them without a remainder. Thus 3 is the common measure of 12, and 15, and the greatest number that will do this is called the *greatest common measure*.

5. Also those numbers, that can be thus divided, are said to be *commensurable*, except in the instance, where unity is the greatest common measure; for in that case, they are said to be *prime* to each other, or *incommensurable*, as are 3, 4; 4, 5, 6.

6. A number, which can be measured by two or more numbers, is called their *common multiple*, and if it be the least number, that can be so measured, it is called their *least common multiple*, so 30, 45, 60, 75, &c. are multiples of 3 and 5, but their least common multiple is 15.

There are two useful problems (and but two, * besides those which may be answered mentally) arising out of the above particulars, and we shall here take the opportunity of inserting them, as a place, which seems as convenient as any for that purpose.

Problem I.

"To find the G. C. M. or greatest common measure (or divisor) of two or more numbers."

† Rule.

"In the case of two numbers, divide the greater by the less, and the less (or last divisor) by the remainder (if any) and the last remainder or divisor

* 'Tis sometimes necessary to know the *components* of a number (as was used in case 6 of multiplication) and whether it be a *prime*, &c. and direct rules might be given for that purpose; but when these enquiries are of use here, they will for the most part be readily discovered without any operation of the pen.

† As numbers may be aptly represented by lines, the reason of the rule will perhaps appear the clearer, if we use that method of illustration. Therefore in the example that follows, suppose the line AB (see figure 1st. of the plate at the end) equal to the greater number 88, and CD equal to the lesser number 24. Then tis plain the greatest line (or number) that measures them both, cannot exceed CD, but it may be equal to it, and if so, will consequently be had once in CD, and some number of times exactly in AB: Suppose it so, and let us try if it will measure AB (which is evidently done by dividing AB 88, by CD 24) and if it succeeds (or leaves no remainder) the question is answered. But let us now suppose there is a remainder, or that CD is not contained in AB a certain number of times exactly (as 3 times) and that the excess or remainder is eB (or 16) then can no number bigger than it measure them both: To prove which, suppose Bf (bigger than Be) will answer; then as it by the supposition measures CD, it will also measure Ae , which is a multiple of (or 3 times) CD, and also what yet remains of the line AB, or eB . But eB is less than fB , consequently the supposition is absurd

divisor, by the present remainder, and so on, till nothing remains; and then is the last divisor of all the G. C. M. of the two given numbers. When this is wanted of three or more numbers it must be first found of two as above, and of that G. C. M. and some other number, and again (if need be) of this last G. C. M. and another, &c. and then is the G. C. M. last found the answer: Observing farther (agreeably to what has been already said) that if unity in any of these operations proves the G. C. M. the given numbers are prime to one another, and found to be incommensurable."

Examples.

1. Required the G. C. M. of 24 and 88, and (2) 784 and 952?

$$24) 88 (3$$

$$\underline{72}$$

$$16) 24 (1$$

$$\underline{16}$$

$$\text{Answer } 8) 16 (2$$

$$\underline{16}$$

$$784) 952 (1$$

$$\underline{784}$$

$$168) 784 (4$$

$$\underline{672}$$

$$112) 168 (1$$

$$\underline{112}$$

$$\text{Answer } 56) 112 (2$$

$$\underline{112}$$

Dd

3. Re-

absurd. Therefore to proceed: Tho' the G. C. M. wanted cannot be greater than eB , yet it may be equal to it, and, as AB is made up of a multiple of CD (i. e. Ae) and the line eB , if it will measure CD it answers the purpose. Therefore the next step is to try it, and if it succeeds the enquiry is determined (which is obviously done by dividing CD , or the last divisor by eB , or the last remainder) but if not, and there now remains something, as gD (or 8) then is gD , for reasons similar to those above, the greatest line, that can now answer the purpose; and if it will measure (or divide) Cg (equal eB the last divisor) it must measure AB , and CD , being both made up, in like manner as before, of multiples of gD i. e. CD by the supposition is made up of the measure gD itself, and the part Cg which it measures; but AB is made up of a multiple of CD , and the part eB equal Cg , consequently the above assertion is true. Now on trying if this line gD (or 8) will divide Cg exactly, it is found to answer, and hence is the G. C. M. of AB and CD required. Now as this reasoning is the same in every step thus far, it follows that the process would still be the same, let there be ever so many remainders and trials, and manifestly proves the truth of the rule in the case of two numbers. And that the directions for finding the G. C. M. of three or more numbers is right, must, after what is said above, in a manner be obvious of itself, for a greater C. M. for three or more numbers than of two of them is absurd, as the same two numbers are included in the supposition; hence taking the G. C. M. for two, and seeing if it will measure the other, must be the first step towards the discovery, and if it does not succeed, the enquiry then goes on in the same manner between it and the third number, &c. as before: Hence in this case the directions of the rule are right.

3. Required the G. C. M. of 918, 1998, and 522?

$$\begin{array}{r} 918) 1998(2 \\ \underline{1836} \end{array}$$

So 54 is the G. C. M. of 918, 1998. Hence

$$\begin{array}{r} 162) 918(5 \\ \underline{810} \end{array}$$

$$\begin{array}{r} 108) 162(1 \\ \underline{108} \end{array}$$

$$\begin{array}{r} 54) 108(2 \\ \underline{108} \end{array}$$

$$\begin{array}{r} 54) 522(9 \\ \underline{486} \end{array}$$

$$\begin{array}{r} 36) 54(1 \\ \underline{36} \end{array}$$

$$\begin{array}{r} 18) 36(2 \\ \underline{36} \end{array}$$

Therefore 18 is the answer required.

* In the same manner the G. C. M. of 468, and 864 is found to be 36; and of 516, 1212, and 3548, 4 is the G. C. M.

Problem II.

"To find the least common multiple (L. C. M.) of two or more numbers."

† Rule.

"In the instance of two numbers first find their G. C. M. by the last problem; then divide either of the given numbers by that measure, and multiply the quotient by the other of these numbers, and the product is the L. C. M. sought. In the case of three or more numbers the L. C. M. of any two of them must be first found, and then after the same manner the L. C. M. of this multiple, and a third number found, and so on, till all the numbers have been taken in, and then will the last found common multiple be the answer required.—From this it appears, that if they be prime to one another, their product will be the L. C. M."

Examples.

* As this problem is not difficult, and will be farther illustrated in problem 4th. of Vulgar Fractions &c. any more practical questions here would be needless.

† By dividing the two given numbers by their G. C. M. we find two others the least possible in the same proportion, from what was said in the contractions of the rule of three (page 90) and the consideration that the greater the divisor is the less must be the quotient. Now as these quotients are the least possible, they must be prime to one another, and hence their product, their L. C. M. But as the numbers under consideration are respectively so many times bigger than those quotient numbers, as the G. C. M. (above found) is bigger than unity, 'tis evident their L. C. M. will be the same number of times bigger than the L. C. M. found of their

Examples.

1. Required the L. C. M. of 24 and 32?

$$\begin{array}{r} 24) 32 (1 \\ \underline{24} \end{array}$$

Then 8) 24 ($3 \times 32 = 96$ the answer.

The G. C. M. 8) 24 (3
 $\underline{24}$

OR 8) 32 ($4 \times 24 = 96$

2. Required the L. C. M. of 24, 32 and 52? (96 being found in the above example to be the L. C. M. of 24 and 32, there needs only to be found the L. C. M. of it and 52. Thus).

$$\begin{array}{r} 52) 96 (1 \\ \underline{52} \end{array}$$

Hence 4) 96 ($24 \times 52 = 1248$ the answer.

$$\begin{array}{r} 44) 52 (1 \\ \underline{44} \end{array}$$

4) 52 ($13 \times 96 = 1248$

$$\begin{array}{r} 8) 44 (5 \\ \underline{40} \end{array}$$

$$4) 8 (2$$

* In the same manner the L. C. M. of 56, 98, and 120 is found to be 5880; and of 8, 12, 9 and 15, the L. C. M. is 360.

Dd 2

Useful

their least proportionals. Therefore multiplying the L. C. M. of these proportionals, by the above found G. C. D. we get the answer. OR (in the example that follows) as

$$3 \times 4 \times 8 = 3 \times 32 = 96$$

$$\text{also } 4 \times 3 \times 8 = 4 \times 24 = 96$$

'tis visibly the same thing, and a little readier, to multiply one of the proportionals by the other given number as directed.—The sum of this reasoning therefore is a demonstration of the rule in the case of two numbers. And when more are concerned, 'tis plain a less common multiple of three or more numbers cannot be found than of two of them; because the same two are included, &c. Hence the process must be carried on (in the same manner as in the case of two numbers) between the multiple of the two above found, and the next number, and so on; and the result will evidently be the answer to the problem, as 'tis very manifest, that a number will measure any multiple, or multiples of the multiple it originally measured.

* See an instance or two in problem 5th. of what follows.

Useful “contractions” of the above rule.

* “If any of the given numbers is an aliquot part, or divisor of any other of the given numbers, that aliquot part may be omitted in the operation. And if in the process, where several numbers are concerned, it be discovered that the L. C. M. to some of the preceding numbers, is also a multiple to some of the following numbers; such following numbers may be omitted in the operation.”

Examples.

1. Required the L. C. M. of 4, 6, 8, 12, and 16?

$$\begin{array}{r} 12) 16 \text{ (1} \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 4) 12 \text{ (3} \\ 12 \\ \hline \end{array}$$

4) 12 ($3 \times 16 = 48$ the answer.

Here 4 and 6 being aliquot parts of 12, and 8 of 16, they are omitted, and the L. C. M. of 12 and 16 found for the answer.

2. Required the L. C. M. of 15, 8, 12, and 16?

8 and 15 being prime to one another, $8 \times 15 = 120$ is their L. C. M.

$$\begin{array}{r} 16) 120 \text{ (7} \\ 112 \\ \hline \end{array}$$

$$\begin{array}{r} 8) 16 \text{ (2} \\ 16 \\ \hline \end{array}$$

8) 16 ($2 \times 120 = 240$ the answer.

Seeing that 120 the L. C. M. of 8 and 15 is also a multiple of 12, 12 is neglected in the operation.

The

* The reason is evident, for the numbers that are thus neglected must necessarily measure any multiple of the numbers they are aliquot parts of; so that whatever is determined for a multiple of those that remain, must answer equally for every one concerned in the problem.

The Doctrine of **VULGAR FRACTIONS.**

* Definition.

“**FRACTIONS**, or broken numbers, are expressions for any assignable part or parts of an unit, whole, or integer.”

† NOTATION.

“**T**HE established method of expressing fractions, or parts of an integer, is by placing below a small line the number of parts the integer is supposed to be divided into, and above the line the number of parts intended by the fraction. Thus if I want to express three fourths of a penny (or three of those parts whereof four make a penny) 'tis wrote thus $\frac{3}{4}$, and eight ninths of a mile (eight of those parts whereof nine make a mile) thus $\frac{8}{9}$; also thirteen twenty sevenths, and fifty six ninety fourths of any thing are thus wrote $\frac{13}{27}$, $\frac{56}{94}$. &c.

Now as from this notation, the number below the line shews (or denominates) how many parts the integer is supposed to be divided into, it is called the Denominator, and the number above the line the Numerator, because it (numerates or) shews, how many of these parts are concerned in the fraction.”

This is the notation in general, but in respect to some other particular circumstances, fractions are distinguished into the following kinds.

Definitions.

1. “A Proper Fraction is when the expression is less than a whole (or integer); or whose numerator is less than the denominator. Thus $\frac{3}{4}$, $\frac{6}{7}$,”

* This short definition we think sufficient to introduce the notation immediately; as what is meant by a fraction cannot but be known to any person the least conversant in ordinary affairs, and much more to an arithmetician thus far advanced. Besides the metaphysical parade, so often introduced here, in order to set the subject in every point of view, instead of instructing only puzzles; and experience tells us, that nothing is more detrimental to the evidence of the sciences, than an ill-judged refinement.

† We have in part anticipated this notation in page 33, nevertheless to make this article intire it must be gone over again.

$\frac{3}{5}$, $\frac{85}{100}$, &c. are proper fractions, and read three fifths, six sevenths, eighty five hundredths, &c.

2. An Improper Fraction is when the numerator is equal to, or greater than the denominator, as $\frac{4}{4}$, $\frac{5}{3}$, &c. (read four fourths, five thirds) and this name is given them from a seeming impropriety there is in the expressions; as instead of denoting a part less than an integer, their usual acceptance, one is equal to, and the other greater than an integer.

3. A Simple Fraction is one single expression referred to the integer. Thus $\frac{9}{7}$, $\frac{8}{3}$, &c. are simple fractions.

4. A Compound Fraction is the fraction of a fraction, consisting of two or more simple fractions joined together with the word of. Thus $\frac{2}{3}$ of $\frac{3}{4}$, and $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{10}$, &c. are compound fractions, and read two thirds of three fourths, and three fourths of four sixths of eight tenths, &c.

5. A Mixt Number (or Fraction) is composed of an integer, and a fraction. Thus $8\frac{1}{2}$, $12\frac{3}{4}$, &c. are mixt numbers, and read eight and one half, twelve and three fourths, &c.

It appears from the above notation, that whenever it is necessary to consider a whole number as a vulgar fraction (i. e. as consisting of a numerator and denominator) it may be done by placing an unit (or 1) under it. Thus 6 may be wrote $\frac{6}{1}$, and $365\frac{3}{4}$, &c."

These things established, the learner perhaps may expect we should next proceed to the Algorithm, as before; but as there are from the nature of the notation, several preparative reductions of expressions from one form to another necessarily required, they must first be taught, as follows.

Reduction of Vulgar Fractions.

Definition.

"UNDER this title are shewn several necessary reductions of fractions from one form to another, preparative to the operations of addition, subtraction, &c."

Problem I.

"To reduce improper fractions to their equivalent whole, or mixt numbers."

Rule.

** Rule.*

“ Divide the numerator by the denominator, and the quotient is the integer, and if any thing remains, it must be placed over the divisor in form of a fraction at the end of it for a mixt number.”

Examples.

1. Reduce $\frac{167}{18}$ to is equivalent whole or mixt number?

$$\begin{array}{r} 18 \overline{) 167} \quad (9\frac{5}{18} \text{ Answer.} \\ \underline{162} \end{array}$$

2. Reduce $1\frac{6426}{27}$ to its equivalent whole or mixt number?

$$\begin{array}{r} 27 \overline{) 16426} \quad (608\frac{10}{27} \text{ Answer.} \\ \underline{162} \\ 226 \\ \underline{216} \\ 10 \end{array}$$

Practical

* This follows obviously from the notation; for as the denominator expresses a whole (or integer) broke into a certain number of parts, and the numerator an equal or greater number of those parts, 'tis evident the fraction must contain a whole (or integer) as often as the divisor above mentioned is contained in the dividend. As to the direction for ordering the remainder so as to make a mixt number, 'tis too evident to need explanation, and besides is what has been all along done in the divisions made in the other part of the work.

It may not be amiss to hint in this place, that all fractions represent a division of the numerator by the denominator, and are taken altogether as proper and adequate expressions for the quotient. Thus the quotient of 2 divided by 3 is $\frac{2}{3}$: For on asking how often 3 may be had in 2 the answer is (0) no times, and 2 remains. Now (agreeably to what was explained page 33) this remainder is to be put over the divisor, and placed at the end of the quotient (0) for an exact quotient, that is in this case $0\frac{2}{3} = \frac{2}{3}$ is the exact quotient.—The same of others. Now this shews that there is something more than an arbitrary choice in the second method of representing division as explained in the introduction; and in algebra this method is of very great use, as from its general manner of expressing quantities no other is often practicable.

Practical Questions.

CLASS I.

1. Reduce $2\frac{4}{7}$ to its proper terms? - - - - - Answer $3\frac{3}{7}$.
2. Reduce $2\frac{6}{17}$ to its proper terms? - - - - - Answer $56\frac{9}{17}$.
3. Reduce $2\frac{4}{9}$ to its proper terms? - - - - - Answer $27\frac{7}{9}$.

CLASS II.

1. Reduce $1\frac{245}{22}$ to its proper terms? - - - - - Answer $56\frac{13}{22}$.
2. Reduce $8\frac{227}{16}$ to its proper terms? - - - - - Answer $514\frac{3}{16}$.

CLASS III.

Reduce $6\frac{21613}{314}$ to its proper terms? - - - - - Answer $120\frac{433}{314}$.

Problem II.

"To reduce a mixt number to an equivalent improper fraction."

** Rule.*

"Multiply the integral part by the denominator of the fraction, and to the product add the numerator; make the sum a numerator to the given denominator, and that is the fraction required."

Examples.

1. Reduce $9\frac{5}{13}$ to an improper fraction?

$$9 \times 13 + 5 = 127 \quad \text{Answer } 127\frac{5}{13}.$$

2. Reduce $608\frac{10}{27}$ to an improper fraction?

$$608 \times 27 + 10 = 16426 \quad \text{Answer } 16426\frac{10}{27}.$$

Practical

* To make the whole number incorporate with the fractional part, every integer in it must be broke into the same number of parts pointed out by the denominator of the fraction given, which is evidently done by multiplying them by this denominator. Then as all the parts both of the whole number and fraction are now made equal in magnitude, they may be added and placed over the given denominator, as a new numerator agreeing with it in kind, for the answer required.

Practical Questions.

CLASS I.

1. Reduce $56\frac{1}{2}$ to an improper fraction? - - - Answer $112\frac{1}{2}$.
2. Reduce $79\frac{1}{9}$ to an improper fraction? - - - Answer $79\frac{1}{9}$.
3. Reduce $183\frac{5}{11}$ to an improper fraction? - - - Answer $2013\frac{5}{11}$.

CLASS II.

1. Reduce $100\frac{1}{9}$ to an improper fraction? - - - Answer $901\frac{1}{9}$.
2. Reduce $514\frac{5}{6}$ to an improper fraction? - - - Answer $3089\frac{5}{6}$.

CLASS III.

Reduce $47\frac{3}{8}\frac{1}{4}\frac{7}{6}$ to an improper fraction? - - - Answer $3979\frac{47}{80}$.

Problem III.

"To reduce a compound fraction to an equivalent simple one."

* Rule.

"Multiply all the numerators continually for a numerator of the simple fraction, and all the denominators continually for a denominator."

Ee

Example.

* That a compound fraction may be represented by a simple one is very evident, as a part of a part must needs be some part of the whole, and in order to prove the truth of the rule for this reduction, it will be necessary first to establish the following premises, which contain several good hints for the right understanding of the nature of vulgar fractions. *Viz.* (I.) Any fraction is multiplied or divided by a whole number, by multiplying or dividing (if possible) its numerator by it.

Thus $\frac{4}{10}$ multiplied by 2 must be as evidently $\frac{8}{10}$ i. e. $\frac{2 \times 4 = 8}{10}$ as twice 4 oxen

(or any other thing) make 8 oxen; and on the contrary the half of $\frac{8}{10}$ is as plainly $\frac{4}{10}$ as the half of 8 oxen is 4 oxen. Again (II.) we must premise, that if the numerator, and denominator be both multiplied or divided by the same number, the value of the fraction is not altered. Thus $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$ &c. in which the numerator and denominator of each preceding fraction is multiplied by 2 to make that immediately following, and $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$ in which the numerator and denominator of the preceding fractions are all along divided by 3, &c. and the reason is evident, as each multiplication and division only breaks and collects (as it were) the parts of the numerator and denominator they find into other different ones, as to magnitude, and consequently their relative value is still the same (i. e. one part out of two of an apple is the same thing as 50 out of 100, and

Examples.

1. Reduce
- $\frac{2}{3}$
- of
- $\frac{3}{4}$
- to a simple fraction?

$$\begin{array}{l} 2 \times 3 = 6 \\ 3 \times 4 = 12 \end{array}$$

Answer $\frac{6}{12}$

2. Reduce
- $\frac{1}{2}$
- of
- $\frac{5}{6}$
- of
- $\frac{3}{4}$
- of
- $\frac{2}{3}$
- to a simple fraction?

$$\begin{array}{l} 1 \times 5 \times 3 \times 2 = 30 \\ 2 \times 6 \times 4 \times 3 = 144 \end{array}$$

Answer $\frac{30}{144}$ *Practical Questions.*

CLASS I.

1. Reduce $\frac{7}{8}$ of $\frac{4}{6}$ of $\frac{2}{10}$ to a single fraction? - - - Answer $\frac{2 \times 2}{4 \times 6 \times 10}$.
 2. Reduce $\frac{5}{9}$ of $\frac{1}{7}$ of $\frac{11}{12}$ to a single fraction? - - - Answer $\frac{2 \times 2 \times 11}{7 \times 9 \times 12}$.
 3. Reduce $\frac{12}{16}$ of $\frac{18}{20}$ of $\frac{3}{5}$ to a single fraction? - - Answer $\frac{1 \times 8 \times 3}{9 \times 6 \times 5}$.

CLASS

and the converse). (III.) From this and the certainty of a fraction's being great or little, not as it is formed of great or little numbers, but as the value of the numerator is near or far off the value of the denominator, &c. we may discover a different, and very useful, way of multiplying a fraction by a whole number, from that already mentioned, i. e. A fraction is multiplied by an integer, if we divide its denominator by that integer, and, on the contrary, 'tis divided by an integer, if we multiply its denominator by that integer. Thus $\frac{4}{12}$ multiplied by 2 is either

$$\frac{4 \times 2 = 8}{12} \text{ or } \frac{4}{12 \div 2 = 6}, \text{ because the results } \frac{8}{12} \text{ and } \frac{4}{6} \text{ are the same by the}$$

last (the terms of the latter being each half of the former). Also $\frac{4}{12}$ divided by

$$2 \text{ is either } \frac{4 \div 2 = 2}{12} \text{ or } \frac{4}{2 \times 12 = 24}, \text{ the results } \frac{2}{12}, \text{ and } \frac{4}{24} \text{ being equal in}$$

value. And this agreement of the latter method, in effect, with the former proves it to be right.

These truths premised (which must be equally valid in all other instances) the reason of the rule will readily appear. Thus in the example that follows $\frac{2}{3}$ of $\frac{3}{4}$ is plainly the same as $2 \times \frac{1}{3}$ of $\frac{3}{4}$ (or twice $\frac{1}{3}$ of $\frac{3}{4}$). Now $\frac{1}{3}$ of $\frac{3}{4}$ from what

$$\text{was said above (III) is } \frac{3}{3 \times 4} = \frac{1}{4}. \text{ But twice this (from II) is } \frac{2 \times 3}{12} = \frac{6}{12},$$

which reasoning agreeing with the directions of the rule shews it to be right, when two numbers are concerned. And as in instances where three or more numbers are given, the two first may be reduced to one (as above) and this one and the next in the same manner to one, and so on, till the whole are taken in; and as also the process in this would evidently be a continual multiplication of the numerators and denominators, the rule is proved true in all instances.

CLASS II.

1. Reduce $\frac{1\frac{1}{2}}{1\frac{1}{2}}$ of $\frac{1\frac{1}{4}}{1\frac{1}{4}}$ of $\frac{2\frac{1}{2}}{2\frac{1}{2}}$ to a single fraction? - - Answer $\frac{3003}{4872}$.
 2. Reduce $\frac{1\frac{1}{3}}{1\frac{1}{3}}$ of $\frac{1}{2}$ of $\frac{2\frac{1}{3}}{3\frac{1}{3}}$ to a single fraction? - - - Answer $\frac{275}{806}$.

CLASS III.

Reduce $\frac{4}{3}$ of $\frac{12}{13}$ of $\frac{6}{9}$ of $\frac{11}{120}$ to a single fraction? Answer $\frac{8928}{81000}$.

As there are many different expressions for fractions of the same value, and as the less they are the better, the following "*contraction*" of the rule in the last problem will be useful upon that account, as well as in abridging the work itself.

* "If any numerator be equal to any denominator, they may be both rejected in the process, and if any numerator and denominator can be divided by the same number, the quotients may be used in their stead."

Example.

1. Reduce $\frac{1\frac{1}{2}}{1\frac{1}{2}}$ of $\frac{8}{3}$ of $\frac{7}{3}$ to a single fraction?

$$\frac{1\frac{1}{2}}{1\frac{1}{2}} \text{ of } \frac{8}{3} \text{ of } \frac{7}{3} = \frac{8}{3} \text{ of } \frac{1}{1} \text{ of } \frac{7}{3} = \frac{1}{2} \text{ of } \frac{1}{1} \text{ of } \frac{7}{1} = \left(\frac{1 \times 1 \times 7}{2 \times 1 \times 1} \right) = \frac{7}{2} \text{ the ans.}$$

OR, more practically thus, writing the new figurers over the old ones as they are cancelled with the dash.

$$\begin{array}{c} 1 \\ 8 \\ 18 \\ 16 \\ 2 \end{array} \text{ of } \begin{array}{c} 1 \\ 8 \\ 3 \\ 1 \end{array} \text{ of } \begin{array}{c} 7 \\ 3 \\ 1 \end{array} = \frac{7}{2}$$

Ee 2

Problem.

* This is evident from what has been often hinted, and particularly in the contraction, page 186. Also 'tis in effect dividing both the numerator and denominator by the same number, therefore the value of the fraction is not altered from what was said in article (II) of the last demonstration.

Problem IV.

"To reduce a fraction to its lowest terms, or to find an equivalent one expressed in the least numbers possible."

* Rule.

"Divide the numerator and denominator, by their G. C. D. and the quotients will be the terms of the fraction required."

Examples.

1. Reduce $\frac{486}{648}$ to its lowest terms?

$$\begin{array}{r} 486) 648 \text{ (1)} \\ \underline{486} \\ \text{G. C. D. } 162) 486 \text{ (3)} \\ \underline{486} \end{array} \qquad \begin{array}{l} 486 \div 162 = 3 \\ 648 \div 162 = 4 \\ \text{Answer } \frac{3}{4}. \end{array}$$

2. Reduce $\frac{137}{283}$ to its lowest terms?

$$\begin{array}{r} 137) 283 \text{ (2)} \\ \underline{274} \\ 9) 137 \\ \underline{9} \\ 15 \text{ (2) } 9 \\ \underline{18} \\ 4 \text{ (1) } 2 \text{ (2)} \end{array}$$

Here the G. C. D. being unity the fraction is already in its lowest terms.

Practical Questions.

CLASS I.

1. Reduce $\frac{84}{176}$ to its lowest terms? - - - - - Answer $\frac{21}{44}$.
 2. Reduce $\frac{192}{378}$ to its lowest terms? - - - - - Answer $\frac{16}{31.5}$.
 3. Reduce $\frac{162}{196}$ to its lowest terms? - - - - - Answer $\frac{81}{98}$.

CLASS

* We have shewn above that a different and less expression of some fractions may be found, of the same value, by a common division of both the numerator and denominator; and tis evident that if the common divisor be the greatest possible (as directed) the quotients, or terms of the new fraction, must be the least possible.—Generally after this problem fractions will be expressed in their lowest terms,

CLASS II.

1. Reduce $\frac{208}{884}$ to its lowest terms? - - - - - Answer $\frac{52}{171}$.
 2. Reduce $\frac{518\frac{1}{2}}{6912}$ to its lowest terms? - - - - - Answer $\frac{3}{4}$.

CLASS III.

Reduce $\frac{2520}{2880}$ to its lowest terms? - - - - - Answer $\frac{7}{8}$.

The work of the above problem may be "*more practically*" done thus:

"Divide the numerator and denominator by any little common divisor, as 2, 3, 4, 5, &c. and the result again in the same manner, and so on, till it can be divided no lower."

Now this method, tho' not so direct as the other, is often readier, as by practice it will be easy to discover these divisors at sight; for 'tis certain any even number will divide by 2, and any number ending with 5 or 0 by 5, * &c.

Examples.

1. Required to reduce $\frac{144}{240}$ to its lowest terms?

$$\begin{array}{r|l} \frac{1}{2} & \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & \frac{1}{2} \\ \hline 144 & 72 & 36 & 12 & 6 & 3 \\ \hline 240 & 120 & 60 & 20 & 10 & 5 \end{array} \quad \text{Answer } \frac{3}{5}$$

2. Re-

* Some add to these directions the following, which are more curious perhaps than useful. (I.) If the last two figures of any number will divide by 4, the whole will divide by 4. (II.) If the last three will divide by 8, the whole will divide by 8: (III.) And if the sum of the digits of any number be a multiple of 3 or 9, the whole is a multiple of 3 or 9.—The reason of the 1st. is, that whatever digit remains from the figure before the two last is so many hundreds, and as any number of hundreds is a multiple of 4, if the following number to be joined to it be also a multiple of 4, the whole is a multiple of 4. The reason of the 2d. is such like. For whatever digit remains from the figure before the three last is so many thousands, and as any number of thousands is a multiple of 8, if the following digits to be joined express a multiple of 8, the whole is a multiple of 8. As to the 3d. it is a direct consequence of what was shewn in the demonstration of the method of proving multiplication, &c. by casting out the 9s. For from the lemma and corollary there demonstrated, it appears that any number, which leaves nothing when the 9s are cast out, will divide by 9, without a remainder. And as 9 is a multiple of 3, the same particular must be true of 3 also.

2. Required to reduce $\frac{1344}{1536}$ to its lowest terms?

$$\begin{array}{r} \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{3} \\ 1344 \mid 672 \mid 336 \mid 84 \mid 21 \mid 7 \\ \hline 1536 \mid 768 \mid 384 \mid 96 \mid 24 \mid 8 \end{array} \quad \text{Answer } \frac{7}{8}$$

Problem V.

"To reduce fractions of different denominations to equivalent ones having all the same denominator, called a common denominator."

* Rule.

"Multiply each numerator into all the denominators continually, except its own, for a new correspondent numerator, and all the denominators continually for a common denominator."

O R,

Having found the common denominator, as above, divide it by each of the given denominators, and multiply the quotients by the correspondent numerators for the new numerators."

Examples.

1. Reduce $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{4}{7}$ to a common denominator?

Per 1st. method.

$$\begin{array}{ll} 2 \times 5 \times 7 = 70 & \text{the common denominator.} \\ 1 \times 5 \times 7 = 35 & \text{the new numerator for } \frac{1}{2}. \\ 3 \times 2 \times 7 = 42 & \text{Do. - - - - - for } \frac{3}{5}. \\ 4 \times 2 \times 5 = 40 & \text{Do. - - - - - for } \frac{4}{7}. \end{array}$$

Therefore the new equivalent fractions are $\frac{35}{70}$, $\frac{42}{70}$, and $\frac{40}{70}$, the answer.

Per

* The reason of the first method will appear by placing the members multiplied in these operations properly under one another, for then it will be seen that the numerator and denominator of every fraction is multiplied by the very same component numbers, consequently their values are not alter'd (by what was said in note page 209)—Thus in the example that follows:

$$\begin{array}{ccc} \frac{1}{2} & \frac{3}{5} & \frac{4}{7} \\ \hline \begin{array}{l} 1 \mid \times 5 \times 7 \\ 2 \mid \times 5 \times 7 \end{array} & \begin{array}{l} 3 \mid \times 2 \times 7 \\ 5 \mid \times 2 \times 7 \end{array} & \begin{array}{l} 4 \mid \times 2 \times 5 \\ 7 \mid \times 2 \times 5 \end{array} \end{array}$$

As to the second method its reason is fully shewn in the process itself, in which such parts of the common denominator are taken for new numerators, as are expressed by the given fractions.

Per 2d. method.

$$\begin{array}{r} 2) 70 \\ \hline 35 \text{ new numerator for } \frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 5) 70 \\ \hline 14 \\ 3 \\ \hline 42 \text{ Do. for } \frac{3}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 7) 70 \\ \hline 10 \\ 4 \\ \hline 40 \text{ Do. for } \frac{4}{7} \text{ the same as before.} \\ \hline \end{array}$$

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{7}{8}$ to a common denominator?

Per 1st. method.

$$\begin{array}{l} 2 \times 3 \times 6 \times 8 = 288 \text{ the common denominator.} \\ 1 \times 3 \times 6 \times 8 = 144 \text{ the new numerator for } \frac{1}{2}. \\ 2 \times 2 \times 6 \times 8 = 192 \text{ Do. } - - - - - \text{ for } \frac{2}{3}. \\ 5 \times 2 \times 3 \times 8 = 240 \text{ Do. } - - - - - \text{ for } \frac{3}{5}. \\ 7 \times 2 \times 3 \times 6 = 252 \text{ Do. } - - - - - \text{ for } \frac{7}{8}. \end{array}$$

The fractions required are therefore $\frac{144}{288}$, $\frac{192}{288}$, $\frac{240}{288}$, and $\frac{252}{288}$.

Per 2d. method.

$$\begin{array}{r} 2) 288 \\ \hline 144 \text{ new numerator for } \frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 3) 288 \\ \hline 96 \\ 2 \\ \hline 192 \text{ Do. for } \frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 6) 288 \\ \hline 48 \\ 5 \\ \hline 240 \text{ Do. for } \frac{3}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 8) 288 \\ \hline 36 \\ 7 \\ \hline 252 \text{ Do. for } \frac{7}{8} \text{ the same as before.} \\ \hline \end{array}$$

Practical

Practical Questions.

CLASS I.

1. Reduce $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}$ to a common denominator? Answer $\frac{32}{64}, \frac{48}{64}, \frac{40}{64}$.
2. Reduce $\frac{4}{9}, \frac{7}{11}, \frac{6}{7}, \frac{1}{2}$ to a common denominator?
Answer $\frac{616}{1386}, \frac{882}{1386}, \frac{1188}{1386}, \frac{693}{1386}$.
3. Reduce $\frac{3}{5}, \frac{2}{7}, \frac{1}{3}, \frac{7}{8}$ to a common denominator?
Answer $\frac{504}{840}, \frac{240}{840}, \frac{280}{840}, \frac{735}{840}$.

CLASS II.

1. Reduce $\frac{2}{9}, \frac{1}{7}, \frac{3}{8}, \frac{4}{5}$ to a common denominator?
Answer $\frac{560}{2520}, \frac{350}{2520}, \frac{945}{2520}, \frac{2016}{2520}$.
2. Reduce $\frac{5}{6}, \frac{7}{8}, \frac{3}{4}, \frac{9}{10}$ to a common denominator?
Answer $\frac{1600}{1920}, \frac{1680}{1920}, \frac{1440}{1920}, \frac{1728}{1920}$.

CLASS III.

- Reduce $\frac{11}{13}, \frac{15}{16}, \frac{9}{11}, \frac{5}{7}$ to a common denominator?
Answer $\frac{13552}{16016}, \frac{1515}{16016}, \frac{13104}{16016}, \frac{11440}{16016}$.

Tho' the above problem finds a common denominator to any number of fractions, yet perhaps it is not the least that might be; and as fractions in their lowest terms are fittest for use, we shall next shew how "to find the lowest common denominator possible" in these cases, thus:

(* Rule 2d.)

"Having each fraction in its lowest term, find the least common multiple of the denominators for a common denominator, and then find the correspondent numerators, by the second method in the last rule."

Examples.

1. Reduce $\frac{7}{12}$, and $\frac{11}{18}$ to their least common denominator?

12)

* The reason of the rule is evident; for as this common denominator is a multiple of all the denominators, it will divide by any of them, and consequently proper parts may be taken for all the numerators, as directed.

$$\begin{array}{r} 12 \overline{) 18} \quad (1 \\ \underline{12} \\ \text{G. C. M. } 6 \overline{) 12} \\ \underline{12} \\ 2 \end{array}$$

6) 12 ($2 \times 18 = 36$ L. C. M. of 12 and 18, or the least common denominator.

Then

$$12 \overline{) 35}$$

$$18 \overline{) 36}$$

$3 \times 7 = 21$ new numer. for $\frac{7}{12}$

$2 \times 11 = 22$ new numer. for $\frac{11}{18}$

Hence we have $\frac{21}{36}, \frac{22}{36}$ the answer.

2. Reduce $\frac{1}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{11}{16}, \frac{17}{24}$ to their least common denominator?

Neglecting 3, 4, 6, and 8 as aliquot parts of 16 and 24 find the L. C. M. of 16 and 24. Thus

$$\begin{array}{r} 16 \overline{) 24} \quad (1 \\ \underline{16} \\ \text{G. C. M. } 8 \overline{) 16} \\ \underline{16} \\ 2 \end{array}$$

8) 16 ($2 \times 24 = 48$ L. C. M. or L. C. Denom.

Then

$$3 \overline{) 48}$$

$$4 \overline{) 48}$$

16 new numer. for $\frac{1}{3}$,

$12 \times 3 = 36$ new numer. for $\frac{3}{4}$,

$$6 \overline{) 48}$$

$$8 \overline{) 48}$$

$8 \times 5 = 40$ new numer. for $\frac{5}{6}$,

$6 \times 7 = 42$ new numer. for $\frac{7}{8}$,

$$16 \overline{) 48}$$

$$24 \overline{) 48}$$

$3 \times 11 = 33$ new numer. for $\frac{11}{16}$,

$2 \times 17 = 34$ new numer. for $\frac{17}{24}$.

Therefore we have $\frac{16}{48}, \frac{36}{48}, \frac{40}{48}, \frac{42}{48}, \frac{33}{48}, \frac{34}{48}$ the answer.

Problem VI.

“To express fractionally what part any applict. number is of any other of the same kind.”

F f

* Rule.

* Rule.

"Make them both of the same name by common reduction, and then the result of the former placed over the latter is the expression required."

Examples.

1. Express 6 farthings in the fraction of a pound?

20s. $\times 12 \times 4 = 960$ the farthings in a £. Hence $\frac{6}{960} = \frac{1}{160}$ is the answer.

2. Express $4\frac{1}{2}d$ in the fraction of a shilling?

12d. $\times 4 = 48$ farthings, and $4\frac{1}{2}d. = 18$ farthings. Hence $\frac{18}{48} = \frac{3}{8}$ is the answer.

3. Express 3s. $2\frac{1}{4}$ in the fraction of a moidore?

27s. $= 1296$ farthings, and 3s. $2\frac{1}{4}d. = 153$ farthings. Hence $\frac{153}{1296} = \frac{17}{144}$ is the answer.

4. Express $16\frac{3}{4}$ drams in the fraction of a lb. avoirdupoise?

16oz. $= 1024$ quarter drams, and $16\frac{3}{4}$ drams $= 67$ quarter drams. Hence $\frac{67}{1024}$ is the answer.

Practical Questions.

CLASS I.

1. Express 8lb. in the fraction of a ton? - - - - Answer $\frac{1}{280}$.
 2. Express $3\frac{1}{2}$ inches in the fraction of a yard? - - Answer $\frac{7}{2}$.
 3. Express 5s. $7\frac{1}{2}d$. in the fraction of a £. - - - - Answer $\frac{9}{32}$.

CLASS II.

1. Express 2dwt. $3\frac{1}{2}grs$. in the fraction of a lb. Troy?
 Answer $\frac{103}{11520}$.
 2. Express 15s. in the fraction of 2s. 10d.? - - - - Answer $\frac{20}{17}$.

CLASS III.

Express 3C. 2Qrs. 5lb. $13\frac{1}{3}oz$. to the fraction of 7C. 3Qrs?
 Answer $(\frac{12226}{41664} =) \frac{11}{24}$.

Problem

* This is evident from the practice.

Problem VII.

"To reduce fractions of one denomination to fractions of another denomination."

* Rule.

"1. If it be from a less to a greater,

Bring a single unit, considered of the same denomination the required fraction is to be of, into the same denomination the given fraction refers to at present; then multiply this result by the denominator of the given fraction, and over it place the numerator for the answer.

2. If it be from a greater to a less.

Bring a single unit, considered of the same denomination the fraction refers to at present, into the same denomination the required one is to be of, then multiply this result by the given numerator, and under it place the denominator for the answer.

OR, in general,

Going through the scale of the subdivisions from the given denomination to the other referred to, make (by way of comparison) every step the member of a compound fraction, and then reduce it to single one for the answer."

This latter method is the most usual, and on perusing a few examples, it will be found easier in practice, than it may seem in precept.

Examples.

1. Reduce $\frac{5}{8}$ of a penny to the fraction of a pound?

Per 1st. method. $20s. = 240d.$ Then $240 \times 6 = 1440$ sixths of a penny. Hence $\frac{5}{1440} = \frac{1}{288}$ is the answer. OR, per general rule form it thus $\frac{5}{8}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{5}{1440}$ as before.

Ff 2

2. Reduce

* 'Tis judged here, that a careful examination of the process in an instance or two, will shew the reason of so easy a rule, better than a tedious account in words, which it would require if attempted.

2. Reduce $\frac{7}{8}$ of an ounce to the fraction of a C. weight?

Per 1st. $112lb. = 1792oz.$ Then $1792 \times 8 = 14336$ eighths of an ounce. Hence $\frac{7}{14336} = \frac{1}{2048}$ is the answer.

OR per general rule $\frac{7}{8}$ of $\frac{1}{16}$ of $\frac{1}{128}$ ($= \frac{7}{8}$ of $\frac{1}{16}$ of $\frac{1}{28}$ of $\frac{1}{4}$) = $\frac{7}{14336}$ as before.

3. Reduce $\frac{7}{9}$ of a £. to the fraction of a penny?

Per 2d. $1l. = 240d.$ Then $240 \times 9 = 2160$. Hence $\frac{1680}{2160} = \frac{14}{18}$ is the answer.

OR per general rule $\frac{7}{9}$ of $\frac{20}{1}$ of $\frac{12}{1} = \frac{1680}{2160}$ as before.

4. Reduce $\frac{3}{5}$ of a yard to the fraction of an inch?

Per 2d. $1yd. = 36$ inches. Then $36 \times 5 = 180$. Hence $\frac{108}{180}$ is the answer.

OR per general rule $\frac{3}{5}$ of $\frac{3}{1}$ of $\frac{12}{1} = \frac{108}{180}$ as before.

Practical Questions.

CLASS I.

1. Reduce $\frac{1}{2}$ of a farthing to the fraction of a shilling? Answer $\frac{1}{96}.$
2. Reduce $\frac{1}{8}$ of a £. to the fraction of a penny? Answer $\frac{40}{3}.$
3. Reduce $\frac{4}{5}$ of a pennywt. to the fraction of a lb. Troy? Answer $\frac{1}{300}.$

CLASS II.

1. Reduce $\frac{5}{1440}$ of a £. to the fraction of a penny? Answer $\frac{5}{144}.$
2. Reduce $\frac{7}{8}$ of a guinea to the fraction of a pound? Answer $\frac{147}{160}.$

CLASS III.

Reduce $\frac{6}{7}$ of a moidore to the fraction of a crown? Answer $\frac{162}{35}.$

Problem VIII.

“To find the value of an applicate fraction in its proper quantity (as of money, weight, measure, &c).”

* Rule.

** Rule.*

"Multiply the numerator by the parts in the next inferior denomination, and divide the product by the denominator for the answer in that denomination, and if any thing remains, multiply it by the next inferior denomination to the last, and divide again by the denominator, and so on, till it is exhausted as far as necessary, and then the quotients placed after one another in their order are the answer."

Examples.

1. What's the value of $\frac{5}{7}$ of a shilling, and (2d.) $\frac{2}{3}$ of a hundred weight?

$$\begin{array}{r} 5 \\ 12 \overline{) 60} \\ \underline{60} \\ 8d. \quad 4 \\ \quad 4 \\ \underline{\quad} \\ 7) 16 \\ \underline{14} \\ 2\frac{2}{7} \end{array}$$

$$\begin{array}{r} 2 \\ 4 \overline{) 8} \\ \underline{8} \\ 2\text{Qrs. } 2 \\ \quad 28 \\ \underline{\quad} \\ 3) 56 \\ \underline{54} \\ 18lb. \quad 2 \\ \quad 16 \\ \underline{\quad} \\ 3) 32 \\ \underline{30} \\ 10\frac{2}{3}oz. \end{array}$$

Answer $8\frac{1}{2}d. \frac{2}{7}$, and 2Qrs. 18lb. $10\frac{2}{3}oz.$

3. What's the value of $\frac{2}{7}$ of a pound sterling, and (4th.) $\frac{15}{17}$ of a lb. Troy?

2

* As the numerator of a fraction may be always considered as a remainder, and the denominator a divisor, of a division, this very easy problem will be found already solved, both in division of money, and the valuation of remainders in the rule of three; so any thing more of explanation than this hint would be superfluous.

$$\begin{array}{r}
 2 \\
 20 \\
 \hline
 7) 40 \\
 \hline
 5s. \quad 5 \\
 \quad 12 \\
 \hline
 7) 60 \\
 \hline
 8d. \quad 4 \\
 \quad 4 \\
 \hline
 7) 16 \\
 \hline
 2\frac{2}{7}
 \end{array}$$

$$\begin{array}{r}
 16 \\
 12 \\
 \hline
 17) 192 \text{ (110z.} \\
 \quad 17 \\
 \hline
 \quad 22 \\
 \quad 17 \\
 \hline
 \quad 5 \\
 \quad 20 \\
 \hline
 17) 100 \text{ (5dwt.} \\
 \quad 85 \\
 \hline
 \quad 15 \\
 \quad 24 \\
 \hline
 17) 360 \text{ (21}\frac{3}{17}\text{grs.} \\
 \quad 6c.
 \end{array}$$

Answer 5s. $8\frac{1}{2}d. \frac{2}{7}$, and 110z. 5dwt. $21\frac{3}{17}grs.$

Practical Questions.

CLASS I.

1. What's the value of $\frac{3\frac{1}{2}}{40}$ of a pound sterling? Answer 15s. 6d.
2. What's the value of $\frac{1\frac{3}{7}}{17}$ of a day? Answer 18h. 21m. $10\frac{1}{7}^{th}$.
3. What's the value of $\frac{5}{9}$ of a lb. avoirdupois? Answer 8oz. $14\frac{2}{9}dr.$

CLASS II.

1. What's the value of $\frac{37\frac{3}{8}}{480}$ of a pound sterling? Answer 15s. $6\frac{1}{2}d.$
2. What's the value of $\frac{2}{13}$ of a ton? Answer 3C. 0Qrs. 8lb. $9\frac{1}{13}oz.$

CLASS III.

What's the value of $\frac{13\frac{1}{6}}{216}$ of a lb. Troy? Answer 7oz. 5dwt. $13\frac{1}{3}grs.$

Problem IX.

"To reduce a fraction to an equivalent one having a given numerator."

* Rule.

** Rule.*

“ Say, as the numerator of the given fraction is to its denominator, so is the numerator supposed to the denominator required.”

Examples.

1. Reduce $\frac{3}{4}$ to an equivalent fraction whose numerator shall be 12?

$$\begin{array}{r} 3 : 4 :: 12 : 16 \\ \quad \quad 4 \\ \hline 3) 48 \\ \hline 16 \end{array} \quad \text{Answer } \frac{12}{16}.$$

2. Reduce $\frac{3}{5}$ to an equivalent fraction whose numerator shall be 17?

$$\begin{array}{r} 3 : 5 :: 17 : 28\frac{1}{5} \\ \quad \quad 5 \\ \hline 3) 85 \\ \hline 28\frac{1}{5} \end{array} \quad \text{Answer } \frac{17}{28\frac{1}{5}}$$

Problem X.

“ To reduce a fraction to an equivalent one having a given denominator.”

Rule.

“ Say, as the denominator of the given fraction is to its numerator, so is the denominator supposed to the numerator required.”

Examples.

1. Reduce $\frac{4}{5}$ to an equivalent fraction whose denominator shall be 30?

$$5 : 4 :: 30 : 24 \quad \text{Answer } \frac{24}{30}.$$

2. Reduce

* The statings in this and the following problem explain themselves: But it may not be amiss to observe here, that they afford an easy method of finding whether two given fractions be equal or not, by trying whether their numerators, and denominators be proportionable or not. Thus $\frac{3}{4}$ and $\frac{12}{16}$ are equal, because 'tis as $3 : 4 :: 12 : 16$ exactly, but $\frac{3}{5}$ and $\frac{17}{28\frac{1}{5}}$ are not equal, because as $3 : 5 :: 17 : 28\frac{1}{5}$ which is less than 30, &c.

2. Reduce $\frac{6}{7}$ to an equivalent fraction whose denominator shall be 15?

$$7 : 6 :: 15 : 12\frac{5}{7} \quad \text{Answer } \frac{12\frac{5}{7}}{15}$$

Scholium.

The two last problems might very well have been spared, nor should we have given them a place, had they not led, in as neat a manner as any, to a kind of fractions not yet taken notice of, such as $\frac{17}{28\frac{1}{3}}$, and $\frac{12\frac{5}{7}}{15}$ (&c.) where the numerator and denominator are themselves fractions. These some authors very properly call *complex fractions*, and they are easily reduced to others of the common form; but as this requires a previous knowledge of the division of fractions, their farther management must be deferred to another place, it being sufficient here to have pointed them out and distinguished them with a name.

Having thus laid down, in as rational a manner as we were able, the nature of vulgar fractions, and their several reductions, we now proceed to the Algorithm, which to a person, who has made himself master of what is thus far delivered, will appear an exceeding easy praxis.

(Problem XI.)

* *Addition of Fractions.*

† *Rule.*

“**PREPARE** your fractions by reducing (1st.) compound ones to single ones; (2^{dly}.) fractions of different denominations to others of the

* These rules addition, subtraction, &c. are understood as defined in whole numbers, except in multiplication, in which (from the nature of fractions) we must say “as much of the multiplicand is to be taken, as is expressed by the multiplier” instead of “The multiplicand is to be taken as often as there are units in the multiplier.” Thus to multiply any number by 2 is to take it twice, but to multiply by $\frac{1}{2}$ is to take half of it.

† The rule is evident; for before fractions are reduced to a common denominator, they are as much dissimilar as any things in nature: Thus $\frac{1}{3}$ and $\frac{2}{7}$ can neither be called 3 fifths nor 3 sevenths, any more than one apple and two pears can be called 3 apples and 3 pears; but when they are reduced to a common denominator ($\frac{7}{21}$, $\frac{6}{21}$) and made parts of the same thing their sum ($\frac{13}{21}$) is then as properly expressed by the sum of the numerators, as the sum of two quantities of apples or pears, by the sum of their individuals.

the same denomination; and (3dly, if need be) the simple fractions given and resulting to a common denominator: Then write the sum of the numerators over the common denominator for the answer, which, if an improper fraction, may then be reduced to a mixt number.—Farther, in the case, where mixed numbers are concerned, instead of reducing them to improper fractions (which might be done) it will be as well to reserve the integral parts to the last, and then add them to what results from the fractions."

Examples.

1. Add $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{4}{7}$ together?—These reduced to a C. D. are $\frac{35}{70}$, $\frac{42}{70}$, and $\frac{40}{70}$: Hence $\frac{35+42+40}{70} = \frac{117}{70} = 1\frac{47}{70}$ is the answer.

2. Add $\frac{2}{3}$ of $\frac{5}{7}$ to $\frac{4}{9}$?— $\frac{2}{3}$ of $\frac{5}{7} = \frac{10}{21}$; and $\frac{4}{9}$, and $\frac{10}{21}$ reduced to a C. D. are $\frac{81}{189}$, and $\frac{90}{189}$: Hence $\frac{81+90}{189} = \frac{171}{189}$ = (when brought to its lowest terms) $\frac{19}{21}$, is the answer.

3. Add $17\frac{1}{4}$, $6\frac{3}{5}$, and $4\frac{13}{14}$ together?— $\frac{1}{4}$, $\frac{3}{5}$, and $\frac{13}{14}$ reduced to a C. D. are $\frac{70}{280}$, $\frac{168}{280}$, $\frac{260}{280}$: Hence their sum is $\frac{70+168+260}{280} = \frac{498}{280} = 1\frac{249}{140}$.—Therefore $17+6+4+1\frac{249}{140} = 28\frac{109}{70}$ is the answer.

4. Add $\frac{3}{10}$ of $\frac{3}{8}$ of a £, to $\frac{1}{3}$ of a penny?— $\frac{3}{10}$ of $\frac{3}{8} = \frac{9}{80}$. Now (per prob. VII.) $\frac{1}{3}$ of a penny is $= \frac{1}{3}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{1}{720}$ of a £. Hence $\frac{9}{80}$, and $\frac{1}{720}$ reduced to a C. D. give $\frac{81}{7200}$, and $\frac{10}{7200}$. Therefore their sum $\frac{6480+80}{7200} = \frac{6560}{7200} = \frac{41}{450}$ of a £. is the answer.

Practical Questions.

CLASS I.

1. Add $\frac{7}{10}$, $\frac{11}{12}$, and $\frac{4}{9}$ together? — Answer $2\frac{11}{180}$.
2. Add $5\frac{2}{3}$, $6\frac{7}{8}$, and $\frac{1}{2}$ together? — Answer $13\frac{1}{24}$.
3. Add $\frac{1}{2}$ of $\frac{7}{8}$, and $\frac{2}{3}$ of $\frac{10}{20}$ together? — Answer $1\frac{17}{240}$.

CLASS II.

1. Add $\frac{4}{5}$ of a ton to $\frac{5}{6}$ of a lb. — Answer $\frac{10757}{13440}$.
2. Add $6\frac{7}{8}$ of $\frac{9}{10}$, $\frac{4}{7}$ of $\frac{1}{2}$, and $7\frac{1}{2}$ together? — Answer $13\frac{1}{112}$.

CLASS III.

Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{7}{8}$ of an ell English?

Answer $2\frac{1}{96}$ yards.

G g

(Problem

(Problem XII.)

Subtraction of Fractions.

* Rule.

“*PREPARE* them as in addition, and then the difference of the numerators placed over the common denominator is the answer. And if whole or mixed numbers be concerned, they may either be thus managed, in the form of an improper fraction, or (which is easier) the whole number parts reserved to the last, if, in case of want in the fractional part of the subtrahend, you borrow to it an integer supposed broken into the parts of the fraction (or, which is the same, as many parts as are in the common denominator) and for that carry one to the whole number, as in common subtraction.”

Examples.

1. From $\frac{1}{3}$ of $\frac{7}{10}$ take $\frac{1}{8}$? ---- $\frac{1}{3}$ of $\frac{7}{10} = \frac{7}{30}$. Then $\frac{7}{30}$, and $\frac{1}{8}$ reduced to a C. D. are $\frac{56}{240}$, $\frac{30}{240}$. Hence $\frac{56-30}{240} = \frac{26}{240} = \frac{13}{120}$ is the answer.

2. From $16\frac{1}{2}$ take $8\frac{3}{5}$? ---- $\frac{1}{2}$, $\frac{3}{5}$ reduced to a C. D. make $\frac{5}{10}$, $\frac{6}{10}$. Hence $16\frac{5}{10} - 8\frac{6}{10} = 7\frac{9}{10}$ the answer. OR thus $16\frac{1}{2} = \frac{32}{2}$, and $8\frac{3}{5} = \frac{43}{5}$ which reduced to a C. D. give $\frac{165}{10}$, $\frac{86}{10}$. Hence $\frac{165-86}{10} = \frac{79}{10} = 7\frac{9}{10}$ as before.

3. From $\frac{3}{4}$ of a £. take $\frac{3}{5}$ of a shilling? ---- $\frac{3}{5}$ of a shilling = $\frac{3}{5}$ of $\frac{1}{20} = \frac{3}{100}$ of a £. Then $\frac{3}{4}$, $\frac{3}{100}$ reduced to a C. D. make $\frac{300}{400}$, $\frac{12}{400}$. Hence $\frac{300-12}{400} = \frac{288}{400} = \frac{18}{25}$ of a £. the answer.

Practical Questions.

CLASS I.

1. From $\frac{5}{8}$ take $\frac{3}{5}$ of $\frac{5}{8}$? - - - - - Answer $\frac{1}{24}$ remains.
2. From $\frac{11}{12}$ take $\frac{3}{4}$? - - - - - Answer $\frac{2}{12}$.
3. From $\frac{1}{2}$ take $\frac{1}{7}$ of $\frac{2}{3}$? - - - - - Answer $\frac{3}{42}$.

CLASS

* The reason must be obvious.

CLASS II.

1. From $71\frac{1}{2}$ take $\frac{17}{9}$? - - - - - Answer $70\frac{23}{18}$ remains.
 2. From $\frac{1}{5}$ of an oz. take $\frac{2}{3}$ of a drwt. (Troy)? Answer $\frac{129}{160}$ of an oz.

CLASS III.

From $\frac{4}{5}$ of a ton take $\frac{5}{6}$ of a lb. - - - - - Answer $\frac{10747}{13440}$ of a ton.

(Problem XIII.)

Multiplication of Fractions.

* Rule.

“REDUCE compound fractions to single ones, and mixt numbers to improper fractions, and then the product of the numerators is the numerator, and the product of the denominators the denominator of the answer.”

Examples.

1. Multiply $\frac{5}{8}$ by $\frac{3}{4}$? $\begin{array}{l} 5 \times 3 = 15 \\ 8 \times 4 = 32 \end{array}$ Answer $\frac{15}{32}$.
 2. Multiply $13\frac{1}{4}$ by $\frac{3}{7}$? ----- $13\frac{1}{4} = \frac{53}{4}$. Hence $\frac{53 \times 3}{4 \times 7} = \frac{159}{28} = 5\frac{19}{28}$
 is the answer.

Practical Questions.

CLASS I.

1. Multiply $\frac{7}{8}$ by $13\frac{2}{3}$? - - - - - Answer $12\frac{13}{8}$.
 2. Multiply $\frac{16}{21}$ by $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{4}{5}$? - - - - - Answer $\frac{16}{49}$.
 3. Multiply $12\frac{3}{4}$ by $\frac{2}{3}$ of 7? - - - - - Answer $29\frac{2}{3}$.

Gg 2

CLASS

* As the multiplication by a fraction implies the taking such a part of the multiplicand, as is expressed by it (see note to addition) the product of any two fractions may be truly expressed by a compound one. Thus $\frac{3}{4}$ of $\frac{5}{8}$ is the answer of the first example that follows; and as the direction of the rule agrees with the method already given to reduce these fractions to simple ones, it is shewn to be right.—This might have been demonstrated many other ways, if the process were not too nearly ally'd to several occasional steps taken in this work, to need any at all.

CLASS II.

1. Multiply $\frac{1}{3}$ of $\frac{4}{5}$ by $\frac{7}{10}$ of $\frac{11}{12}$? - - - - - Answer $\frac{77}{50}$.
 2. Multiply $\frac{4}{5}$ of 91 by $7\frac{1}{2}$? - - - - - Answer $520\frac{1}{5}$.

CLASS III.

Multiply $4\frac{1}{2}$, $\frac{3}{4}$ of $\frac{1}{7}$, and $18\frac{4}{5}$ continually? - - - Answer $9\frac{2}{140}$.

(Problem XIV.)

Division of Fractions.

* Rule.

“*PREPARE* them as in multiplication, and then multiply the numerator of the divisor into the denominator of the dividend for a denominator, and the denominator of the divisor into the numerator of the dividend for a numerator of the answer. OR, in short (which is the very same) invert the terms of the divisor, and then multiply like names together (as in multiplication) for the answer.”

Examples.

* In the example that follows $\frac{2}{3}$ is the divisor, and $\frac{4}{7}$ the dividend. Now $\frac{2}{3}$ is 2 divided by 3 (or $\frac{1}{3}$ of 2), and to argue out the reason of the rule in this instance, we will consider the division in the first place to be made by 2, then (per note prob. III.) $\frac{4}{7}$ divided by 2 gives $\frac{4}{2 \times 7}$ for the quotient; but as this divisor is 3 times as big, as it must be, the quotient will be but $\frac{1}{3}$ of what it ought to be, therefore if it be multiplied by 3 it will be the true quotient, i. e. (per the same note) $\frac{3 \times 4}{2 \times 7} = \frac{12}{14}$ is the answer required. Now this process agrees exactly with the rule, and proves it to be right.

We may add farther, as this is the same thing as inverting the divisor (i. e. making $\frac{2}{3}$, $\frac{3}{2}$) and then multiplying it into the dividend, that not only the multiplication and division of fractions by whole numbers may be effected by their contraries (i. e. multiplication by division, and division by multiplication, See note to prob. III.) but that in a similar manner the multiplication and division of fractions by fractions may be made to do each others business.

It appears also, as another consequence arising from the nature of these two operations just mentioned, that in multiplication the product may be less than the multiplicand, and in division the quotient greater than the dividend; effects just contrary to what are brought about by them in whole numbers. Now these effects in both algorithms agree respectively to the proportion the multipliers and divisors have to unity, remembering to esteem them properly, according to what is observed of them above.

Upon the whole, as the student becomes more and more acquainted with this curious part of arithmetic, he will find a great deal of neatness, and harmony in the whole system, and that it depends upon fewer principles, and contains fewer difficulties, than he may at first imagine.

Examples.

1. Divide $\frac{4}{7}$ by $\frac{2}{3}$? --- $\frac{3}{2} \times \frac{4}{7} = \frac{12}{14} = \frac{6}{7}$ is the answer.
2. Divide $\frac{3}{5}$ of $\frac{7}{8}$ by $4\frac{1}{6}$? --- $\frac{3}{5}$ of $\frac{7}{8} = \frac{21}{40}$, and $4\frac{1}{6} = \frac{25}{6}$. Hence $\frac{21}{40} \times \frac{6}{25} = \frac{126}{1000} = \frac{63}{500}$ is the answer.

Practical Questions.

CLASS I.

1. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$? --- Answer $\frac{2}{3}$ quotient.
2. Divide $4\frac{5}{9}$ by $\frac{5}{6}$ of 4 ? --- Answer $2\frac{1}{6}$.
3. Divide $672\frac{2}{3}$ by $13\frac{5}{6}$? --- Answer $48\frac{1}{3}$.

CLASS II.

1. Divide $\frac{1}{5}$ of 19 by $\frac{2}{3}$ of $\frac{3}{4}$? --- Answer $7\frac{3}{4}$.
2. Divide $7935\frac{2}{7}$ by $18\frac{3}{4}$? --- Answer $430\frac{1}{10}$.

CLASS III.

Divide $5205\frac{2}{10}$ by $\frac{4}{5}$ of 91 ? --- Answer $71\frac{1}{2}$.

Scholium.

In a former scholium (page 224) we in some sort gave the rise and nature of *complex-fractions*, and hinted that they might be reduced to simple ones by the help of division. Having therefore now got this preparative, we shall apply it to an instance or two, as follows.

Required to reduce $\frac{17}{18\frac{1}{3}}$, and $\frac{12\frac{6}{7}}{15}$ to simple fractions?

As every fraction represents the division of the numerator by the denominator, these expressions may be considered as questions of that kind stated thus. Divide 17 by $18\frac{1}{3}$, and $12\frac{6}{7}$ by 15, and as these enquiries agree exactly with those in division, they are reduced to simple ones in the same manner (i. e. $\frac{51}{33}$, and $\frac{90}{15}$). Also if others more complicated were proposed, they might be thus taken to pieces

and solved; for instance, suppose $\frac{2\frac{1}{3}}{6}$ was given to be reduced to $3\frac{1}{2}$

a sim-

a simple expression (in which the double line parts the absolute numerator from the denominator). First $2\frac{1}{3}$ must be divided by 6, and then that quotient by $3\frac{1}{2}$ for the answer. i. e. (after reducing the mixt numbers $2\frac{1}{3}$, $3\frac{1}{2}$ to improper fractions, and placing an unit under the whole number 6 for a denominator) the multiplications would be as pointed out by the curves in what follows.

$$\left. \begin{array}{r} 2\frac{1}{3} \\ \hline 6 \\ \hline 3\frac{1}{2} \end{array} \right\} = \left\{ \begin{array}{l} \frac{7}{3} \\ \frac{6}{1} \\ \frac{7}{2} \end{array} \right\} = \frac{7}{18} = \frac{1}{2\frac{4}{6}} = \frac{1}{9} \text{ the answer.}$$

Another instance, with the steps of the solution may be as under, in which something of order may be observed in the directing curves, &c.

$$\left. \begin{array}{r} 2\frac{1}{2} \\ 3 \\ \hline 4\frac{1}{3} \\ 5\frac{1}{3} \\ \hline 7 \\ 6\frac{1}{4} \\ \hline 2 \end{array} \right\} = \left\{ \begin{array}{l} \frac{5}{2} \\ \frac{3}{1} \\ \frac{21}{5} \\ \frac{16}{3} \\ \frac{7}{1} \\ \frac{25}{4} \\ 2 \end{array} \right\} = \left\{ \begin{array}{l} \frac{5}{6} \\ \frac{63}{80} \\ \frac{28}{55} \\ \frac{28}{50} \end{array} \right\} = \frac{400}{378} = \frac{20000}{10584} = 1\frac{177}{1325} \text{ the answer.}$$

Tho' these fractions are seemingly more curious, than useful, yet they may not be amiss in trying the dexterity of the artist, who perhaps may gather some useful hints from the process, preparative to an acquaintance with algebra, in which a thorough knowledge of vulgar fractions is indispensibly required,

Praxis.

Praxis.

AS all the different * rules of arithmetic are effected by the primary ones, 'tis needless to illustrate the above doctrine under any particular head, every thing continuing the same, except the management of the algorithm. Therefore all that now remains is to collect a few promiscuous questions as follows, and that rather to inure the student in the method of calculation, than instruct him how to apply it.

1. Add $\frac{1}{2}$ of a penny to $\frac{2}{3}$ of a pound, and value the sum?

$$\frac{1}{2} \text{ of } \frac{1}{12} \text{ of } \frac{1}{20} = \frac{1}{480}.$$

$$\frac{2}{3} + \frac{1}{480} = \frac{960+3}{1440} = \frac{963}{1440} = \frac{107}{160} \text{ of a } \text{£}.$$

$$\text{Hence } \begin{array}{r} 107 \\ 20 \\ \hline \end{array}$$

$$16|0) 214|0$$

$$13s. \quad 60$$

$$12$$

$$16|0) 72|0$$

Answer 13s. $4\frac{1}{2}d.$ (as is plain independent of the calculation).

$$4d. \quad 80$$

$$4$$

$$16|0) 32|0$$

$$2$$

2. Add $\frac{1}{2}$ of a lb. Troy to $\frac{7}{12}$ of an ounce, and value the sum?

$$\frac{7}{12} \text{ of } \frac{1}{12} = \frac{7}{144}$$

$$\frac{1}{2} + \frac{7}{144} = \frac{144+14}{288} = \frac{79}{144}$$

* The reader need scarce be told, that in the rule of three after inverting the first term, they are (from the directions for multiplying and dividing) then all multiplied together for the answer.

$$\begin{array}{r}
 79 \\
 12 \\
 \hline
 144) 948 \text{ (6oz.} \\
 \underline{864} \\
 84 \\
 20 \\
 \hline
 144) 1680 \text{ (11 awt.} \\
 \underline{144} \\
 \hline
 \text{Answer 6oz. 11dwt. 16grs.} \quad \begin{array}{r} 240 \\ 144 \\ \hline 96 \\ 24 \\ \hline \end{array} \\
 144) 3804 \text{ (16grs.} \\
 \underline{3520} \\
 284
 \end{array}$$

3. From $\frac{3}{5}$ of a league take $\frac{1}{8}$ of a mile, and value the remainder.

$$\frac{3}{5} \text{ of } \frac{1}{3} = \frac{1}{24} \quad \frac{3}{5} - \frac{1}{24} = \frac{72-5}{120} = \frac{67}{120}$$

$$\begin{array}{r}
 67 \\
 3 \\
 \hline
 12|0) 20|1 \\
 \hline
 1m. 81 \\
 8 \\
 \hline
 12|0) 64|8 \\
 \hline
 \text{Answer 1m. 5fur, 88yds.} \quad \begin{array}{r} 5fur. 48 \\ 220 \\ \hline 12|0) 1056|0 \\ \hline 88yds. \end{array}
 \end{array}$$

4. From $\frac{1}{3}$ of a day take $\frac{2}{5}$ of an hour, and value the remainder?

$\frac{2}{5}$ of

$$\frac{2}{3} \text{ of } \frac{1}{24} = \frac{2}{120} = \frac{1}{60} \quad \frac{1}{3} - \frac{1}{60} = \frac{60-3}{180} = \frac{57}{180} = \frac{19}{60}$$

$$\begin{array}{r} 19 \\ 24 \\ \hline 6 \overline{) 45} 6 \end{array}$$

Answer 7h. 36m.

$$\begin{array}{r} 7h. \ 36 \\ \quad 60 \\ \hline 60 \end{array} \quad (36m.$$

5. If $\frac{1}{3}$ of a yard cost $\frac{7}{12}$ of a £. what will $\frac{6}{15}$ of a yard cost?

$\frac{3}{5}y. : \frac{7}{12}l. :: \frac{6}{15}y. : * \frac{8 \times 7 \times 6}{3 \times 12 \times 15} = \frac{7}{3 \times 2 \times 3} = \frac{7}{18}$ which valued is 7s. 9 $\frac{1}{3}$ d. the answer.

6. If I give $1\frac{1}{2}l.$ for a C. weight, what does $3\frac{1}{3}lb.$ stand me in?

$112lb. : 1\frac{1}{2}l. :: 3\frac{1}{3}lb. OR \ 112 : \frac{3}{2} :: \frac{10}{3} : \frac{3 \times 10}{112 \times 2 \times 3} = \frac{10}{112 \times 2}$
 $11\frac{5}{12}l.$ which valued is 10 $\frac{5}{7}$ d. the answer.

7. If $\frac{1}{2}$ of a yard cost $\frac{1}{3}$ of a pound, what will $\frac{1}{4}$ of an ell English cost?

$\frac{1}{4}$ of $\frac{5}{4} = \frac{5}{16}$ of a yard --- $\frac{1}{2} : \frac{1}{3} :: \frac{5}{16} : \frac{2 \times 1 \times 5}{1 \times 3 \times 16} = \frac{5}{3 \times 8} = \frac{5}{24}$, which valued is 4s. 2d. the answer.

8. If when a bushel of wheat is sold for $4\frac{2}{3}s.$ the penny loaf weighs $5\frac{2}{3}$ ounces; what must it weigh when the bushel is sold for $6\frac{3}{10}s.$?

(Inverse) $4\frac{2}{3}s. : 5\frac{2}{3}oz. :: 6\frac{3}{10}s. OR \ \frac{14}{3} : \frac{27}{5} :: \frac{63}{10} : \frac{10 \times 14 \times 27}{63 \times 3 \times 5} = \frac{2 \times 2 \times 9}{9} = 4oz.$ the answer.

Hh

9. If

* Placing the sign of multiplication between the factors in such operations as these, before we actually multiply them, is very useful, as we may generally, in like manner as observed in the contractions page 186 and 211, abridge the work considerably by cancelling similar ones.

9. If $3\frac{1}{2}$ times $3\frac{1}{2}lb.$ of tea cost $1\frac{1}{2}$ times $1\frac{1}{2}l.$ (of money), what will $\frac{1}{2}$ times $\frac{1}{3}lb.$ and the $\frac{1}{4}$ of $12\frac{1}{4}lb.$ amount to?

$$3\frac{1}{2} = \frac{7}{2}, \quad \frac{7}{2} \times \frac{7}{2} = \frac{49}{4}, \quad 1\frac{1}{2} = \frac{3}{2}, \quad \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}, \quad \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}, \quad \frac{1}{4} \text{ of } 12\frac{1}{4} = \frac{31}{10}.$$

$$\frac{1}{6} + \frac{31}{10} = \frac{16 + 294}{90} = \frac{310}{90}. \text{ Then } \frac{49}{4}lb. : \frac{9}{4}l. :: \frac{310}{90}lb. : \frac{4 \times 9 \times 310}{49 \times 4 \times 90} =$$

$$\frac{9 \times 155}{49 \times 48} = \frac{1395}{2352} = \frac{465}{784}l. \text{ which valued gives } 11s. 10\frac{17}{49} \text{ the answer.}$$

10. *A young hare starts 5 rods before a greyhound, and is not perceived by him, till she has been up 34 seconds; she scuds away at the rate of 12 miles an hour, and the dog on view makes after her at the rate of 20: How long will the course hold, and what ground will be run, beginning with the out-setting of the dog?

$$1h. = 3600'' : 12m. :: 34'' : \frac{34 \times 12}{3600} = \frac{34}{300}m. = 199\frac{7}{5} \text{ yards the hare}$$

had run before seen, which added to 5 rods (i.e. $5\frac{1}{2}yds. \times 5 = 27\frac{1}{2}yds.$ gives $226\frac{29}{30} = \frac{6809}{30}$ yards the distance the dog had to overtake when he started. But $20m - 12m. = 8$ miles is what the dog gains per hour. Hence $8m. = 14080yds. : 60' :: \frac{6809}{30}yds. : \frac{60 \times 6809}{14080 \times 30} =$

$$\frac{6809}{7040} \text{ minutes, which valued give } 58\frac{1}{32} \text{ seconds, the time the course will hold. Again. } 3600'' : (20m. =) 35200yds. :: \frac{6809}{32}'' (= 58\frac{1}{32}'') :$$

$$\frac{35200 \times 1857}{3600 \times 32} = \frac{88 \times 1857}{9 \times 32} = \frac{22 \times 619}{2 \times 8} = \frac{13616}{8} = 1702 = 567\frac{1}{3} \text{ yards}$$

the distance the dog runs.

11. The minute pointer of a clock goes once round in an hour; the hour pointer only once in 12 hours: Now supposing them both together at 12 o'Clock of the day, what time will it be, before they meet again?

'Tis evident that at 1 o'clock the minute pointer will be at 12, therefore according to their different speeds, we must see how long after this it will be, before the quicker pointer overtakes the slower. Now as the minute pointer runs 12 times as fast as the other, we have $12 - 1 = 11$ the spaces (as we may call the hours) it runs more than the other in an hour; hence this proportion.

11

* One demand in this question is the same as that in the question page 89, but as the manner of the solution here given is different to that, and more accurate, the example may perhaps merit the reader's perusal, as much as one with more novelty in it.

11 spaces : 1 h. :: 1 space : $\frac{1}{11}$ h. = 5' 27 $\frac{3}{11}$ ", time past one when the meeting required will happen.

12 My water-tub holds 147 gallons, the pipe usually brings in 14 gallons in 9 minutes. The tap discharges at a medium 40 gallons in 31 minutes. Suppose these both carelessly to be left open, and the water to be turned on at two in the morning. The servant at 5, finding the water running, shuts the tap, and is solicitous in what time the tub will be filled after this accident, in case the water continues flowing from the main?

$31' : 40 \text{ gal.} :: 9' : \frac{9 \times 40}{31} \text{ gal.} = 3\frac{50}{31}$, what the tap discharges in 9'.

Hence $14 \text{ gal.} - 3\frac{50}{31} \text{ gal.} = 7\frac{4}{31} \text{ gal.}$ is what the water-tub fills in 9', when both pipes are open, and at this rate it fills for 3 hours (i. e. 5-2).

Therefore $9' : 7\frac{4}{31} :: (3 \text{ h.} =) 180' : \frac{74 \times 180}{9 \times 31} = 74 \times \frac{20}{31} = 47\frac{23}{31}$ gal-

lons, the quantity in the tub when the tap was shut. Now $147 - 47\frac{23}{31} = 99\frac{6}{31} \text{ gal.}$ is what is yet to fill at the rate of 14 gallons per 9 minutes. Hence $14 \text{ gal.} : 9' :: (99\frac{6}{31} =) 3\frac{277}{31} : \frac{3077 \times 9}{14 \times 31} =$ (when solved)

$63' 48\frac{14}{31}''$ is the time it will take in filling, or it will be full at 3 minutes 48 seconds after 6.

Practical Questions.

CLASS I.

1. Two men have shares in a ship, A hath $\frac{1}{7}$, and B $\frac{2}{5}$; I demand whose is the greater share and how much? Answer B's by $\frac{1}{63}$.

2. In 3 boxes are $67\frac{3}{4}$ yards of cambric, viz. in No. 1 are $21\frac{1}{2}$ yards, in No. 2, $22\frac{1}{4}$ yards: What yards are in No. 3?

Answer $23\frac{5}{8}$.

3. How much is $\frac{1}{2}$ of $\frac{2}{3}$, and $\frac{7}{8}$ of $\frac{9}{10}$ of a C. weight?

Answer 1 C. 0 $\frac{2}{15}$ rs. $13\frac{8}{15}$ lb.

4. A certain person having $\frac{3}{5}$ of a coal mine sells $\frac{1}{4}$ of his share for 17 $\frac{1}{2}$ l. what is the whole mine worth? Answer 380 l.

5. If $\frac{1}{8}$ of a lb. cost $2\frac{1}{4}$ d. what will $\frac{3}{4}$ cost? Answer $13\frac{1}{2}$ d.

6. A mercer bought $3\frac{1}{2}$ pieces of silk, each containing $24\frac{1}{2}$ yards at 6s. 0 $\frac{1}{2}$ d. per yard, I demand what the whole comes to?

Answer £. 25-14-6 $\frac{1}{2}$ $\frac{1}{3}$.

CLASS II.

1. If $27\frac{1}{2}$ yards of cloth cost £.3-19 $\frac{3}{4}$ what will $1\frac{1}{4}$ yards cost?
Answer $3\frac{5}{8}s.$
2. A merchant had $5\frac{8}{9}C.$ of sugar at $6\frac{3}{4}s.$ per *lb.* which he would barter for tea at $8\frac{3}{8}s.$ per *lb.* I demand how much tea must be given for the sugar?
Answer $43\frac{1}{10}lb.$
3. A father devised $\frac{3}{4}$ of his estate to one of his sons, and $\frac{3}{8}$ of the residue to another, and the surplus to his relict for her life; the children's legacies were found to be £.257-3-4 different: Pray what money did he leave the widow the use of?
Answer £.534-2-8.
4. A lad having got 4000 nuts, in his return was met by Mad-Tom, who took from him $\frac{5}{8}$ of $\frac{2}{3}$ of his whole stock: Raving-Ned light on him afterwards and forced $\frac{2}{5}$ of $\frac{5}{8}$ of the remainder from him: Unluckily Positive-Jack found him, and required $\frac{7}{10}$ of $\frac{1}{20}$ of what he had left: Smiling-Dolly was by promise to have $\frac{3}{4}$ of a quarter of what nuts he brought home: How many then had the boy left?
Answer $575\frac{5}{8}.$

CLASS III.

1. Suppose a dog, a wolf, and a lion were to devour a sheep, and that the dog could eat up the sheep in an hour, the wolf in $\frac{3}{4}$ of an hour, and the lion in half an hour; now if the lion begins to eat $\frac{1}{8}$ of an hour before the other two, and afterwards all three eat together, the question is in what time the sheep will be devoured?
Answer $\frac{3\frac{1}{4}}{10\frac{1}{4}}$ of an hour.
2. A cistern in a certain conduit is supplied with water by one pipe of such bigness, that if the cock A, at the end of the pipe be set open, the cistern will be filled in half an hour; moreover at the bottom of the cistern two other cocks B and C are placed, whose capacities are such, that by the cock B set open alone (all the rest being stoped, and the cistern supposed to be full) it will be emptied in $2\frac{1}{3}$ hours; now because more water will be infused by the cock A, than can be expelled by both the cocks B and C in one and the same time, the question is to find in what time the cistern will be filled, if the said three cocks be set open at once?
Answer in $1\frac{2}{3}$ hours.

The Doctrine of DECIMAL FRACTIONS.

Definition.

“*A* Decimal fraction is a fraction whose denominator is 10, 100, 1000, &c. or in general, an unit with some number of cyphers annexed.”

NOTATION, &c.

“*I*N notation, part the first, we observed, that in any number, the value of every digit was altered according to the place it possessed, and that this alteration was in a tenfold proportion, of decrease, at every remove from left to right; or, which is the same thing, its value was always equal to its product (as a digit) when multiplied into an unit with so many cyphers annexed, as there were figures placed after it to the right. Now it has occurred to the * inventors of decimals, that according to this law, parts of an integer might be easily (and most conveniently) denoted in the continuance of this scale of descent, provided that the integer was broke into tenths, hundredths, thousandths, &c. parts, by placing them after the units figure (with a point before them for distinction) agreeably to the supposed value of the fraction. Thus in the following table (which of itself with a very little consideration might unfold the whole mystery) the first perpendicular column shews the digit 5 (expressing different values as to place) divided by 10 for its value in the next inferior place: The second shews its value compared with its distance from unity, and the third the common notation resulting from this law, till 5 be in the units place; and afterwards, the notation arising from the continuance of the same law, when considered as decimal parts, &c.

$$\begin{array}{r} 50000 \\ \hline 10 \end{array}$$

* The doctrine of decimals is but a modern improvement in fractions, the first hint seeming tacitly to have been started (as Mr. Malcolm observes) by Regiomontanus about the year 1464, and the first express treatise on this subject appears to have been wrote, by Simon Stevinus, about the year 1582, since which time they have been generally known and practised.

$\frac{50000}{10}$	$= 1000 \times 5 = 5000$	----- read 5 thousand.
$\frac{5000}{10}$	$= 100 \times 5 = 500$	----- 5 hundred.
$\frac{500}{10}$	$= 10 \times 5 = 50$	----- 5 tens, or fifty.
$\frac{50}{10}$	$= 1 \times 5 = 5$	----- 5 units, or five.
$\frac{5}{10}$	$= \frac{1}{10} \times 5 = .5$	----- 5 tenths.
$\frac{5}{10 \times 10}$	$= \frac{1}{100} \times 5 = .05$	----- 5 hundredths.
$\frac{5}{10 \times 10 \times 10}$	$= \frac{1}{1000} \times 5 = .005$	----- 5 thousandths.
		&c.

All this is very evident, and it appears (with some degree of beauty) that as the multipliers 10, 100, 1000, &c. of the whole numbers are understood according to the place of the digit (and not expressed) so the divisors, or denominators of the decimal fractions 10, 100, 1000, &c. may be, and are, also understood and omitted in the notation.

We have shewn in the last article treated, that there may be an infinite number of different expressions formed for the value of any given fraction, and in problem X. page 223 is laid down a rule to reduce any fraction to an equivalent one of a given denominator; hence any part of any denomination of things may be reduced to a decimal expression (by forming an adequate vulgar one of it the first) so as to enter into the notation. For instance $\frac{1}{2} = \frac{5}{10}$, $\frac{1}{4} = \frac{25}{100}$, $\frac{1}{8} = \frac{125}{1000}$, &c. give us these decimals respectively .5, .25, .125, &c. But this article must be treated of particularly hereafter; however in the mean time it will be necessary to observe, in order to give the student a perfect notion of decimals, that in all cases (or given vulgar fractions) there cannot be an exact equivalent decimal expression found, as the numerator discovered for the assumed denominator may be itself a fraction.

Thus $\frac{1}{7}$ turned into a decimal with 10 for its denominator is $\frac{1\frac{3}{7}}{10}$, or with

100, $\frac{14\frac{2}{7}}{100}$, with 1000, $\frac{142\frac{2}{7}}{1000}$, with 10000, $\frac{1428\frac{4}{7}}{10000}$, &c. which wrote

decimally must only be .1, .14, .142, .1428, &c. as the fractions ($\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, &c.) cannot possibly enter into the decimal notation; from which circumstance the conclusions of this doctrine are not always perfectly correct. But as by taking the assumed decimal denominators 10, or 100, or 1000, &c. we can approximate nearer, and nearer the truth (as will

be evident hereafter) they are in these instances capable of sufficient exactness, in all ordinary enquiries, and most others, where very great nicity is not wanted.

By supplying the imaginary denominator of any decimal expression, it may be easily read by looking then upon it as a vulgar one; and from the above notation, and table, the denominator appears to be always an unit with as many cyphers annexed, as there are figures in the given decimal. * Thus .25 is $\frac{25}{100}$ (read twenty five hundredths) .785 is $\frac{785}{1000}$ (read seven hundred and eighty five thousandths) .05 is $\frac{5}{100}$ (read five hundredths) .0075 is $\frac{75}{10000}$ (read seventy five ten thousandths) &c.

We may yet add to the particulars above mentioned (which indeed would occur to any person who understands the table, &c.) that as cyphers on the right hand of whole numbers increase their value, and on the left are insignificant; so on the contrary, cyphers on the left hand of decimals decrease their value, and on the right hand are insignificant."

This ample account of the nature of decimals premised, we must next shew some necessary reductions, &c. and then proceed to the algorithm, which will be found exceeding easy and very little different from that of abstract whole numbers.

Reductions in Decimal Fractions.

Problem I.

"TO reduce a vulgar fraction to its equivalent decimal one."

† Rule.

Divide the numerator with as many cyphers annexed to it, as may be necessary, by the denominator, and the quotient is the answer, except the figures

* Or these fractions may be taken into parts and read separately, thus $\frac{25}{100} = \frac{20}{100} + \frac{5}{100} = \frac{2}{10} + \frac{5}{100}$ (is read two tenths, five hundredths), and $\frac{785}{1000} = \frac{700}{1000} + \frac{80}{1000} + \frac{5}{1000} = \frac{7}{10} + \frac{8}{100} + \frac{5}{1000}$ (is read seven tenths, eight hundredths, five thousandths) &c. and in ordinary calling them over after other figures, &c. the digits are only named in their order: Thus 27.125 is read twenty seven, one, two, five, and 1728.0625 is read one thousand seven hundred and twenty eight, nothing, six, two, five, &c.

† What was said in the notation and problem X. of vulgar fractions makes the method plain, as this adding of cyphers to the numerator is only multiplying the second term in that stating by 10, 100, 1000, &c. the assumed denominator in

figures in it be not equal in number to the cyphers, that were subjoined; in which case they must be made so by prefixing to their left hand, as many cyphers, as they fall short, for the required decimal.—Observe farther, that when the division does not come clear without a remainder (or gives not an exact decimal) before there be five or six places in the quotient, no farther notice need be taken of it, as these make a decimal sufficiently accurate in most cases.”

Examples.

1. Required the equivalent decimal of $\frac{3}{4}$?

$$\begin{array}{r} 4 \overline{) 300} \\ \underline{.75} \text{ Answer.} \end{array}$$

2. Required

in the third, before it be divided by the first, or given denominator. Or, as every decimal fraction is made up of 10ths, 10ths. of these 10ths. or 100ths. &c. the process is only an obvious way of valuing the fraction according to these subdivisions, by multiplying the remainder by the parts of the inferior denominations, and dividing by the old divisor.

As to the directions for placing cyphers in some cases before the quotient figures, the reason will be manifest on a little examination. For as one cypher added (or an assumed denominator of 10) gives (when possible) the first figure in the decimal place (or those of tenths), two cyphers (or the denominator 100) the second figure (or hundredths), three cyphers (or the denominator 1000) the third figure (or thousandths), &c. tis plain, that if those quotient figures that should be in the 100ths, 1000ths. &c. places, be not thrown there by the figure or figures that precede them (if any) they must have such cyphers prefixed, as will give them these places, and the inspection of an example or two will shew (as well as ever so many words) that this must be done as directed.

That a decimal carried to 5 or 6 places, though not quite equal the vulgar one, will be sufficiently so in cases of very great exactness, appears on considering, that a digit in the 5th or 6th. place represents so many hundred thousandths and millionths of an unit respectively, and as all that could be gained by the decimal by a farther division, could not equal one of these parts, (i. e. all the digits subjoined by future divisions would be short of a time more going in these places) it is evidently too inconsiderable to be regarded. And in the application of decimals to business, 3 or 4 places at most will be sufficient; as they will express the value of the given vulgar fraction to within one thousandth or one ten-thousandth part of an unit.

To this observation we may add, that when decimals are to be used as multipliers or divisors, they will require a place or two more, than are necessary, when only to be used in additions and subtractions, as any little error in these cases may affect the result very sensibly. But after all, we think it not altogether adviseable in what relates to business, to use the method of decimals in any calculation, where divisors and factors are not perfect ones.

The inverse of this problem, or the finding a vulgar fraction equal to a decimal one is more easy perhaps than useful, and done by placing under the decimal its denominator, and then reducing the expression to its lowest terms. Thus $.75 = \frac{75}{100} =$ (per reduction) $\frac{3}{4}$. &c.

2. Required the equivalent decimal of $\frac{1}{25}$?

$$\begin{array}{r} 25 \overline{) 100} \text{ (.04 Answer.} \\ \underline{100} \end{array}$$

3. Required the equivalent decimal of $\frac{579}{2384}$?

$$\begin{array}{r} 2384 \overline{) 57900000} \text{ (.24286 \text{ \&ccaron} the answer.} \\ \underline{4768} \\ 10220 \\ \underline{9536} \\ 6840 \\ \underline{4768} \\ 20720 \\ \underline{19072} \\ 16480 \\ \underline{14304} \\ 2176 \\ \text{\&ccaron} \end{array}$$

4. Required the equivalent decimal of $\frac{3}{1728}$?

$$\begin{array}{r} 1728 \overline{) 30000000} \text{ (.001736 \text{ \&ccaron} the answer.} \\ \underline{1728} \\ 12720 \\ \underline{12096} \\ 6240 \\ \underline{5184} \\ 10560 \\ \underline{10368} \\ \text{\&ccaron} \end{array}$$

Practical Questions.

CLASS I.

1. What's the decimal of $\frac{3}{8}$? - - - - - Answer .375.
 2. What's

ii

2. What's the decimal of $\frac{5}{26}$? - - - - - Answer .192307 &c.
 3. What's the decimal of $\frac{15}{960}$? - - - - - Answer .015625.

CLASS II.

1. What's the decimal of $\frac{275}{3842}$? - - - - - Answer .071577 &c.
 2. What's the decimal of $\frac{3}{848}$? - - - - - Answer .003537 &c.

CLASS III.

What's the decimal of $\frac{2}{61812}$? - - - - - Answer .000113 &c.

Problem II.

"To reduce the different denominations of money, weights, measures, &c. to their equivalent decimal values."

* Rule.

"Express them in the form of a vulgar fraction, [By reducing the integer, and given denominations, into the lowest names mentioned in them, and placing the latter result over the former] and then turn it into a decimal by the last problem."

O R, more practically thus:

First write the given denominations, or parts, orderly under one another; the inferior or least parts being uppermost: Let these be dividends. Next against each part, on the left hand, write the number thereof contained in one of its next superior: Let these be divisors. Then beginning with the highest, write the quotient of each division, as decimal parts, on the right hand of the dividend next below it: Divide this mixt number by its divisor, and so on, till they are all used, and the last quotient will be the decimal sought."

Examples.

* The first method needs no explanation, and the reason of the other will appear by attentively considering the process, which only reduces the lower denominations into decimals of the next superior, and the sum of these and that superior into the next superior, &c. till the whole is in the decimal of the denomination required. Thus in example second that follows (which will illustrate the whole) 3 farthings is obviously turned into the decimal of a penny by dividing by 4, hence the result .75 may be properly wrote after the given pence making 9.75 for $9\frac{3}{4}d$. then this mixt number being so many 12ths. of a shilling, is consequently turned into the decimal of a shilling by dividing by 12, and the result of it .8125 wrote after the shillings gives in the same manner 15.8125 for an expression of the whole 15s. $9\frac{3}{4}d$. But as the expression is wanted in the decimal of a £. and is at present but so many 20ths. it must therefore be divided by 20 for the answer.

Examples.

1. Reduce
- $4\frac{3}{4}d.$
- to the decimal of a shilling?

 $4\frac{3}{4} = \frac{19}{4}$, hence 48) 19000000 (.395833 $\mathcal{C}.$ is the answer.

$$\begin{array}{r} 144 \\ \hline 26 \\ \mathcal{C}. \end{array}$$

$$\text{OR } \begin{array}{r|l} 4 & 3.00 \\ 12 & 4.75 \\ \hline \end{array}$$

.395833 &c. the answer as before

2. Reduce
- $15s. 9\frac{3}{4}d.$
- to the decimal of a pound?

 $15s. 9\frac{3}{4}d. = \frac{753}{80}$, hence 960) 759000000 (.790625 is the answer.

$$\begin{array}{r} 6720 \\ \hline 870 \\ \mathcal{C}. \end{array}$$

$$\text{OR } \begin{array}{r|l} 4 & 3 \\ 12 & 9.75 \\ 20 & 15.8125 \\ \hline \end{array}$$

.790625 the answer as before.

3. Reduce 9oz. 17dwt. 14grs. to the decimal of a lb. Troy?

9oz. 17dwt. 14grs. $= \frac{4742}{5760}$,hence 5760) 4742000000 (.823263 $\mathcal{C}.$ is the answer. $\mathcal{C}.$

$$\text{OR. Instead of } 24 \left\{ \begin{array}{r|l} 2 & 14 \\ 12 & (7) \\ 20 & 17.583333 \\ 12 & 9.879166 \\ \hline \end{array} \right.$$

.823263 $\mathcal{C}.$ the answer as before.

4. Reduce 2Qrs. 9lb. to the decimal of a ton?

 $2Qrs. 9lb. = \frac{65}{224}$, hence 2240) 65000000 (.029017 $\mathcal{C}.$ is the ans. $\mathcal{C}.$

112

OR,

OR. Instead of 28 $\left\{ \begin{array}{l} 4 \\ 7 \\ 4 \\ 20 \end{array} \right| \begin{array}{l} 9 \\ (2.25) \\ 2.321428 \\ .580357 \end{array}$

.029017 &c. the answer as before.

Practical Questions.

CLASS I.

1. Reduce 10s. 8d. to the decimal of a £? Answer .533333 &c.
2. Reduce 52 days to the decimal of a year? Answer .142465.
3. Reduce 3 *Qrs.* 16*lb.* 12*oz.* to the decimal of a *lb.* avoirdupoise? Answer .899553 &c.

CLASS II.

1. Reduce 19s. 5½*d.* to the decimal of a £? Answer .972916 &c.
2. Reduce 10*oz.* 18*dwt.* 16*grs.* to the decimal of a *lb.* Troy? Answer .911111 &c.

CLASS III.

Reduce 48' 37'' 54''' to the decimal of a degree?
Answer .810527 &c.

As decimals for money in particular are often wanted, the following "*method of finding by inspection (as tis called) the decimal of any number of shillings pence and farthings to a pound, true to three places of figures,*" will be found very useful.

* Rule.

"When the number of shillings are even, take half of them for the first place, and let the second and third be filled up with the farthings contained in the remaining pence and farthings, remembering to add one, when this result is 24 or above: But if the shillings are not even, take half of the even number next less, and encrease the second figure (gained as above) by 5 for the odd shilling."

Examples.

* As shillings are so many 20ths. of a pound, half of them must be so many tenths, and consequently take the place of tenths in the decimal; but when they are odd, their half will always consist of two figures, the first of which will be half of the even number, the next less, and the second a 5 (as will be evident on trial, thus $\frac{13}{2} = 6.5$, &c.) and this confirms the rule, as far as it respects shillings. Now 960 farthings make a pound, and had it so fallen out, that 1000 were equal to

Examples.

What's the decimal of 16s. $4\frac{1}{2}d.$ and 13s. $10\frac{1}{2}d.$?

$\begin{array}{r} 16s. \quad 4\frac{1}{2}d. = .8 \\ 4\frac{1}{2}d. = 18 \text{ farthings.} \\ \hline 16s. \quad 4\frac{1}{2}d. = .818 \text{ the answer.} \\ \hline \end{array}$	$\begin{array}{r} 13s. \quad 10\frac{1}{2}d. = .6 \\ 10\frac{1}{2}d. = 42 \text{ farthings.} \\ \quad 1 \text{ for the 24 in 42.} \\ \quad 5 \text{ for the odd shilling.} \\ \hline 13s. \quad 10\frac{1}{2}d. = .693 \text{ the answer.} \\ \hline \end{array}$
--	---

More examples for inspection.

$\begin{array}{r} s. \quad d. \\ 19 \quad 11\frac{1}{4} = .996. \\ 6 \quad 2 = .308. \\ 12 \quad 8\frac{3}{4} = .636. \end{array}$	$\begin{array}{r} s. \quad d. \\ 1 \quad 10\frac{1}{4} = .092. \\ 0 \quad 0\frac{3}{4} = .003. \\ 0 \quad 1\frac{1}{4} = .005. \end{array}$
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Problem III.

"To find the equivalent value of a given decimal in inferior denominations."

** Rule.*

"Multiply the given decimal by the number of parts in the next less denomination."

to one, 'tis plain any number of farthings would have been so many thousandths, and might have taken their place in the decimal accordingly at once. Farther as $11\frac{3}{4}d.$ (i. e. the greatest number of pence and farthings under a shilling) is but 47 farthings, or takes up but two places, 'tis also plain, that these decimal farthings would have fallen in the second and third place of the required decimal. Now as 960 is but 40 short of 1000, we shall find by division (i. e. $\frac{960}{40} = 24$) that if 960 were but encreased with $\frac{1}{4}$ of itself (i. e. 40) it would be equal to 1000, and consequently that any number of farthings encreased by their $\frac{1}{4}$ part will be an exact decimal expression for them. Hence the remaining part of the rule is found right.

From this it appears farther, that there is no occasion to stop at the third figure for want of a true rule, but for convenience, as the work may be carried on to any required number of places, if to the farthings in the given pence and farthings, there be added their $\frac{1}{4}$ (taken decimally by bringing down cyphers, &c.); OR, which is the same, if, after deducting 6d. or 24 farthings when they are above that sum, you take $\frac{1}{2}$ of half what remains for the subsequent figures. Thus $11\frac{3}{4}d.$ will be found to stand .046875, which the reader may examine by rule, if he thinks it worth his pains.

* As every decimal fraction is a vulgar one, whose denominator is one with as many cyphers prefixed, as it contains figures, this rule will be found to consider the fraction in that light, and hence to proceed in every respect as in problem VIII. of vulgar fractions; the cutting off as directed at every step being an obvious division by the denominator, &c.

denominations, and from the product cut off (for a remainder) as many places on the right hand, as there are in the given decimal. Multiply the remainder by the parts in the next inferior denomination, and from what results cut off as before. Proceed thus till 'tis brought into the least known parts of an integer, and then the several denominations standing on the left hand, make the answer."

Examples.

1. Required the value of .37623, and (2d.) .032764 of a £.

$.37623 = 7s. 6\frac{1}{4}d.$ the answer.	$.032764 = 7\frac{1}{4}d.$ the answer.
20	20
7.52460	0.655280
12	12
6.29520	7.863360
4	4
1.18080	3.453440

3. Required the value of .798645 of a C. weight, and of (4th.) .87628 of a lb. Troy?

$.798645 = 32rs. 5lb. 7oz.$ the ans.	$.87628 = 10oz. 10dwt.$
4	12 7grs. the ans.
3.194580	10.51536
28	20
1556640	10.30720
389160	24
5.448240	122880
16	61440
7.171840	7.37280

Practical Questions.

CLASS I.

1. What's the value of .378125 of a £? - - - Answer 7s. 6 $\frac{3}{4}$ d.
2. What's the value of .625 of an C. weight? Answer 22rs. 14lb.
3. What's the value of .74 of a lb. Troy?
Answer 8oz. 17dwt. 14 $\frac{2}{3}$ grs.

CLASS

CLASS II.

1. What's the value of .6725 of an C. weight?
Answer 22rs. 19lb. 5 $\frac{3}{4}$ oz.
2. What's the value of .61 of a tun of wine?
Answer 2hhd. 27gal. 2qts. 1.76pts.

CLASS III.

What's the value of .3 of a year? - - - - Answer 109d, 12h.

"By the following rule the value of any decimal of a pound may be determined by inspection, so as not to err one farthing."

* Rule.

"Double the first figure (or place of tenths) for shillings, and if the second be 5 or exceed 5 reckon one shilling more. Then call the figures in the second and third places (after 5 is deducted out of the second, when possible) so many farthings, abating one for every 24, which result made pence &c. and joined to the shillings, is the answer."

Examples.

Required the value of .718, and .876 of a £?

$$\begin{array}{r} \text{s.} \\ 7 \times 2 = 14 \quad \text{d.} \\ \frac{18}{4} = \quad 4\frac{1}{2} \\ \hline .718 = 14 \quad 4\frac{1}{2} \end{array}$$

$$\begin{array}{r} \text{s.} \\ 8 \times 2 = 16 \\ 5 \text{ in } 7 \text{ gives } 1 \quad \text{d.} \\ 26 \text{ farthings, the remainder } \} = 6\frac{1}{4} \\ \text{with one deducted is } \} \\ \hline .876 = 17 \quad 6\frac{1}{4} \end{array}$$

Examples for inspection.

s.	d.	s.	d.
.927 = 18	6 $\frac{1}{2}$.	.061 = 1	2 $\frac{3}{4}$.
.351 = 7	0 $\frac{1}{4}$.	.020 = 0	5.
.203 = 4	0 $\frac{3}{4}$.	.009 = 0	2 $\frac{1}{2}$.

Scholium.

* This rule being the exact reverse of the last given for inspection must be founded on the same reasoning.

Scholium.

If we calculate, by the help of problem II. (and other means hereafter mentioned) the decimal of every inferior value to any integer of money, weight, measure, &c. and dispose of them in their natural order, against the quantities they came from, we shall have what are called *decimal tables*; the intent of which is, to find by inspection the decimal of any quantity given, and on the contrary the quantity answering to any proposed decimal. But as printed tables may err, and a suspicion of that renders what depends upon them unsatisfactory, and also as such decimals are not difficult to find at all times by the above-mentioned problem, we shall insert here no more than one, by way of specimen. And farther, as we may reasonably suppose persons, who may frequently want compendiums of this kind in any particular affair, will only trust to those they have constructed or examined themselves, tis hoped that the following hint for making them, &c. will be looked on as all that is proper in a treatise of this kind.

General Method of constructing decimal tables.

Find the decimal of an unit of the lowest species to 8 or 9 places of figures by problem II. and then by doubling of it, trebling of it, &c. OR, (after the two first decimals are got) adding the last found, to the first found for the next succeeding &c. you may easily find the decimal of all the given quantities in the scale of ascent from the lowest to the highest, which digested in the manner of the following specimen make the tables required.

The method is too obvious to need any thing more by way of illustration; only we shall observe. that in the way last hinted of constructing them (which is the most easy) it will be sometimes expedient to continue the first decimal (or *root* as it may be called) 2 or 3 places farther, than the decimals in the table are intended to go, in order that the truth of some of the right hand digits may not be affected from neglecting the carriages arising from adding the subsequent ones.—We must also observe farther, that when several 9s happen to follow one another, they may be truly neglected, if the figure immediately preceding them be encreased with an unit, as will be evident * hereafter.

A decimal

* See the next Scholium.

A decimal table of avoirdupoise weight.

No	Pounds.	Ounces.	No	Pounds.	Ounces.
1	.00892857,	.000558035,	16	.14285714,	.008928571,
2	.01785714,	.001116071,	17	.15178571,	
3	.02678571,	.001674107,	18	.16071428,	
4	.03571428,	.002232142,	19	.16964285,	No Qrs.
5	.04464285,	.002790178,	20	.17857142,	
6	.05357142,	.003348214,	21	.1875.	
7	.0625.	.00390625.	22	.19642857,	1 .25.
8	.07142857,	.004464285,	23	.20535714,	
9	.08035714,	.005022321,	24	.21428571,	
10	.08928571,	.005580357,	25	.22321428,	2 .5.
11	.09821428,	.006138392,	26	.23214285,	
12	.10714285,	.006696428,	27	.24107142,	
13	.11607142,	.007254464,	28	.25.	3 .75.
14	.125.	.0078125.			
15	.13392857,	.008370535,			

The following rules for the use of the table will readily occur.

1. To find the decimal of any given quantity.

Rule.

Add the decimals that stand against each separate denomination together for the answer.

Example.

What's the decimal of 3Qrs. 12lb. 9oz. to an C. weight?

Against 3Qrs. is - - - - .75
 Against 12lb. is - - - - .10714285
 Against 9oz. is - - - - .00502232

Their sum - - - - .86216517 is the answer.

Kk

2. To

2. To find the quantity of any given decimal.

Rule.

If the decimal agree precisely with any in the table, the answer is against it; but if not, take the next less in the table, and note down the quantity it points out, as part of the answer, then subtract this tabular decimal from the given one, and seek what remains or a decimal next less in the table in the same manner, and against it you have another part of the answer in a lower species. Go on thus exhausting as far as you can, and you'll have the answer as near as is practicable.

Example.

What's the value of .86216517 of a C. weight?

The next less is $\begin{array}{r} .86216517 \\ .75 \end{array}$ for 3*rs.*

The next less to the above is $\begin{array}{r} .11216517 \\ .10714285 \end{array}$ for 12*lb.*

And opposite to this $\begin{array}{r} .00502232 \end{array}$ is 9*oz.*

Sum. $\begin{array}{r} 3 \\ 12 \\ 9 \end{array}$ the answer.

(Problem IV.)

Addition of Decimals.

* *Rule.*

"*LIKE* places being wrote under like places (as units, tens; tenths, hundredths, &c.) begin at the right hand column, and add as in whole numbers, and in the result make a decimal point, under the points of the given numbers for the answer. Or (which is the same) prick off a number of places from the right hand equal to the most in any of the given decimals, and you have an expression of the sum."

Examples.

* As the tenfold proportion of decrease (from left to right) is carried on from one place to another, without interruption (out of whole numbers into decimals, &c.) the reason of this rule is as obvious, as that of common addition.

Examples.

$$\begin{array}{r}
 376.25 \\
 86.125 \\
 637.4725 \\
 6.5 \\
 358.865 \\
 41.02 \\
 \hline
 1506.2325
 \end{array}$$

$$\begin{array}{r}
 .25 \\
 .378 \\
 .895 \\
 .5 \\
 .9356 \\
 .6827 \\
 \hline
 3.6413
 \end{array}$$

$$\begin{array}{r}
 2.45 \\
 16. \\
 3.78 \\
 12.623 \\
 9. \\
 18.45 \\
 \hline
 62.303
 \end{array}$$

Practical Questions.

CLASS I.

1. Add 50.007, 2.0071, 59.4, 3207.1, together?
Answer 3318.5141.
2. Add 560.2, 13.675, .257, 62.125, together?
Answer 636.257.
3. Add 3.5, 47.25, 927.01, 2.0073, 1.5, together?
Answer 981.2673.

CLASS II.

1. Add 27.2, 163.219, 4.2, 1637.5, .125, together?
Answer 1832.244.
2. Add 3275, 27.514, 1.005, .725, 7.32, together?
Answer 3311.564.

CLASS III.

- Add 276, 54.321, .65, 112, 12.5, .0463, together?
Answer 412.0573.

*(Problem V.)**Subtraction of Decimals.*** Rule.*

“**PLACE** the numbers as in addition, and subtract, as in whole numbers, every figure in the subtractor from that, which stands over it in the subtrahend (imagining cyphers at the end of the decimals in the subtrahend, when tis short of the number of digits in the subtractor). and

K k 2

place

* This is also obvious from the notation.

place the decimal point in the result as in addition, and you have an expression of the remainder."

Examples.

From	6213.725	219.5	426.15
Take	162.25	162.375	3.4185
Remains	<u>6051.475</u>	<u>57.125</u>	<u>422.7315</u>

Practical Questions.

CLASS I.

1. What's the difference of 127.62, and 13.725?
Answer 113.895.
2. What's the difference of 6.1293, and .0672?
Answer 6.0621.
3. What's the difference of 1562, and 3.721?
Answer 1558.279.

CLASS II.

1. What's the difference of 6213.21, and 561?
Answer 5652.21.
2. What's the difference of 51.27, and .0063?
Answer 51.2637.

CLASS III.

- What's the difference of 3760.279, and 423.0076?
Answer 3337.2714.

(Problem VI.)

Multiplication of Decimals.

* Rule.

"PLACE the factors and multiply them as in whole numbers, and from the right hand of the product cut off as many places for a decimal, as there are fractional digits in both factors together, remembering, when the figures in the product fall short of the sum, to supply the defect with cyphers on the left hand, and you have an expression of the product required."

Examples

- * In the first example that follows, by supposing the decimal denominators restored

Examples.

64.25	.2365	.02534
3.653	.2435	.03256
19275	11825	15204
32125	7095	12670
38550	9460	5068
19275	4730	7602
234.70525	.05758775	.0008250704

Practical Questions.

CLASS I.

1. Multiply 79.347 by 23.15? - - - Answer 1836.88305.
2. Multiply .63478 by .8264? - - - Answer .524582192.
3. Multiply 276.38256 by 7.985? - - Answer 2206.91474160.

CLASS

stored, and proceeding with the new equivalent expressions, as in vulgar fractions, we shall readily see the reason of the whole. Thus $64.25 = 64\frac{25}{100}$, and $3.653 = 3\frac{653}{1000}$. Now these two mixt numbers reduced to improper fractions in order to be multiplied will be found to be $\frac{6425}{100}$, and $\frac{3653}{1000}$, in which we may observe that the numerators are the very same digits with those of the given decimal expressions respectively. Hence (as we are to multiply the numerators for a numerator, &c.) their product will be the same, as that of the decimal digits, i. e. 23470525, which placed over the product of the denominators 100 and 1000 gives the improper fraction $\frac{23470525}{100000}$ for the answer. Now as the cyphers in the denominators of the two mixt numbers are equal to the sum of the decimal digits in the given fractions (per notation) 'tis well known the cyphers in the product will be so too; consequently in reducing this improper fraction to a mixt number by division (which is the next step) we must cut off in the numerator for a remainder (or numerator of the fractional part) as many digits as there are cyphers in the denominator (per case 1st. of contractions in multiplication part I.) i. e. (as above observed) a number equal to the sum of the decimal digits originally given. Hence we have $234\frac{70525}{100000} = 234.70525$ for the product.

The agreement of this reasoning with the directions of the rule sufficiently demonstrate its truth, and the process also shews in a striking manner, the great advantage decimals have over vulgar fractions, with regard to simplicity and dispatch.

CLASS II.

1. Multiply .0096 by .0025? - - - Answer .000024.
2. Multiply 3684.7928 by 84.216? - Answer 310318.5104448.

CLASS III.

Multiply .385746 by .00463? - - - Answer .00178600398.

As in multiplications, where many decimal places are used in the factors, the product must contain several, which by reason of their very small value may be looked on as superfluous, the following "method of contracting the work, so as to retain as many decimal places in the product, as may be thought necessary," will be found of good use.

* Rule.

"Under that decimal place in the multiplicand, thought necessary to be retained in the product, write the units place of the multiplier, and then invert (or dispose in a contrary order to what is common) all its other places. Next begin to multiply (figure by figure as usual) and omit those places in the multiplicand to the right of the multiplying digit, and place the first figures of every product directly under one another, observing only to increase this first digit in every line by what would arise from that immediately following on a supposition of carrying (in that place only) one from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. and the sum of these lines will give the product generally exact to the required place."

A single example done both ways will be sufficient illustration.

Required the product of 245.378263 by 72.4385, with only four of the first decimal digits preserved?

Contracted

* As we find nothing to the contrary in any author it appears probable, that the rule for carrying from the right hand digit mentally to the first put down, has only been gained from trial and experience: And as to the particular of inverting the multiplier, &c. the reason of it may be full as well apprehended from a view of the work at large, as any thing that could be offered here.—If the reader thinks proper, he may practice himself farther in this method, by working over those questions answered before the other way.

Contracted way.

$$\begin{array}{r}
 245.378263 \\
 5834.27 \\
 \hline
 171764784. \\
 4907565.. \\
 981513... \\
 73613.... \\
 19630..... \\
 1226..... \\
 \hline
 17774.8331 \text{ Answer,} \\
 \hline
 \end{array}$$

Common way.

$$\begin{array}{r}
 245.378263 \\
 72.4385 \\
 \hline
 1226 \overline{) 891315} \\
 19630 \overline{) 26104} \\
 73613 \overline{) 4789} \\
 981513 \overline{) 052} \\
 4907565 \overline{) 26} \\
 171764784 \overline{) 1} \\
 \hline
 17774.8333 \overline{) 043255} \\
 \hline
 \end{array}$$

(Problem VII.)

Division of Decimals.

* Rule.

"IF the dividend have not so many places of decimals as are in the divisor, make as many at least, by annexing decimal cyphers. Then divide as in whole numbers (adding still more cyphers to the dividend and bringing them down as wanted) to any required exactness. Next cut off in the quotient a number of figures for a decimal, equal to the difference of the fractional digits used in the dividend, and those contained in the divisor, if there be as many in the quotient; if not, cyphers must be prefixed to the left hand to supply the defect, and this gives a true expression of the quotient, as far as 'tis carried."

Examples.

* The reason of the rule for cutting off the difference between the decimals in the dividend, and those of the divisor, in the quotient is obvious, as the quotient must be such an expression, as multiplied into the divisor is to make the dividend, i. e. the sum of their decimals must be equal to those in the dividend, as it is in effect their product. And with regard to the precaution of having as many decimals at least in the dividend as in the divisor, it is drawn from what is just observed; as the neglect of it (on applying the above rule) would give more decimals in the product of the divisor and quotient, than there ought to be. This considered, and also that division is in every respect the reverse of multiplication, 'tis hoped the trouble of running an example over for farther illustration, in a manner similar to that used in the demonstration of multiplication, may be spared.

Examples.

$$64.25) 234.70525 \text{ (3.653.} \\ 192 \text{ 75}$$

$$\begin{array}{r} 41 \text{ 955} \\ 38 \text{ 550} \\ \hline \end{array}$$

$$\begin{array}{r} 3 \text{ 4052} \\ 3 \text{ 2125} \\ \hline \end{array}$$

$$\begin{array}{r} 19275 \\ 19275 \\ \hline \end{array}$$

$$.7854) 14.0000000 \text{ (17.825 \&c.} \quad .00325) .08765000 \text{ (26.66 \&c.}$$

$$7 \text{ 854}$$

$$6 \text{ 1460}$$

$$5 \text{ 4978}$$

$$64820$$

$$62832$$

$$19880$$

$$15708$$

$$41720$$

$$39270$$

$$2450 \text{ \&c.}$$

$$650$$

$$2165$$

$$1950$$

$$2150$$

$$1950$$

$$2000$$

$$1950$$

$$150 \text{ \&c.}$$

Practical Questions.

CLASS I.

1. Divide 743.574 by 75? - - - - - Answer 9.91432.
2. Divide 48 by .0675? - - - - - Answer 711.111 \&c.
3. Divide 2 by 28.74? - - - - - Answer .069589 \&c.

CLASS II.

1. Divide .8727587 by .162? - - - - - Answer 5.38739 \&c.
2. Divide .0081892 by .347? - - - - - Answer .0236

CLASS III.

- Divide 48173.6 by 2187? - - - - - Answer 22.027251 \&c.

"When

"When the divisor contains many decimal places the work may be usefully contracted by the following"

* Rule.

"In the course of the division at any step of it, or even from the first, if you will, you may desist from bringing down the next decimal figure, and suppose what remains all along a new dividend, provided that at every neglect of this kind you point off, as useless, one figure from the right hand of the divisor, and also in multiplying the remaining figures a due regard is had to the last figure cut off, as in contracted multiplication."

The following example done both ways will sufficiently illustrate its nature.

Contracted way.		Common way.	
3.14159)	12.169825 (3.87377	3.14159)	12.169825 (3.87377
.....	9 42477		9 42477
	2 74505.		2 74505 5
	2 51327.		2 51427 2
	23178..		23178 30
	21991..		21991 13
	1187...		1187 170
	942...		942 477
	245....		244 6930
	220....		219 9113
	25.....		24 78170
	22.....		21 99113
	3.....		2 79057

Scholium.

In the course of the reader's practice, he will meet with several decimal expressions, in which the same figure would be repeated, *ad infinitum*, as for instance, in reducing these vulgar fractions to
L1
decimals,

* By comparing the process with the work at large, as much may be seen of its reason, as can be taught, after what was said in contracted multiplication is taken for granted.---The learner may practise himself in any of the foregoing examples done the common way.

decimals, viz. $\frac{1}{9} = .1111$ *℥c.* $\frac{1}{3} = .3333$ *℥c.* $\frac{2}{3} = .6666$ *℥c.* Also he will find, that in some reductions the figures of the decimals will return over and over again in a certain order, as 54, 54, 54, *℥c.* 259, 259, 259, *℥c.* which decimals are drawn from these vulgar fractions, $\frac{5}{11}$, $\frac{7}{27}$, *℥c.*—The first of these kinds is by arithmeticians aptly called *repetends*, and the second *circulates*, as those which are not *finite*, and neither repeat nor circulate, are called *approximates*. Now when either of the two first sorts fall in the way (with some others of a mixt nature) methods have been discovered of managing them so as to give a result in computation as true as if they were finite; the advancing of which praxis has discovered several very curious, and remarkable properties of decimals. But as the subject is very copious, and the multiplicity of devices used in their management has in a great measure destroyed their intended brevity, we shall refer those who are desirous of having a farther knowledge of these things, to the * writers who have professedly taught them; giving it as our opinion (more for the satisfaction of those who have neither leisure nor opportunity for these enquiries, than to under-value so curious an arithmetical subject) that when the extreme niceness of repetends and circulates is wanted, vulgar fractions seem to be to the full as eligible.

While talking upon this head, it may not however be amiss to observe, that all repetends, and circulates are precisely equal to some vulgar fraction, and that in many instances it may be proper to know this particular. Now this may be easily inferred from the generation of these decimals; for as $\frac{1}{9}$ gives .1111 *℥c.* $\frac{2}{9} = .2222$ *℥c.* $\frac{3}{9} (= \frac{1}{3}) = .3333$ *℥c.* $\frac{4}{9} = .4444$ *℥c.* or, in general, any simple fraction which has a 9 (or may be made equal to one having 9) for its denominator, gives a repetend of the same figure as its numerator; we may be certain, when we see any pure repetend, that the repeating digit placed over 9 is an equivalent vulgar fraction. Thus .3333 *℥c.* $= \frac{3}{9} = \frac{1}{3}$, .6666 *℥c.* $= \frac{6}{9} = \frac{2}{3}$, *℥c.* and from this it is argued by analogy, that when 9 repeats it is $= \frac{9}{9}$ or an unit, which last consideration accounts for what was observed of this property in the last scholium concerning decimal tables.

Farther, as the digits in a pure circulate (from the above) are ninth parts, its equivalent vulgar fraction is found by using for a denominator to a period, as many 9s as it contains digits. Thus .54, 54, 54, *℥c.* $= \frac{54}{99} = \frac{6}{11}$, .259, 249, &c. $= \frac{259}{999} = \frac{7}{27}$, &c.

Praxis.

* *Cunn's* treatise of fractions contains the whole art, and *Wilson* in his arithmetic has entered very ingeniously into their nature; also in *Robertson's* mensuration may be found a well wrote system, and in *Malcolm's* arithmetic, they are very learnedly handled, and the rules, &c. demonstrated.

THE algorithm of decimals attained, all that now remains is to add a question or two, solved by their help, for the perusal of the learner; observing, that tho' these fractions are often very usefully applied in business, yet they will be found to be of the greatest, and most constant use in *mensuration, gauging*, and other mathematical subjects.

1. If a yard of cloth is worth 6s. 9d. what come fifty nine yards to at that rate?

ence 6.75
59
 6075
3375
 20) 39|8.25
60
 £.19-18-3 the answer.

2. In £.680-8 how many piftoles at 18s. each?

$$\begin{array}{r} 18s. = .9 \\ 8s. = .4 \end{array} \quad \begin{array}{r} .9) 680.4 \\ \hline 756 \text{ Answer.} \end{array}$$

3. If $2\frac{1}{2}$ lb. of tobacco cost 3s. 3d., what will $234\frac{3}{4}$ lb. cost?

OR

OR 2.5 : 3.25s. :: 234.75 : the answer.

3.25

117375
46950
70425

2.5) 762.9375 (305.175 = (valued) £.15-5-2.
75

129
125

43
25

187
175

125
125

4. If a man spends £.4-2-3 in a week, what is that per year?

2s. 3d. = .1125 of a £. ---- 7d. : 4.1125 :: 365d. : Answer,

365

205625
246750
123375

7) 1501.0625

214.4375 = (per inspection prob. III.)
£.214-8-9.

5. In £.1469-12-6 Flemish how many pounds sterling, exchange at 34s. Flemish per £ sterling?

1.7l. : 1l. :: 1469.625l. : Answer.

$$\begin{array}{r}
 1,7) 1469.625 \quad (864.485 = \text{£}.864-9-8\frac{1}{2}. \\
 \underline{136} \\
 109 \\
 \underline{102} \\
 76 \\
 \text{\&c.}
 \end{array}$$

6. In £.586-18-6 how many Portugal millreas, exchange at 5s. 6d. per millrea?

$$.275l. : 1m. :: 586.925l. : \text{Answer.}$$

$$\begin{array}{r}
 .275) 586.925 \quad (2134^{mr}. 273r. \\
 \underline{550} \\
 369 \\
 \underline{275} \\
 942 \\
 \text{\&c.}
 \end{array}$$

7. Bought $36\frac{1}{2}$ yards of fine broad cloth at 10s. 6d per yard; sold thereof $17\frac{1}{4}$ yards at 12s. 3d per yard, and the rest at 13s. 9d, what shall I have gained by the sale thereof?

$$1y. : 10.5s. :: 36.5y.$$

$$\begin{array}{r}
 10.5 \\
 \underline{} \\
 1825 \\
 3650
 \end{array}$$

$$20) 38|3.25$$

$$\text{£}.19-3-3 \text{ what it cost.}$$

$$\begin{array}{r}
 36.5 \\
 17.25 \text{ First part sold.} \\
 \underline{}
 \end{array}$$

$$\begin{array}{r}
 19.25 \text{ Second Do.} \\
 \underline{}
 \end{array}$$

$$1y. :$$

$$1y. : 12.25s. :: 17.25y.$$

$$\begin{array}{r} 12.25 \\ \hline 8625 \\ 3450 \\ 3450 \\ 1725 \end{array}$$

$$20) 21|1.3125$$

$$\underline{\underline{\pounds.10-11-3\frac{3}{4}}} \quad \text{First part fold for.}$$

$$1y. : 13.75s. :: 19.25y.$$

$$\begin{array}{r} 13.75 \\ \hline 9625 \\ 13475 \\ 5775 \\ 1925 \end{array}$$

$$20) 26|4.6875$$

$$\underline{\underline{\pounds.13-4-8\frac{1}{4}}} \quad \text{Second Do.}$$

Therefore $\pounds.10-11-3\frac{3}{4} + \pounds.13-4-8\frac{1}{4} - \pounds.19-3-3 = \pounds.4-12-9$ the answer.

3. A in company with B and C, puts in $\pounds.12-5$, B puts in $\pounds.20-15$, and C puts in $\pounds.32-10$, and they gained $\pounds.11-9-3$; required each person's share?

$\pounds.12-5 + \pounds.20-15 + \pounds.32-10 = \pounds.65-10$ the whole stock: Hence the statings are

$$65.5 : 11.4625 :: 12.25 : \text{A's share.}$$

$$65.5 : 11.4625 :: 20.75 : \text{B's share.}$$

$$65.5 : 11.4625 :: 32.5 : \text{C's share.}$$

But $11.4625 \div 65.5 = .175$ a common multiplier, therefore

$$12.25 \times .175 = 2.14375 = \pounds 2-2-10\frac{1}{2} = \text{A's share.}$$

$$20.75 \times .175 = 3.63125 = \pounds 3-12-7\frac{1}{2} = \text{B's share.}$$

$$32.5 \times .175 = 5.6875 = \pounds 5-13-9 = \text{C's share.}$$

$$\underline{\underline{11.4625}} = \pounds 11-9-3 \quad \text{Proof.}$$

Haying

Having limited every formal illustration of the use of decimals to this head, and as most of the cases in practice may be solved conveniently by them, it might here be reasonably expected we should give an instance or two in each; but as there is one easy rule which seems very well to comprehend them all, we shall omit the particular ones and insert it as follows.

*"A general * rule for solving questions in practice decimally."*

"Annex to the highest denominations of the things given the decimal of the inferior parts (if any) and multiply this mixed number by the pounds in the price, when 'tis so much: Then point off the units place of the given quantity (which is only dividing by 10) for the amount of the whole at 2 shillings, and from it deduce the value of what odds may be in the price, by taking it such times, parts of times, &c. as they import, and the sum of these results is the answer decimally expressed."

Examples.

1. $847\frac{1}{2}$ ells ($=847.5$ ells) at $8\frac{1}{4}d$.

		£.
6d.	$\frac{1}{4}$	84.75, value at 2s.
2d.	$\frac{1}{2}$	21.1875
$\frac{1}{2}$	$\frac{1}{4}$	7.0625
$\frac{1}{4}$	$\frac{1}{2}$	1.7656
		.8828
		30.8984 = (per inspection) $\text{£. } 30-17-11\frac{1}{2}$ the answer.

2. $793\frac{5}{8}lb.$ ($=793.625lb.$) at $18s. 7d$.

6d. $\frac{3}{4}$

* The reason is so very obvious from what was said in note to case 4th. of practice, &c. that the reducing it into words is almost superfluous, and the reader will observe, that this method seems to be the best apply'd to questions, where the given quantity contains odd denominations easy to turn into decimals.

$$\begin{array}{r|l}
 6d. & \frac{1}{4} \quad \begin{array}{l} \text{£.} \\ 79.3625 \text{ value at } 2s. \\ \hline 9 \end{array} \\
 1d. & \frac{1}{6} \quad \begin{array}{l} 714.2625 \\ 19.8406 \\ \hline 3.3067 \end{array} \\
 \hline
 & 737.4098 = \text{£.} 737-8-2\frac{1}{4} \text{ the answer.}
 \end{array}$$

3. 427oz. 17dwt. 13grs. (=427.877oz.) at £.3-11-6.

$$\begin{array}{r}
 \text{£.} \\
 427.877 \text{ value at } 1l. \text{---Value at } 2s. = 42.7877. \\
 \hline 3 \\
 1283.631 \\
 213.9385 = 42.7877 \times 5 \text{ for } 10s. \\
 6d. \text{ is } \frac{1}{2}) \quad 21.3938 = \frac{1}{2} \text{ of } 42.7877 \text{ for } 1s. \\
 \hline 10.6969 \\
 \hline 1529.6602 = \text{£.} 1529-13-2\frac{1}{2} \text{ the answer.}
 \end{array}$$

4. In £.436-15 sterling how many pounds Flemish exchange at 36s. 6d. Flemish per £ sterling?

$$\begin{array}{r}
 6d. \text{ is } \frac{1}{4}) \quad \begin{array}{l} \text{£.} \\ 43.675 \text{ value at } 2s. \\ \hline 8 \end{array} \\
 \hline 349.400 \\
 10.918 \\
 \hline 436.75 \\
 \hline 797.068 = \text{£.} 797-1-4\frac{1}{2} \text{ the answer.}
 \end{array}$$

5. In 4360m.r. 325r. of Portugal how many pounds sterling, exchange at 5s. 6d per millrea?---As 1000 reas make a millrea, the reas are natural decimals.

Hence

Hence 1s. is $\frac{1}{2}$) $\overset{\text{£.}}{436.0325}$ value at 2s.
 $\underline{\hspace{1.5cm}}$

872.0650
 6d. is $\frac{1}{2}$) $\underline{218.0162}$
 $\underline{109.0081}$

$\underline{\underline{1199.0893}} = \text{£.}1199-1-9\frac{1}{2}$.

6. In 2627 *pez.* 10s. 8d. of Genoa how much sterling, exchange 4s. 8d. per pezzo?

8d. is $\frac{1}{3}$) $\underline{262.7533}$ value at 2s.
 $\underline{\hspace{1.5cm}}$

525.5066
 $\underline{87.5844}$

$\underline{\underline{613.0910}} = \text{£.}613-1-9\frac{3}{4}$.

*Practical Questions (for the * exercise of decimals.)*

CLASS I.

1. If $5\frac{1}{4}$ ells of cloth cost $\text{£.}2-16-8\frac{3}{4}$ what will $278\frac{1}{2}$ cost at the same rate?
 Answer $\text{£.}150-9-4$.

2. A merchant bought 356oz. 13dwt. 15grs. of gold plate for 1160l. the question is what he paid for an ounce?
 Answer $\text{£.}3-5-0\frac{1}{2}$.

3. What will $326\frac{1}{4}$ lb. of tobacco come to at 3s. 6d. for $1\frac{1}{2}$ lb?
 Answer $\text{£.}38-1-3$.

M m

4. Lent

* Those who are desirous of applying decimals to the solution of questions in business, may exercise themselves farther in some of the examples already gone past, in the rule of three, practice, &c.

4. Lent my friend 34*l.* for $\frac{5}{8}$ of a year, how much ought he to lend me $\frac{2}{12}$ of a year, to requite my kindness?

Answer £. 28-6-8.

CLASS II.

1. What is the worth of 371 $\frac{1}{2}$ stone of goods at 7*s.* 10 $\frac{1}{2}$ *d.* per stone?

Answer £. 146-5-6 $\frac{3}{4}$.

2. What is the worth of 827 $\frac{3}{4}$ yards of painting at 10 $\frac{1}{2}$ *d.* per yard?

Answer £. 36-4-3 $\frac{1}{4}$.

3. In 6273*m.r.* 627*r.* of Portugal how many pounds sterling, exchange at 5*s.* 3*d.* per millrea?

Answer £. 1646-16-6 $\frac{1}{4}$.

CLASS III.

1. Suppose four men A, B, C, and D, trade in company; A put in 50*l.* B 16*l.* C 25*l.* and D £. 18-10; they gained £. 20-15, what was each man's part?

Answer, A's share = £. 9-9-5 $\frac{3}{4}$, B's = £. 3-0-7 $\frac{1}{4}$, C's = £. 4-14-8 $\frac{3}{4}$, and D's = £. 3-10-1 $\frac{1}{4}$.

2. Suppose the length of the tropical year; (or space of time in which the sun, running through the whole ecliptic, consisting of 360 degrees, is returned to the same equinoctial, or solstitial point, from whence he departed) to consist of 365 days 5 hours and 49 minutes, the question is to know the sun's mean or equal motion for one day, to wit, what part of 360 degrees the sun moves through in a whole day?

Answer 59' 8" (59*c.*)

Of Raising Powers.

Definition.

"A Power is a number, which is the product of one and the same number multiplied into itself continually a certain number of times.

Thus $2 \times 2 = 4$ is the 2d. power of 2 usually called the square.

$2 \times 2 \times 2 = 8$ is the 3d. power of 2 usually called the cube.

$2 \times 2 \times 2 \times 2 = 16$ is the 4th. power of 2.

$2 \times 2 \times 2 \times 2 \times 2 = 32$ is the 5th. power of 2.

&c.

That is, in general, the power is denominated 2d. 3d. 4th. 5th. &c. according as the factor, that produces it, is 2, 3, 4, 5, &c. times concerned, as is obvious from the above. And when this operation is denoted symbolically, 'tis done by drawing a line over the given number to be raised, and at the right hand of it placing a figure (called the index) which refers to the name of the power. Thus 8^2 signifies 8 raised to the 2d. power or squared, 56^3 signifies 56 raised to the 3d. power or cubed, &c."

These distinctions premised, the praxis is so very easy, that an example or two will be * sufficient.

1. Required the square of 144? --- $144 \times 144 = 20736$ the answer.

2. Required the cube of 3.5? --- $3.5 \times 3.5 \times 3.5 = 42.875$ the answer.

3. Required the fourth power of $\frac{3}{4}$? --- $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{81}{256}$ the answer.

Mm 2

Of

* As 'tis to be hoped the reader will not now need to be beat into every thing merely by the force of a number of practical questions, we shall not for the future, where we think it unnecessary, take up room with that part of the plan.

Of Extracting Roots.

Definition.

“*A Root is a number, by whose continual multiplication into itself a power is produced: And a number is denominated the square, cube, 4th. 5th. &c. root of another, if it is, when raised to the 2d. 3d. 4th. 5th. &c. power, equal to that other. Thus 2 is the square root of 4, cube root of 8, and 4th. root of 16; 5 the square root of 25; 4 the cube root of 64; 3 the 4th. root of 81, &c. because these roots, raised to the power indicated by their names, make the numbers they are said to be the roots of.*

Before we give rules for discovering the roots of given numbers we must observe, that tho' there is no number of which we cannot find any power exactly, yet there may be many numbers given, of which a precise root can never be determined: However, by the help of decimals, we can approximate towards it, as near as we think proper, or as may be convenient. Thus the square or any other root of 3, 5, 6, 7, &c. can never be exactly discovered.

The roots, which thus approximate, are called surd roots, to distinguish them from those which are perfectly adequate, termed sometimes rational roots.

The usual character for denoting this extraction is a radix ($\sqrt{\quad}$) placed over the given number, with a figure in the V part, referring to the name of the root, except it be the square root, and then the figure is understood and omitted. Thus $\sqrt[2]{5}$ or $\sqrt{5}$ signifies the square root of 5, $\sqrt[3]{9}$ signifies the cube root of 9, &c.”

*These things premised we shall next shew how to extract the square and cube root of any number; * neglecting those of higher powers, as they are not often wanted in common affairs.*

Problem

* They are likewise some of them very difficult to come at by common arithmetic, and best referred to the superior methods of algebra, and logarithms, (where they are too the most required), which last invention affords an easy and universal method of performing these operations.

Problem I.

To extract the SQUARE ROOT.

* Rule.

“Distinguish the given numbers into periods of two figures, by putting a point over the units place, and over every other figure from thence, to the left hand for integers, and to the right hand for decimals (when they are concerned). † Then seek the greatest square number in the left hand period (whether it consists of two figures or an odd one) and place the root of it down like a quotient figure in division. This done subtract the square from the period, and to the remainder bring down the next period (as in division) for what is called a resolvend. To the left of the resolvend write the double of the root for a divisor, and then ask, how oft this may be had in the resolvend, with its last figure supposed cut off; the ‡ answer to which (remembering that it can never go above 9 times) place after the other figure in the root, and also to the right of the divisor. This done, multiply the increased divisor by the last figure in the root, and subtract the product from the resolvend; then is what remains with the next period brought down, a new resolvend, which must be managed in a similar manner, by doubling all the root for a divisor, finding a new figure by asking as in division, annexing it to the divisor before multiplication, &c. Proceed thus till all the periods are brought down, and you have the root required, which if not perfect by reason of a remainder, may be carried to any degree of accuracy by bringing down periods of cyphers, and proceeding as before.

When there are decimals in the root, they are easily distinguished in the process; for the last period in the whole numbers must give the last integral figure in the root; or there is one decimal figure in the root for every two (or period) of them in the given square.

As to finding the square root of vulgar fractions, 'tis evident from the method of raising their powers, that the square root of the numerator and denominator (tried always in their lowest terms) will (when perfectly got)

* The demonstration is referred to the latter end of this article.

† This table shews the square of the nine digits.

1.	2.	3.	4.	5.	6.	7.	8.	9	roots.
1.	4.	9.	16.	25.	36.	49.	64.	81	squares.

‡ When it goes 0 times, a cypher must be put in the quotient, and another period brought down, &c. as is evident.

got) be a numerator and denominator of the root: But as they can seldom be determined exactly, 'tis the best to reduce them to decimals, and then find the roots of these equivalent expressions as above.

The truth of these roots may be easily proved by raising them to the given power and taking in the remainder, when there is any, as in the proof of division."

Examples.

1. Required the square root of 5499025?

$$\begin{array}{r}
 \begin{array}{c} \cdot \cdot \cdot \cdot \\ 5499025 \end{array} \begin{array}{l} (2345 \text{ the answer.} \\ 4 = \text{the greatest square in } 5. \end{array} \\
 2 \times 2 = 4 \quad \underline{3} \\
 43) 149 \text{ Refolvend.} \\
 23 \times 2 = 46 \quad \underline{4} \quad 129 = 43 \times 3 \\
 464) 2090 \text{ Refolvend.} \\
 234 \times 2 = 468 \quad \underline{5} \quad 1856 = 464 \times 4 \\
 4685) 23425 \text{ Refolvend.} \\
 \underline{23425}
 \end{array}$$

2. Required the square root of 472738?

$$\begin{array}{r}
 \begin{array}{c} \cdot \cdot \cdot \\ 472738 \end{array} (687.559 \text{ \&c. the answer.} \\
 \underline{36} \\
 128) 1127 \\
 \underline{1024} \\
 1367) 10338 \\
 \underline{9569} \\
 13745) 76900 \\
 \underline{68725} \\
 137505) 817500 \\
 \underline{687525} \\
 1375109) 12997500 \\
 \underline{12375981} \\
 621519 \text{ \&c.}
 \end{array}$$

3. Required

3. Required the square root of 76395820.7163?

$$\begin{array}{r}
 \begin{array}{c} \cdot \cdot \cdot \cdot \cdot \cdot \\ 76395820.7163 \end{array} \text{ (8740.47027 \text{ \textit{E}c.} the answer.} \\
 \underline{64} \\
 167) 1239 \\
 \underline{1169} \\
 1744) 7058 \\
 \underline{6976} \\
 174804) 822071 \\
 \underline{699216} \\
 1748087) 12285563 \\
 \underline{12236609} \\
 174809402) 489540000 \\
 \underline{349618804} \\
 1748094047) 13992119600 \\
 \underline{12236658329} \\
 1755461271 \text{ \textit{E}c.}
 \end{array}$$

4. Required the square root of .0312125?

$$\begin{array}{r}
 \begin{array}{c} \cdot \cdot \cdot \cdot \cdot \\ 0.0312125 \end{array} \text{ (.17667 \text{ \textit{E}c.} the answer.} \\
 \underline{1} \\
 27) 212 \\
 \underline{189} \\
 346) 2312 \\
 \underline{2076} \\
 3526) 23650 \\
 \underline{21156} \\
 35327) 249400 \\
 \underline{247289} \\
 2111 \text{ \textit{E}c.}
 \end{array}$$

* "When

* “When a root will have a great many places (and some of them decimals) after the better half of the intended quantity are discovered, the rest may be obtained by only dividing the next resolvend by the next divisor, either in the common manner or that of contracted division.” Thus,

Required the square root of 7 to nine places of figures ?

$$\begin{array}{r}
 7 \text{ (2.6457} \\
 4 \\
 \hline
 46) 300 \\
 276 \\
 \hline
 524) 2400 \\
 2096 \\
 \hline
 5285) 30400 \\
 26425 \\
 \hline
 52907) 397500 \\
 370349 \\
 \hline
 529145) 2715100 \text{ (5131 \&c.} \\
 \dots 2645725 \\
 \hline
 69375 \\
 52915 \\
 \hline
 16460 \\
 15874 \\
 \hline
 686 \\
 529 \text{ \&c.} \\
 \hline
 \end{array}$$

Hence the answer is
2.64575131 &c.

Practical

* The reason is, as the divisors and resolvends become larger and larger at every step, what alteration of relative value is made in them, by adding the new figures of the root, and bringing down fresh periods, is not of much consideration, only reaching to a few right hand figures: Hence as the *times going* are determined by the value of the left hand figures (still continuing the same) this method of contraction (which only deals in them) must be right,

Practical Questions.

CLASS I.

1. What's the square root of 106929? - Answer 327.
2. What's the square root of 368863? - Answer 607.34092 &c.
3. What's the square root of 75967.96? - Answer 275.6228 &c.
4. What's the square root of $\frac{810}{12344}$? - Answer $\frac{6}{7}$.
5. What's the square root of $\frac{3108}{6192}$? - Answer .71528 &c.
6. What's the square root of $51\frac{2}{3}$? - Answer $7\frac{1}{3}$.

CLASS II.

1. What's the square root of 119550669121? - Answer 345751.
2. What's the square root of 3.1721812? - Answer 1.78106 &c.
3. What's the square root of .00032754? - Answer .01809 &c.
4. What's the square root of 4.000067121? Answer 2.000016 &c.

CLASS III.

1. What's the square root of $6\frac{2}{3}$? - Answer 2.5298 &c.
2. What's the square root of 10? - Answer 3.1622776602 &c.

Problem II.

To extract the CUBE ROOT.

* Rule.

"Distinguish the given number into periods of three figures, by putting a point over every third place from units, to the left hand for integers, and to the right for decimals (if any). † Then seek the great-
N n est

* The demonstration at the latter end.

† In this table is shewn the cube of the 9 digits.

1.	2.	3.	4.	5.	6.	7.	8.	9.	roots.
1.	8.	27.	64.	125.	216.	343.	512.	729.	cubes.

est cube in the left-hand period; place the root like a quotient figure, and subtract the cube from the period, to which remainder bring down the next period for a resolvend. Under this draw a line, and beneath it write the triple square of the root, so that units in the latter may stand under the place of hundreds in the former. Also below this write the triple root, removed one place to the right hand, and then the sum of these lines is a divisor. Next ask how oft this divisor may be had in the resolvend, its right hand place excepted, writing the result in the quotient (and remembering that it can never be above 9). Then under the divisor place the product of the triple square of the root by the last quotient figure, in such a manner, that the units place of this line may fall under that of tens in the divisor; also beneath this write the product of the triple root by the square of the last figure, one place removed to the right hand: And again, one place to the right of this last line subscribe the cube of the same quotient figure; the sum of which three lines will be the subtrahend. Subtract the subtrahend from the resolvend, and to the remainder bring down the next period for a new resolvend, which manage as before; adding (properly placed) the triple square of the whole root to the triple of the root for a divisor, and for a subtrahend the product of the last figure in the root by the last triple square, the square of the last figure by the last triple root, and the cube of the last figure, &c. Proceed thus till all the periods are brought down, and you have the root required, which if a surd may be carried on to any required exactness, by bringing down periods of cyphers.

Decimals in the root are easily distinguished in the process, as was hinted in the square root, and what was said there of vulgar fractions may be understood in like manner in this article."

Examples.

1. Required the cube root of 934007.359375?

$$\begin{array}{r}
 \overset{\cdot}{9} \overset{\cdot}{3} \overset{\cdot}{4} \overset{\cdot}{0} \overset{\cdot}{0} \overset{\cdot}{7} \overset{\cdot}{.} \overset{\cdot}{3} \overset{\cdot}{5} \overset{\cdot}{9} \overset{\cdot}{3} \overset{\cdot}{7} \overset{\cdot}{5} \quad (97.75 \text{ the answer.}) \\
 \underline{729} \quad \quad \quad = \text{cube of } 9. \\
 205007 \text{ Resolvend.} \\
 \hline
 \end{array}$$

205007

* We have here put down the factors of the multiplications, &c. at every step, but in the examples that follow, the results are only given, being judged (with the rule) sufficient information to any attentive peruser.

205007 Resolvend.

$$243 = 9 \times 9 \times 3 \text{ the triple square of the root.}$$

$$27 = 9 \times 3 \text{ the triple root.}$$

2457 Divisor.

$$1701 = 243 \times 7 \text{ last triple square by last quotient figure.}$$

$$1323 = 27 \times 7 \times 7 \text{ last triple root by square of 7 the last quot. fig.}$$

$$343 = 7 \times 7 \times 7 \text{ cube of last quotient figure.}$$

183673 Subtrahend.

21334359 Resolvend.

$$28227 = 97 \times 97 \times 3.$$

$$291 = 97 \times 3.$$

282561 Divisor.

$$197589 = 28227 \times 7.$$

$$14259 = 291 \times 7 \times 7.$$

$$343 = 7 \times 7 \times 7.$$

19901833 Subtrahend.

1432526375 Resolvend.

$$2863587 = 977 \times 977 \times 3.$$

$$2931 = 977 \times 3.$$

28638801 Divisor.

$$14317935 = 2863587 \times 5.$$

$$73275 = 2931 \times 5 \times 5.$$

$$125 = 5 \times 5 \times 5.$$

1432526375 Subtrahend.

0

2. Required the cube root of 865635?

A T R E A T I S E

$$\begin{array}{r} 865635 \text{ (95.304 \&c.} \\ 729 \end{array}$$

$$136635 \text{ Resolvend.}$$

$$\begin{array}{r} 243 \\ 27 \end{array}$$

$$2457 \text{ Divisor.}$$

$$\begin{array}{r} 1215 \\ 675 \\ 125 \end{array}$$

$$128375 \text{ Subtrahend.}$$

$$8260000 \text{ Resolvend,}$$

$$\begin{array}{r} 27075 \\ 285 \end{array}$$

$$271035 \text{ Divisor.}$$

$$\begin{array}{r} 81225 \\ 2565 \\ 27 \end{array}$$

$$8148177 \text{ Subtrahend.}$$

$$111823000 \text{ Resolvend.}$$

$$\begin{array}{r} 2724627 \\ 2859 \end{array}$$

$$27249129 \text{ Divisor.}$$

$$111823000000 \text{ Resolvend.}$$

$$\begin{array}{r} 272462700 \\ 28590 \end{array}$$

$$2724655590 \text{ Divisor,}$$

$$\begin{array}{r} 1089850800 \\ 457440 \\ 64 \end{array}$$

$$108989654464 \text{ Subtrahend.}$$

$$2833345536 \text{ Remainder,}$$

2. Re-

3. Required the cube root of 2?

2 (1.2599 &c. the answer.

1

1000 Refolvend.

3

3

33 Divisor

6

12

8

728 Subtrahend.

272000 Refolvend.

432

36

4356 Divisor.

2160

900

125

225125 Subtrahend.

46875000 Refolvend.

46875

375

469125 Divisor.

421875

30375

729

42491979 Subtrahend.

4383021000 Refolvend.

4755243

3777

47556207 Divisor.

42797187

305937

729

4282778799 Subtrahend.

100242201 Remainder.

4. Required the cube root of .000485613?

0.0004

0.000485613 (.078 &c. the answer.

343

142613 Resolvend.

147

21

1491 Divisor.

1176

1344

512

131552 Subtrahend.

11061 Remainder.

Practical Questions.

CLASS I.

1. What's the cube root of 48228544? - - - Answer 364.
2. What's the cube root of 673373097125? - Answer 8765.
3. What's the cube root of 7164523.145083? Answer 192.77 &c.
4. What's the cube root of 7121.1021698? - Answer 19.238 &c.
5. What's the cube root of $\frac{1924}{4608}$? - - - - Answer $\frac{3}{4}$.
6. What's the cube root of $31\frac{1}{343}$? - - - - Answer $3\frac{1}{7}$.

CLASS II.

1. What's the cube root of 51686.703125? - Answer 37.25
2. What's the cube root of 219365327791? Answer 6031.
3. What's the cube root of $\frac{4}{9}$? - - - - - Answer $\frac{2}{3}$ &c.
4. What's the cube root of $8\frac{1}{7}$? - - - - - Answer 2.057 &c.

CLASS III.

1. What's the cube root of 427362? - - - Answer 75.8237 &c.
2. What's the cube root of .0069761218? - Answer .19107 &c.

Scholium.

It appears from these examples, that the carrying of the cube root to any great length becomes a very troublesome task, and to render the

the operation a little easier, there are some other different methods invented; but as they are not often so direct, as this common one, and require a little more judgment, perhaps, in their application, we think it not worth while to insert them in this treatise. For such like reason, we likewise omit giving a general rule for all roots, as any thing of that sort is very tedious, and often depends a good deal upon tryal. However, while in hand with the subject, it may not be improper to observe to the inquisitive reader, that several roots denominated by an even number may be extracted by the help of the two rules already delivered: For as the fourth power of 3 (or any other number) i. e. $3 \times 3 \times 3 \times 3$ may be considered as 9×9 , or the square of the square of 3, so the fourth root of any number may be found by extracting the square root of its square root. Again as the 6th. power of 2, i. e. $2 \times 2 \times 2 \times 2 \times 2 \times 2$, is the same as 8×8 or the square of the cube of 2, so the 6th. root may be found of any number by extracting the cube root of its square root. And, in general, if of these two indexes (2 and 3) a product can be compounded equal to the index of any required root, the root may be determined by first extracting a root out of the given number pointed out by one of the indexes concerned, and then a root of this root agreeing with another index, and so on continually, till all the factorial indexes have been extracted for. But the practice of this we leave to the reader, if he thinks it worth attempting.

Before we put an end to this subject, we shall give a method, sometimes found useful, of guessing pretty near at the root of a vulgar fraction (not to be perfectly extracted in its lowest terms) by limiting it between two other fractions, viz. "Multiply both the numerator and denominator by a power of the denominator, less by one degree than that of the root required, and then, as the denominator is consequently thus made a perfect power of that root, we may take the denominator for the denominator of the answer, and find the root of the numerator in two round numbers, one bigger and the other less than right for the limits of the numerator of the answer." Thus let it be required to limit fractionally the square root of

$\frac{5}{9}$? $\frac{5}{9} = \frac{5 \times 9}{9 \times 9} = \frac{45}{9 \times 9}$: Now the square root of 45 is between 6 and 7,

hence the true root lies between $\frac{6}{9}$ and $\frac{7}{9}$. Also the cube root of $\frac{5}{9}$ will be found in the same manner to be between $\frac{8}{12}$ and $\frac{9}{12}$ or $\frac{2}{3}$ and

$\frac{3}{4}$: For $\frac{5 \times 12 \times 12}{12 \times 12 \times 12} = \frac{720}{12 \times 12 \times 12}$, and the cube root of 720 is bigger than 8 and less than 9. It appears from the practice, that the denominator need not be multiplied, as directed, it being only mentioned here to shew the reason of the method. In the same manner (by the help of tables of powers of the digits, or a few trials, when a more direct way does not offer) the limits of any root of any single fraction may be determined.

A demonstration of the preceding rules.

Before we enter upon particulars, it will be necessary to observe to the learner, that as a number may be represented by a line, so the product of two numbers may be represented by a superficies (limited by right lines perpendicularly situated to one another) called a *parallelogram* or *rectangle*, whereof one side is equal to one of the factors, and the other equal to the other. Also the product of three numbers may be represented by a solid (bounded by parallel plains perpendicularly placed) called a *parallelopipedon*, whereof the three dimensions of length, breadth, and depth, agree with three given factors. Thus for the first particular, if in the rectangle *abcd* Fig. II. the length $ab = (cd =) 4$, and $ac = (bd =) 3$, then will the area or plain superficies $abcd = 4 \times 3 = 12$: For 'tis evident to the eye, that if the lines *ab*, *ac* be divided each into a number of parts equal to that of their units (i. e. 4 and 3) the superficies will contain as many (superficial) units, as are in the product 12, of the given numbers 4 and 3. Now when the factors or sides are both alike as 3 and 3, the area then agreeing with their product is called a *square* (Fig. III.) hence the name of square for the second power in the article of raising powers, &c. Again, for the other particular, if in the parallelopipedon *abcdefg*, Fig. IV. the length or side $ab = (cd = fg =) 4$, *ac* or breadth $= (bd = ef =) 3$, *ae* the depth $= (ef = dg =) 2$, then will the content of the solid $abcdefg = 4 \times 3 \times 2 = 24$: For it is easy to see, that if the solid was divided according to the lines in the figure (which agree with a supposed division of the numbers 4, 3, 2 into units) the number of (solid) units contained in it would be equal to the product of the given factors, i. e. 24. Now when the factors (or dimensions of the the figure) are alike, as 3, 3, and 3, the solid agreeing with their product is called a *cube* (Fig. V.) and hence the name of cube for the third power in the article of raising powers, &c.

These general notions of the agreement betwixt the products of numbers, and geometrical figures attended to (which, besides being of use here, may furnish good hints to the reader, before he enters upon mensuration, &c.) we shall proceed to account for the reason of the first rule, or that for extracting the square root; in order to which we must yet premise the following lemmas.

Lemma I.

If the area of a rectangle be divided by a number equal to one of its sides, the quotient will be the other side.

Demonstration.

Demonstration.

This is evident; for as by the above remarks the area of any rectangle is equal to the product of two numbers equal to its two dimensions, or sides, so from the nature of multiplication and division, if the product of two factors be divided by either of them, the quotient is the other. Thus 12 the area of the above parallelogram divided by 4 and 3, gives 3 and 4 respectively for their contrary dimensions.

Lemma II.

The product of any two numbers can but at most, consist of as many digits as are in them both, and at least but of one fewer.

Demonstration.

This will be obvious from a single instance. Suppose 123456 were multiplied by 4321. Now in the adjoining example, which shews the position of the first and last lines in the product, according to the usual method of placing them, &c. (the intermediate ones being useless in our reasoning here) 'tis evident that the product would consist of so many digits, as are on the right of the perpendicular lines in the higher row, joined to those on the left of the same lines in the lower row, and besides one more which would come from the addition of the two digits included by them. But the number of those on the right hand are still equal to those in the multiplier, save one; to which if we add one for that included by the perpendiculars we have a number of digits equal to those in the multiplier. Again, the digits in the lower line on the left hand side of these perpendiculars must be always one less than the number in the multiplicand (because of that it loses between the perpendiculars) or equal to them, just as the last figure in the multiplicand happens to produce one or two in the multiplication (which last number it evidently has a chance to do, and this increase is here denominated by *a*). Hence, as the digits in the product must by this reasoning be equal to the extent of these lines, and these lines are proved to be always equal to, or but one less than, the sum of those in the factors, the lemma is manifest.

$$\begin{array}{r}
 123456 \\
 4321 \\
 \hline
 \begin{array}{|c|c|c|}
 \hline
 12 & 3 & 456 \\
 \hline
 a & 49382 & 4 \\
 \hline
 \end{array}
 \end{array}$$

Corollary I.

Hence it follows, that if the factors are equal, as they are in raising a root to a square, then the square or product can never have more places than twice the number of places in the root, nor fewer than that number save one.

Oo

Corollary

Corollary II.

And from this it follows again, that if a square number be divided into periods of two figures, as directed, the number of points must be equal to the number of places in the root, and also that, as there will be one digit for every period, there must be one for that on the left hand, tho' it should happen to consist of but one figure.

These things premised, let us now endeavour to shew the reason of the rule.---Suppose the line AB *Fig. VI.* $=258$, and that the part of it $Ac=200$, the part $cd=50$, and $dB=8$. Make in the square $ABCD$ of the given dimensions, Df , Dg , and $Ch=dB$, and draw the lines df and kg ; also make ne , nh , and $ki=cd$, and draw the lines ce , and ih , then will im , cm , and $iA=Ac$. Now tis visible to the eye (and the matter of fact needs no other demonstration) that $ABCD$ the square of the whole line AB is made up of the following areas, *viz.* 1st. The square Am , whose side is Ac . 2d. the gnomon-like-figure $cmiknd$ (made up of the two equal rectangles ch , *ie* whose lengths and breadths are equal Ac , cd , and the square mn , whose side is equal cd), and 3d. the gnomon $dnkCDB$ (composed of the two equal rectangles dg , kf , whose lengths and breadths are equal $Ac+cd$, and dB , and the square nD , whose side is equal dB). Now as lines and areas may be referred to factors and products (from what is already said), we shall shew for the farther satisfaction of the reader, that these three areas expressed in numbers, according to the suppositions at the first, will just equal the square of AB , or the whole area in numbers. Thus

$$\left. \begin{array}{l} \text{The square } Am = 200 \times 200 \text{ ---} = 40000 \\ \text{The 1st. gnomon} = 200 \times 50 + 200 \times 50 + 50 \times 50 = 22500 \\ \text{The 2d. gnomon} = 250 \times 8 + 250 \times 8 + 8 \times 8 \text{ ---} = 4064 \end{array} \right\} \text{as is evident.}$$

The sum of which --- 66564 is exactly equal to 258×258 , or the complete square of AB .

This composition of a square understood, we shall soon see how to find its root, by running the process back again. Thus if we want to find the square root of 66564 $=$ the area $ABCD$ (which per lemma II. will consist of three digits) we must first take 40000 $=$ the area Am , the greatest square possible out of it, and reserve the root $200=Ac$ for the first term or part of the root AB required (i.e. the root of 6 the first period with cyphers annexed according to its place). Now the remainder 26564 is evidently the area of the gnomon $cmiCDB$, the most valuable parts of which are the areas ch , *ie*, and mn , as the stripes round the edges kD , ge , are but of little moment because of their small breadth, which represents only
a digit

a digit in the lowest place. Let us therefore suppose these stripes omitted, as likewise the square mn ; then as the gnomon last mentioned is now supposed to consist of the rectangles ch , ie only, and as they make, when laid out at the end of each other, a rectangle whose length is $cm + im$, or $200 \times 2 = 400$ we may find the other dimension cd of the gnomon, on this supposition, by division (agreeably to lemma I. which may be observed to account for the method of doubling the root for a divisor, in order to get the next figure, &c.) i. e. by asking how oft 400 may be had in 26564, which appears to be about 60 times (i. e. 6 times with cyphers annexed according to its place) therefore we take $cd = 60$, and try it [For on account of the little stripes kD , dg , and square mn , being omitted, and the common chance of going wrong, as in any division, we cannot be very certain we have a right term, till we prove it, &c.] by finding by its means the areas ch , ie , and mn . Now from what was said above, these areas $= 400 \times 60 + 60 \times 60$, or, which is the very same thing, $460 \times 60 = 27600$ (which accounts for the method of putting the quotient figure at the end of the divisor before it be multiplied, &c.) but this number being too big, shews 60 is too much for cd , therefore we try 50 (the next less digit to 6 with a cypher on account of its place) i. e. $400 \times 50 + 50 \times 50$, or rather $450 \times 50 = 22500$, which being less than 26564 proves 50 to be the next term in the root. This acquired, in order to get the remaining term dB , 'tis plain that if $22500 =$ the gnomon $cmiknd$, be taken from 26564 $=$ the gnomon $cmiCDB$, there will remain the gnomon $dnkCDB =$ (per actual subtraction) 4064. Now the sides of the rectangle dg , and kf are evidently equal to the sum of the terms before found, i. e. $Ac + cd = 250$, twice which (i. e. $250 \times 2 = 500$) is the length of them both together. Therefore (supposing the square nD omitted) in order to get from this dimension 500 of the area 4064 (per lemma I.) the other dimension dB or last term of the root, we ask how oft 500 in 4064, the result of which is 8 times, and on tryal answers exactly i. e. $(500 \times 8 + 8 \times 8 =) 508 \times 8 = 4064$ precisely.

Thus we have exhausted the square by piece-meal, and found the parts of its sides or root from the areas bounded by them, &c. to be 200, 50, and 8 (i. e. 258). And as every step gained from the preceding reasoning (when obvious allowances are made all along for the admission of superfluous cyphers) agrees with the directions of the rule, it is fully demonstrated for three figures in the root. But as the process in the two gnomons, from which the last figures were gained, was the same, we have no reason to doubt, let their number have been ever so many (i. e. whatever number of figures were in the root) but they would all not only have put on a similar appearance (see the pricked lines added to the figure) but also have been handled in a similar manner, in order to attain the root required.

With regard to approximating to a surd root by decimals, there is

no more difficulty in accounting for it, than in the approximations of division, &c. by their help; all being but a valuation of remainders, by the same standard, that determined the whole numbers, &c.

A Demonstration of the Rule for extracting the Cube Root.

To facilitate which it will be proper to premise these lemmas.

Lemma I.

If the solidity of a parallelopipedon be divided by the area of any two of its dimensions, the quotient will be the other dimension.

Demonstration.

This is manifest; for as by what was said before the solidity of any parallelopipedon is equal to the product of its three dimensions, so if its solidity be divided by the product (or area) of two of them, the quotient must consequently be the other, per the nature of multiplication and division. Thus 24 the solidity of the parallelopipedon before going, divided by $(3 \times 4 =) 12$, $(2 \times 4 =) 8$, and $(2 \times 3 =) 6$ the area of its three different plains the quotients, 2, 3, and 4, are the other dimensions of it respectively.

Lemma II.

If any three numbers be multiplied together continually, the product can but at most have as many digits in it, as are in all the three factors, and at least but two fewer.

Demonstration.

This is evident from the second lemma in the foregoing demonstration; for as in the first multiplication of two factors, there is a chance either of falling a digit short, or of having the same number in the product as are in the factors; so in the second multiplication of the last product and other factor there is the same hazard: But if the first lose one and the last one, the sum is but two upon the whole, and if neither of them lose, the last product does not lose; therefore the lemma is true.

Corollary I.

Hence it follows, that if the three factors are equal, as they are in

in raising a root to a cube, then the cube can never consist of more than thrice so many digits, as are in the root, nor fewer than that number save two.

Corollary II.

It also follows, that if a cube number be divided into periods of three figures as directed, the number of points must be equal to the number of places in the root, and that also, as there will be one digit for every period, there must be one for that on the left hand, tho' it should happen to consist of but one or two figures, &c.

Having premised these things, we will now attempt the demonstration. As we observed in the square root that the process for the third digit, was the same as the second, and so on, we will here in order to avoid confusion in the figure but suppose two digits in the root, hoping that if the reason of getting these appear, it will be granted by analogy, when more are concerned, and especially when 'tis considered, how near akin the methods of arguing are in both cases, as will be evident.

Let therefore AB, *Fig. VII.* = 25, the part of it Ac = 20, and cB = 5: Then in the cube ABCDEFG *q* of the same dimensions of the line AB, make Ap, Bn, Di, Co; Dg, Fm, Gr; Df, Fl, Es, Ct, all = BC, and draw the lines pn, ni, io, op; cg, gm, mr, rc; and fl, ls, st, tf; Also conceive the solid cut into parcels by plains passing thro' these lines: Then (in order to enumerate these sections) 'tis visible, that the whole cube is made up of these four different parcels, *viz.* 1st. The cube xqp, whose side is = Ac. 2d. Three similar parallelopipedons eAp, hGr, and kEs, whose greatest superficies are each = the square of Ac, and thickness cB. 3d. Three similar parallelopipedons fBy, gCz, and iFx, whose ends are all equal the square of Bc, and 4th. The cube eDi, whose side is equal cB. This is evident, but for farther satisfaction, we shall illustrate the division by numbers, according to our supposition at first. Thus

The greater cube - - - - = $20 \times 20 \times 20$ = 8000

The 3 greatest parallel. - = $20 \times 20 \times 5 \times 3$ = 6000

The 3 less Do. - - - - = $5 \times 5 \times 20 \times 3$ = 1500

The less cube - - - - = $5 \times 5 \times 5$ - - = 125

The sum of which - - - - - 15625 is equal $25 \times 25 \times 25$, the cube of the whole line AB.

This composition of a cube understood, we will now resolve it back again in order to find its root, and shew the reason of the rule.

Taking

Taking for an example the above number $15625 =$ the solid ADF (which per lemma II. will have two digits in its root) it appears we must first lessen it by $8000 =$ the cube xqp the greatest it contains, and reserve its root $20 = Ac$ (or 2 the root of the first period 15 with a cypher for its place) for the first portion of the root AB wanted. The remainder or resolvend 7625 is then evidently the sum of the solidities of the three parallelopipedons eAp , hGr , kEs ; the three *ditto* fBy , gCz , iFx , and the cube eDi . Now the first set of parallelopipedons, laid at the end of each other, make a solid whose length is $= 3$ times $Ac = 60$, and breadth equal $Ac = 20$, and depth equal cB , tho' at present a dimension supposed unknown; hence its surface is $20 \times 20 \times 3 = 20^2 \times 3 = 1200$. The second set also, placed at the end of each other, make a solid, whose length is 3 times $Ac = 20 \times 3 = 60$, and breadth and depth equal to the depth of the above, and consequently likewise unknown, which dimension also limits the little cube eDi . However, in order to guess what this unknown dimension is (or the next figure in the root) let us not regard what would have been made by multiplying 60 by it, in order to get the surface of the latter set of parallelopipedons, nor what we should have farther gained from having the area of one of the sides of the little cube, as defects of no very great moment; then will the surface of the solid that remains on this supposition be equal to the sum of 1200, and $60 = 1260$ (which accounts for the method of forming a *divisor* in the rule) and on asking how oft it may be had in 7625 in order get the other unknown dimension cB (per lemma I.) it appears to go 6 times, which on tryal is found too much; therefore we must see what 5 will do, and on examination it is found to answer exactly. Now the method of trying 5 (one instance being sufficient) is illustrated below, and fully shews the reason of the method of forming a *subtrahend*. Thus if $cB = 5$

Then the 1st. set of parallel. $= (20^2 \times 3 \times 5 =) 1200 \times 5 = 6000$

The 2d. Do. $= (20 \times 3 \times 5^2 =) 60 \times 5 \times 5 = 1500$

The little cube $= (5^3 =) 5 \times 5 \times 5 = 125$

The sum of which subtrahend $= 7625$

being the same as the resolvend above, proves $5 = cB$ precisely; hence the cube root required is 20 and 5 (i. e. 25). Now (cyphers allowed for) the agreement of this reasoning with the directions of the rule for making divisors and subtrahends (all the variety in the process after the first step) demonstrates it to be right.

What was farther said at the end of the demonstration of the square root, may be properly understood here.

Scholium.

Scholium.

Some readers may possibly think the above demonstrations exceeding difficult, and * tedious, and be tempted to pass them over with a very slight perusal, or none at all; but no learner should be discouraged with what at first may appear difficult in any thing, much less in these sciences. For tho' the truths about which they are conversant, require often a good deal of sagacity to comprehend them, yet a serious attention and perseverance, till the subject grows warm will chiefly render that an easy task, which at first appeared so formidable. And with regard to the particulars just gone over, for encouragement we shall observe to the young arithmetician, that there is virtually contained in them the foundation of the rules for measuring all rectilineal superficies and solids; two noted propositions, one in *Euclid*, and the other in *Ramus*, and a tolerable specimen of mathematical reasoning; and farther, that if he can get master of, and fully see the whole drift of the demonstrations, 'tis ten to one he has a head very well turned for geometry and the sciences.

PROGRESSION.

Definition.

"*A Progression is a rank or series of numbers (above two) increasing or decreasing by a constant law, and is divided into two sorts, distinguished by the terms arithmetical and geometrical, as follows.*"

I. *Arithmetical Progression (A. P.)**Definition.*

"*This is when the terms of a series of numbers encrease or decrease by a common difference, as by the continual addition or subtraction of some equal number. Thus 1. 3. 5. 7. 9. 11, and 45. 40. 35. 30. 25. 20*
are

* In order that the reader may judge more favourably of the above reasoning with regard to these particulars, it may not be amiss to inform him, that two of our best late authors on the theory (*Malcolm and Donn*) have each with the advantage of algebra taken up above twice the room it does (i. e. one 20 octavo pages, and the other 11 large quarto) nor do we apprehend they have at last made the thing any more clear and intelligible, than is done by this method.

are two series of numbers in A. P. the one encreasing by the equal difference of 2, and the other decreasing by the equal difference of 5,

According to this definition in any A. P. the five following parts are naturally included.

1. The first term commonly understood the least.
 2. The last term commonly understood the greatest.
 3. The number of terms.
 4. The equal difference (which some call ratio).
 5. The sum of all the terms.
- } called also extremes.

Any three of which five being given, the other two may be found, which admits of twenty problems; but as several of them would require the assistance of algebra to demonstrate, and besides seem more curious than useful, we shall confine ourselves to a few of the most ordinary as follows."

Problem I.

"The first term, the last term, and number of terms given to find the sum of all the series."

* Rule.

"Multiply the sum of the extremes by the number of terms, and half of the product is the answer."

Example.

The first term of an A. P. is 2, the last term 53, and number of terms 18, required the sum of all the series?

$$53 + 2$$

* The reason of this will soon appear, whether the number of terms be odd or even, by supposing another series of the same kind with the given one placed under it in an inverse order; for then will the sum of every two perpendicular terms be alike and equal to the sum of the first and last: Hence one of these sums multiplied by the number of them (or the number of terms in the series) must be their whole sum; but as there were in this process two series used instead of one, we must take half of the product for the true sum of that given. The series, &c. below may be perused for farther satisfaction.

$$\begin{array}{cccccccc} 1. & 2. & 3. & 4. & 5. & 6. & 7. \\ 7. & 6. & 5. & 4. & 3. & 2. & 1 \end{array}$$

$$8 + 3 + 8 + 8 + 8 + 8 + 8 = 8 \times 7 = 56. \quad \text{Hence}$$

$$1 + 2 + 3 + 4 + 5 + 6 + 7 = \frac{55}{2} = 28.$$

$$\frac{53 + 2 \times 18}{2} = 55 \times 9 = 495 \text{ the answer.}$$

Practical Questions.

1. How many strokes do the clocks of Venice, which go to 24 o'clock, strike in the compass of a day? Answer 300.

2. If 100 eggs were placed in a right line, exactly a yard asunder from one another, and the first a yard from a basket; what length of ground does that man go, who gathers up these eggs singly, returning with every one to the basket to put it in?

Answer 5 miles 1300 yards.

3. A merchant sold 1000 yards of linnen at 2 pins for the first yard, 4 for the second, 6 for the third, &c. encreasing 2 pins every yard; I demand how much the linnen produced, when the pins were afterwards sold at 12 for a farthing? Answer £.86-17-10.

Problem II.

"The first term, last term, and number of terms given to find the equal difference."

* Rule.

"The difference of the extremes divided by the number of terms less one, gives the equal difference sought."

Example.

The extremes of an A. P. are 2 and 53, and the number of terms 18, required the equal difference?

$$\frac{53 - 2}{18 - 1} = \frac{51}{17} = 3 \text{ the answer.}$$

P p

Practical

* Taking the first term out of the last shews what the first gained before it became equal to the last, by all the subsequent additions, and as it gained equally at every remove (per def.) which were evidently in number one time less than the number of terms, 'tis plain that that gain divided by the number of removes must give the equal gain or difference at every one separately. Hence the rule is manifest.

Practical Questions.

1. A man had 8 sons, the youngest was 4 years old, and the eldest 32, they encrease in A. P. what was the common difference of their ages? Answer 4 years.

2. A debt is to be discharged at 16 several payments in A. P. the first payment is to be 14*l.* the last 100*l.* what is the equal difference, and the sum of the whole debt?

Answer, the equal difference is £. 5-14-8, and whole debt 912*l.*

3. A man is to travel from London to a certain place in 12 days, and to go but 3 miles the first day, encreasing every day by an equal excess, so that the last days journey may be 58 miles; what is the daily encrease, and how many miles distance is that place from London?

Answer, daily encrease 5; distance 366 miles.

Problem III.

“The first term, the last term, and equal difference given to find the number of terms.”

** Rule.*

“Divide the difference of the extremes by the equal difference, and the quotient encreased by one is the number of terms required.”

Example.

The extremes of an A. P. are 2 and 53, and the equal difference 3; what is the number of terms?

$$\frac{53-2}{3} + 1 = 18 \text{ the answer.}$$

Practical Questions.

1. A man going a journey, his first days travel was five miles, his last

* This is evident from the last problem. For as there the difference of the extremes divided by the number of terms less one, gave the common excess, so tis clear the same dividend divided by the common difference must give the number of terms less one; hence this quotient augmented by one must be the answer to the question. 'Tis obvious that by first finding the number of terms by this problem, the sum of the series may be found by problem first from the same data, which makes another problem of the twenty.

last days travel was 35 miles; he encreased his journey every day by 3 miles, how many days did he travel? Answer 11 days.

2. A man being asked how many sons he had, said that the youngest was 4 years old, and the eldest 32, and that he encreased one in his family every 4 years; how many had he? Answer 8 sons.

Problem IV.

“the first term, the number of terms, and equal difference given to find the last term.”

* Rule.

Multiply the number of terms less one by the equal difference, and the product added to the first term gives the last.”

Example.

The first term of an A. P. is 2, the number of terms 18, and equal difference 3, what is the last term?

$$\overline{18-1} \times 3 + 2 = 53 \text{ the answer.}$$

Practical Questions.

1. A grocer buys 50 pounds of spices, and agrees to pay 10 times as much for them as the last pound would come to on the supposition of giving 4*d.* for the first pound, 7*d.* for the second, &c. in A. P. Required what that was? Answer £.6-5-10.

2. A certain man bought 20 ells of Holland, to be paid in 17 weeks, agreeing to pay the first week 1*s.* 8*d.* the second 3*s.* 4*d.* and so on in A. P. for the number of weeks mentioned. Required what the Holland stood him in? Answer £.12-15.

Pp 2

Problem

* This is very clear from the foregoing problems; for as the number of terms, less one, shews how many are affected by the equal difference, so that number multiplied by the equal difference must give the encrease from first to last: Hence the first term added to that encrease must be the last term.---From the same data 'tis plain, the sum may be found by problem I. after having discovered the last term by the above. This also makes another of the 20 problems.

Problem V.

"The last term, the number of terms, and equal difference given to find the first."

** Rule.*

"Multiply the number of terms less one by the equal difference, and the product taken from the last term gives the first."

Example.

The last term of an A. P. is 53, the number of terms 18, and the equal difference 3; what is the first term?

$$53 - \overline{18 - 1} \times 3 = 2 \text{ the answer.}$$

Practical Questions.

1. A man takes out of his pocket, at 6 several times, six several numbers of shillings, every one exceeding the former by 6: The last was 42; what was the first? Answer 12s.
2. A man in ten days went from London to a certain town in the country, every days journey exceeding the former by 4 miles, and the last he went was 46, what was the first? Answer 10 miles.

Problem VI.

"The first term, last term, and sum of all the series given to find the number of terms."

† Rule.

"Divide double the sum of the series by the sum of the extremes, and the quotient is the number of terms."

Example.

* This is evident from the last problem; for if the product of the number of terms less one, and equal difference, with the first term added, give the last, the last term made less by this product must give the first.---'Tis evident by finding the number of terms according to this problem, that the sum of the series may be determined by the first problem, which makes another of the 20 problems.

† This is evident from the first problem. For as we there, in order to get the sum

Example.

The first term of an A. P. is 2, the last term 53, and the sum of all the series 495, what is the number of terms?

$$\frac{495 \times 2}{53 + 2} = \frac{990}{55} = 18 \text{ the answer.}$$

Practical Questions.

1. A person received 85 pounds of different men, but of how many he has forgot; yet he remembers that the first gave him 7*l.* and the last 27*l.* and that every payment rose in A. P. Pray how many men had paid him money? Answer 5 men.

2. A person disbursed 685 pounds to a certain number of men, yet could not tell how many they were; however he remembers the payments were in A. P. and that the first had 19, and the last 118 pounds; 'tis required from this to tell the number of payments, and what they constantly exceeded one another?

Answer, Number of payments 10, and equal difference 11*l.*

Problem VII.

"The first term, the number of terms, and the sum of the series given to find the last term."

* *Rule.*

"Divide the double of the sum by the number of terms, and from the quotient subtract the first term, and the remainder is the last."

Example.

sum of the series, took half the product of the sum of the first and last terms into the number, so, on the contrary, the double of the sum divided by the sum of the first and last terms must be the number of terms.--It will appear from this, that if the equal difference be required from the same data, it may be found by problem II. after the number of terms is determined by this, which is another of the 20 problems.

* Dividing double of the sum by the number of terms must give, from problem I. the sum of the extremes, hence the first term taken from that quotient must leave the last.--If the equal difference be required from this data, it may be found by first finding the extreme required as above, and then having recourse to problem II. which is another of the 20 problems.

Example.

The first term of an A. P. is 2, the number of terms 18, and the sum of all the series 495, what is the last term?

$$\frac{495 \times 2}{18} - 2 = 53 \text{ the answer.}$$

Practical Questions.

1. A traveller went 100 leagues in 8 days; and every day he travelled equally farther than the preceding. Now it being discovered that the first day he travelled two leagues, the question is what he travelled the last day?

Answer 23 leagues

2. A gentleman spent at a tavern £.99-7-6 in 50 days, and every succeeding days expence rose equally above the preceding one; the first day cost him 3s. what was the equal advance?

Answer 18 pence,

Problem VIII.

"The last term, the number of terms, and sum of all the series given to find the first term."

** Rule.*

"Divide the double of the sum by the number of terms, and from the quotient subtract the last term, and the remainder is the first."

Example.

The last term of an A. P. is 53, the number of terms 18, and the sum of all the series 495, what is the first term?

495

* This is so easy and like the last, it scarce needed to have been made a new problem, but for the sake of order; for if the quotient got by the division, mentioned in the rule, be the sum of both the extremes, tis evident either of them taken from it must leave the other.---If the equal difference be required from this *data*, it may be found by the second problem, when the extremes are determined as above, and this is another of the 20 problems.

This problem is the same as *Hill's* 6th. and last (see page 128 of his arithmetic) in which there seems to be a mistake, as the *data* in it are not referred to in the subsequent rule and example.

$$\frac{495 \times 2}{18} - 53 = 2 \text{ the answer.}$$

Practical Questions.

1. A man is to receive 300*l.* at 12 several payments; the last payment is to be 47*l.* 'tis required from this to find the first payment?

Answer 3*l.*

2. A young industrious damsel resolved to try her fortune in Paul's church-yard in the shape of an orange-wench, having gained credit for her barrow and a proper shew of fruit, 'till she found herself in a capacity to pay for them. She follows the business 15 weeks, and then finds, besides maintaining herself, she is possessed of £.7-16-2 in ready money, and that her barrow and stock in trade (both in the mean time honestly paid for) were worth 11*s.* 4*d.* Now she remembers in the course of this trade (from being better known and the like) that her clear gains per week advanced equally one above another from the first, and that the last amounted to 18*s.* 9*d.* 'Tis required from these particulars to find how much our fair adventurer cleared the first week, and what the above-mentioned weekly increase after that might be?

Answer, the first weeks gain was 3*s.* 7*d.* and the equal difference 13 pence.

Scholium.

Thus we have gone over 14 problems (i. e. 8 in the text and 6 in the notes) which perhaps may be as many to the full, as will in a general way be found useful; however as they are very easy, and something curious, we have endeavoured to order them in a manner, that will shew their reason and dependance (as far as they go) perhaps rather better, than is commonly to be met with in books of arithmetic.

Before we leave this subject, it will be proper to observe, that in any arithmetical progression, consisting of three terms, the double of the middle term is equal to the sum of the extremes. Thus in 3. 6. 9 --- $6 \times 2 = 3 + 9 = 12$, as is evident: Hence half of the sum of the extremes is equal to the middle term. And when a middle number is thus found betwixt two given ones for certain purposes (as in reducing the various breadths of a field to an equal one, &c.) it is called an *arithmetical mean*. It will farther appear, that in any A. P. of 4 terms the sum of the two means is equal to the sum of the extremes; hence if from either of these sums a term of the other be taken, it must leave its yokefellow in the addition.---Thus in 2. 4. 6. 8---
2+8

$2+8=4+6=10$; hence $10-8=2$, and $10-6=4$. &c. Farther in a series of ever so many terms the sum of the two extremes must be equal the double of the mean, if odd, and that of every other two terms equally distant from them.---Thus in 1. 3. 5. 7. 9--- $1+9=3+7=2\times 5=10$. And in 2. 6. 8. 10. 12. 14. 16--- $2+16=4+14=6+12=8+10=18$; all which is evident from the nature of an A. P. and the demonstration of problem I.

II. Geometrical Progression (G. P.)

Definition.

"This is when the terms of a series of numbers gradually exceed, or fall short of one another, a certain number of times, as by the constant multiplication or division of some equal number.---Thus 4. 8. 16. 32. 64, and 243. 81. 27. 9. 3. 1 are two series in G. P. the one increasing by a constant multiplication by 2, and the other decreasing by a constant division by 3.

In this progression the same number of particulars might be considered as in arithmetical, which would also admit of a like variety of problems; but as the greatest part of the rules can only be demonstrated by algebra, and are likewise very tedious and seldom wanted, we shall only here consider two of them, observing that the names of the particulars are alike in both series, except it be, that here the common multiplier, or divisor is called ratio."

Problem I.

"The first term, last term, and common ratio given to find the sum of all the series."

* Rule.

"Multiply the last term into the ratio, and from the product subtract the first term; the remainder divided by the ratio less unity, gives the sum of all the series."

Example.

The first term of a G. P. is 1, the last term 2187, and the ratio 3, what is the sum of the series?

$$3 \times 2180$$

* The demonstration at the end of the article.

$$\frac{3 \times 2187 - 1}{3 - 1} = 3280 \text{ the answer.}$$

Practical Questions.

1. The extremes of a G. P. are 1 and 65536, the ratio 4, what is the sum of the series? Answer 87381.

2. The extremes of a G. P. are 1024 and 59049, the ratio $1\frac{1}{2}$ what is the sum of the series? Answer 175099.

Problem II.

"The first term, ratio, and number of terms given to find the last term, or any other assigned."

** Rule.*

"Over a few of the first terms (made and put down for that purpose) place the terms of an A. P. (called indices) differing by an unit, and beginning at one, if the first term of the G. P. is equal to the ratio, and in all other instances with a cypher. Then in the first case if you add up any of the indices together one after another, so as to make the number of terms required, and multiply continually the terms in the geometrical series that stand under them, the last product will be the term required. But in the second case you must, in the same manner, work for a term one short of that wanted, and divide the product of every multiplication in the G. P. by the first term, as you go on, and the last quotient will be the term required."

Examples.

1. The first term of a G. P. is 2, the number 13, and the ratio 2, what is the last term?

1. 2. 3. 4. 5 Indices.
2. 4. 8. 16. 32 Leading terms.

Then $\begin{cases} 4+4+3+2=13 \text{ the number of terms.} \\ 16 \times 16 \times 8 \times 4 = 8192 \text{ the answer.} \end{cases}$

2. The first term of a G. P. is 5, the number of terms 16, and the ratio 2, what is the last term and also their sum?

Q9

O. 1.

* See the latter end of this subject.

O. 1. 2. 3. 4 Indices.
5. 10. 20. 40. 80 Leading terms.

$$\begin{aligned} \text{Then } \left\{ \begin{array}{l} 4+4+4+3=15 (=16-1) \\ 80 \times 80 \times 80 \times 40. \end{array} \right. & \text{ But } \frac{80 \times 80}{5} = 1280, \frac{1280 \times 80}{5} = \\ 20480, \frac{20480 \times 40}{5} & = 163840 \text{ the last term. Hence } \frac{2 \times 163840}{2-1} = \\ & = 327675 \text{ the sum.} \end{aligned}$$

Practical Questions.

1. The first term of a G. P. is one, the ratio 2, and the number of terms 23, what is the last term? *Answer* 4194304.

2. One at a country fair had a mind to a string of 20 fine horses; but not caring to take them at 20 guineas per head, the jockey contented, that he should, if he thought good, pay but a single farthing for the first, doubling it only to the 19th. and he would give the 20th. into the bargain: This being presently accepted, how were they sold? *Answer* at £. 27-6-1 $\frac{7}{8}$ a piece.

3. A nobleman dying, left ten sons; to whom, and to his executor he bequeathed his estate in manner following; viz. To his executor; for seeing his will performed, he left 1024 crowns, the youngest son was to have as many, and half as many as the executor; and so every son to exceed the next younger by the equal ratio of 1 $\frac{1}{2}$. The question is what the eldest son's portion was? *Answer* 59049.

4. A coffee-man, upon the signing the last peace (in the way of wagering) for fifty guineas down, agreed to pay the first day one coffee-berry; the second 2, the third 4, and so on to double the quantity every day, till the same was proclaimed. What number of berries would it amount to, supposing the time 60 days; and what would the value be, supposing 1000 berries to the pound, and the pound to be sold for 5 shillings? *Answer* 11529215046066346975 berries, and £. 288230376151711-10 value.

A demonstration of problem I.

We thought once to have desired the reader to take the truth of this rule for granted, as it must be demonstrated after the algebraic manner of reasoning, but considering that we had already familiarized the signs, and that every step in the process refers no farther than to common sense, we judged it might be no unentertaining exercise for the young student to endeavour to follow us through a demonstration of

of that kind; nor will it be found an ill judged piece of curiosity, if ever he intends to engage with these studies, as he may from this specimen form some notion of its method of arguing, and perhaps gather up a few serviceable hints.

But before we enter upon our task we must premise the following useful lemmas, relating to the nature of these series, which ought not to be passed over on other accounts.

Lemma I.

In any G. P. of three terms as 2. 6. 18 the square of the mean term is equal to the product of the extremes.---Thus $6 \times 6 = 2 \times 18 = 36$.

Demonstration.

For whatever be the ratio, the last term must always be the square of it into the first, i. e. here $3 \times 3 \times 2 (=18)$ which multiplied into the first is $3 \times 3 \times 2 \times 2$, and the second (or mean) term must always be the first multiplied by the ratio, i. e. $3 \times 2 (=6)$; but this squared is $3 \times 2 \times 3 \times 2 = 3 \times 3 \times 2 \times 2 (=6 \times 6 = 36)$ consequently as the component factors of the product of the extreme terms are constantly the same as those of the square of the mean, the results of each must be equal. Hence the lemma is true.

Corollary.

Hence also the square root of the product of the extremes must be the middle term. And when for certain uses a middle number is thus found betwixt two other given ones (as in finding the side of a square equal in area to a rectangle, &c.) it is called a *geometrical mean*.

Lemma II.

In any G. P. of four terms as 3. 6. 12. 24 the product of the mean is equal to the product of the extremes.---Thus $3 \times 24 = 6 \times 12 = 72$.

Demonstration.

For from the nature of multiplication if one factor be encreased as many times as the other is lessened, their products will still be the same. Hence in the above series taking the middle terms as factors, they give $6 \times 12 = 72$ for their product, and as from what we have

Q q 2

just

just said, if we make one factor or 12 twice as much as it is, i. e. 24, and the other 6 half as much as it is, or 3, their products will still be equal; but this alteration has evidently given the extremes of the series, and as this practice might always be supposed, whatever be the ratio, the lemma is manifest.

Corollary.

It will not be difficult to see from what is said above, that in any G. P. consisting of an even number of terms, the product of the means will be equal to the product of the extremes, or any other pair equally distant from them. Also that in any G. P. containing an odd number of terms, the square of the mean will be equal to the product of the adjoining extremes, or any two equally distant from it.---Thus agreeably to the first case in 5. 10. 20. 40. 80. 160--- $5 \times 160 = 10 \times 80 = 20 \times 40 = 800$, and with respect to the second in 6. 12. 24. 48. 96.--- $24^2 = 12 \times 48 = 6 \times 96 = 576$.

Scholium.

As the word *ratio* (see the notation of ratios a little farther on) is understood among mathematicians to mean "the relation of two things when compared as to quantity" or how many times the greater exceeds the less, and as in any 4 terms of a geometrical series (let them follow one another regularly or not) the 2d. and 4th. exceed the 1st. and 3d. respectively the same number of times, the two first and two last have consequently the same ratio one to another. But *proportionality* (see an article called Common properties of proportionality towards the end) is again understood to be "the similitude of ratios;" hence as the ratio of the two first terms in the series of lemma II. is by what is there proved, the same as the ratio of the two last, according to this idea they are proportionable, i. e. $3 : 6 :: 12 : 24$; and as we have likewise proved, independent of this, in the same lemma, the products of the extremes to be equal to the products of the means, we have proved this to be the case in four terms that are proportionable; hence the truth of the golden rule is set in a more general light than it was before. For if the product of the extremes of four terms in proportion be equal to the product of the means, 'tis exceeding evident, that the product of the means (or 2d. and 3d. terms) divided by either of the other (as the 1st. suppose) must give the remaining one (or 4th. term), which is the practice of the rule.

Definitions.

In the two terms of any ratio, that, which is compared to the other, is called the *antecedent*, and the other, to which it is compared, is

is called the *consequent*. Thus in the last example $3:6::12:24$, 3 and 12 are antecedents, and 6 and 24 consequents. And when the terms of two ratios making a proportion succeed one another, as so many terms of a G. P. the proportion is said to be *continued*, as in the above; but when it is broken, or the ratios are taken between such pairs of numbers as do not stand together in a G. P. the proportion is then said to be *discontinued*, as in these $1:3::12:36$ or $2:3::10:15$; and as the terms of the ratios of the rule of three are mostly thus disjoined, 'tis sometimes in reference to that called *disjunct proportion*.—It follows from these particulars, that in any G. P. all the terms, but the first and last, are both antecedents, and consequents, as there are always some terms both before and after them to form a comparison with.

Lemma III.

In any G. P. it holds, as any one antecedent is to its consequent, so is the sum of all the antecedents to the sum of all the consequents. Thus in 2. 4. 8. 16. 32. 64. 'tis as $2:4::2+4+8+16+32$ (the sum of the antecedents) $:4+8+16+32+64$ (the sum of the consequents).

Demonstration.

For in the above (or any ascending series) 'tis plain every term in the added consequents must always exceed the correspondent one in the added antecedents, as many times as is equal to the ratio, or what number of times any single consequent exceeds its antecedent. Hence as every one term in the added antecedents bears the same ratio to their respective ones in the added consequents, the total of all in each of the series must be in the same ratio: Therefore the lemma is true.

These things premised, we will enter upon the demonstration. If besides the first term, last term, and ratio given, we actually knew what the sum of the G. P. was (which we now enquire after) from the last lemma, we could easily find the sum of all the antecedents, as they would be the sum of the whole series made less by the last term; and also the sum of all the consequents, as they would be the total sum made less by the first term.---Therefore if the extremes of a G. P. be 2 and 1458, and the ratio 3 we may find its sum by the following reasoning. Suppose the total wanted to be really known, and that 'tis equal to the letter x , then will the sum of the antecedents be by the above (in signs) $x-1458$ and the sum of the consequents $x-2$. Again 2×3 must be the second term of the series; therefore by the last lemma, we have this proportion $2:2 \times 3::x-1458:x-2$. Now as the product of the extremes of any proportion

tion is equal to the product of the means (per the last scholium) we infer that $2 \times 3 \times x - 1458 = 2 \times x - 2$. And as these two expressions are equal, if we deprive them both of the common factor 2, they will still be equal, from what has been often hinted; therefore $3 \times x - 1458 = x - 2$. But as the product of the difference of any two numbers (as $x - 1458$) by another number (3) must be equal to the difference of their products by the same number, the above left hand expression may be wrote as here $3 \times x - 3 \times 1458 = x - 2$. Now as $3 \times x - 3 \times 1458$ intimates that 3×1458 is to be taken from $3 \times x$, 'tis plain, that if we neglect that deduction, we in effect give that expression so much, but if we encrease it by 3×1458 we must encrease its opposite equal one the same quantity in order to preserve their equality; hence we may write $3 \times x = x - 2 + 3 \times 1458$, which, by setting the terms only in a different order, may stand thus $3 \times x = x + 3 \times 1458 - 2$. Again if we deduct from both expressions the same quantity x (whatever it be) they must still be equal, therefore we may assert $3 \times x - x = 3 \times 1458 - 2$. Now $3 \times x - x$ (or which is the same $3 \times x - 1 \times x$) may be thus wrote $3 - 1 \times x$ (by the reverse of what was intimated before about the multiplication of two numbers to be subtracted from one another) therefore our expressions may stand in this manner $3 - 1 \times x = 3 \times 1458 - 2$; but if we take the factor $3 - 1$ from the left hand expression, we consequently divide it by it; therefore to keep both our expressions still equal, the right hand side one must also be divided by it. Hence we conclude $x = \frac{3 \times 1458 - 2}{3 - 1}$, which actually brought out gives $x = 2186$.

Now having, by the above chain of reasoning, found what x is equal to in known terms, we have discovered what the sum of the series (which it all along represented) is equal to. And if what is imported by the terms and signs of this expression $\frac{3 \times 1458 - 2}{3 - 1}$ (which algebraists would call a *theorem*) be turned into words, they will make the very rule we wanted to demonstrate, i. e. first it appears we must multiply the last term 1458 by the ratio 3, and from the product subtract the first term 2, and then divide what remains by the ratio 3, made less by 1 for 2186 the sum of the series. And as the like parts of any other progression would have stood in the same form at the end of a similar process, the rule is shewn to be universally true.

A demonstration of the 2d. problem.

The agreement, that there is between the addition of the terms of an A. P. with that of the multiplication of the correspondent ones in

in a G. P. as mentioned by the rule, may be thus illustrated. In this series

1. 2. 3. 4. 5. 6 &c.
2. 4. 8. 16. 32. 64 &c.

where the first term is equal to the ratio, 'tis evident every term after it, is some power of the ratio, and that an A. P. beginning with unity, placed over it, as directed, points out at every term the number of factors, of which it is composed. And as from the nature of multiplication any power may be produced without gradually ascending from the root, by the multiplication of other numbers that are composed of the same factors the same number of times; 'tis evident the addition of indices, here spoken of, must always point out the right numbers for this purpose, which is all the advantage this device has over an actual raising of the first term to the power referred to by the last term.

As to the second case, when the G. P. does not begin with the ratio, as 5. 10. 20. 40. 80. &c. it appears that every term after the two first contains a power of the ratio multiplied into the first term, and that the whole series may be thus properly taken into factors---

0. 1. 2. 3. 4 &c.
2. 4. 8. 16
× × × ×
5. 5. 5. 5. 5 &c.

Hence 'tis visible, that on placing the A. P. over the higher row of pure powers, as before, a cypher will be thrown over the first term of the given series, which accounts, with what is said before, for the reason of that part of the rule. And as every term of the G. P. is just as many times bigger than its corresponding pure power, as there are units in the first term (i. e. 5 here), and as the multiplication of any two of them (in order to get some other one by the rule), would bring the first term as a factor *twice* into the product, 'tis evident the product must be divided by the first, in order to take that superfluous factor out again. It also appears, that as the A. P. begins with a cypher every figure must fall a place farther over the G. P. than in the other case; hence the direction for working for a term less than the number given is evident.

Scholium.

From this conformity between numbers in A. P. as indices, placed over a G. P. the first notion of *logarithms* has arose, which are "certain numbers in tables placed opposite the natural order of numbers 1. 2. 3. &c. as far as 10000, of such a nature, that the addition of any

any two or more of them, refers to another, against which is the product of the natural numbers they were taken for." Hence the subtraction of these logarithms must discover in the same manner the quotient of the numbers they are the logarithms of. Hence also (as by this means where the factors are still the same, the logarithm will only be as often taken, as is denominated by the number of factors) the logarithm of any number, multiplied by the index of any power, must give the logarithm of the power of that number; and on the contrary the logarithm of any number, divided by a number, answering the index of any root, must give the logarithm of the root: And as these operations are very easy with respect to those, which produce the same effect by natural numbers, they are justly looked upon as one of the greatest, and most useful inventions that have ever been added to the science of figures. We are indebted for them to Lord *Napier* (a Scotch nobleman or baron) who 'tis said with the assistance of Mr. *Briggs* Savilian professor of geometry at Oxford, about the year 1624, with incredible labour, perfected these tables, as they now stand.

To make this hint a little plainer, in the two series below,---

0.	1.	2.	3.	4.	5.	6.	7.	8	Indices (or Logs.)
1.	2.	4.	8.	16.	32.	64.	128.	256	G. P.

2 the log. of 4, + 4 the log. of 16, = 6, the log. of 64, which is the product of 4, and 16, their natural numbers.

Again. 8 the log. of 256, - 3 the log. of 8, = 5, the log. of 32, the quotient of 256 by 8. Farther. 2, the log. of 4, $\times$ 3, the index of the third power, = 6, the log of 64, which is the cube of 4.

Also. 8, the log. 256, $\div$ 4, the index of the 4th. root, = 2 the log. of 4, which is the 4th. root of 256. &c.

This praxis is very obvious from what has been already said; but the reader will easily observe there lies a defect in these series, inasmuch as the G. P. does but take in a very few of the natural numbers, i. e. a log. for 3 is wanted betwixt 2 and 4. a log. of 5, 6, and 7 betwixt 4 and 8, &c. And tho' other G. P. might be used, yet they would still leave out the greatest part of the natural numbers. Now in discovering the logarithms of these intermediate numbers the whole art has laid, and as they must be somewhere in value betwixt 0. 1. 2. 3, &c. 'tis plain they will be decimals, which is the appearance they have in the tables.

Mathematicians take into consideration several other sorts of progressions and series; but as they are too difficult to be accounted for properly

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properly on such limited principles as ours, and not wanted in ordinary affairs, 'tis the best to omit them, as also the rules called *Combination, Election, Permutation, &c.* of numbers, except indeed the last, in which an instance or two may not be amiss, as in the article following.

PERMUTATION.

Definition.

"THE rule here teaches to discover, how many several ways any given number of things may be changed in respect to their places; as how many changes may be rung upon a given number of bells, &c."

* Rule.

"Multiply the natural series of numbers from one (i. e. 1, 2, 3, &c.) up to the given number of things together, and the last product is the answer."

Example.

Suppose the ten stringed instrument mentioned in the Psalms gave ten different sounds, how many changes could be made upon it, with regard to succession of notes?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 = 3628800 \text{ the answer.}$$

R r

Practical

* If there be two things or letters, as *a, b*, 'tis plain there can be but two positions or changes made upon them, i. e. *a, b*, and *b, a* $= 1 \times 2$. Add to these a third *c*, and suppose it to possess in order, a first, second, and third place; then as at each remove the other two may make two changes in respect to one another, as before, they must at the end of all the removes, with respect to the constant variety added by the new letter, have made 6 changes, or $2 \times 3 = 1 \times 2 \times 3$. Again suppose another letter *d* to be added, and that it also goes through four different places, then in like manner as above, while in each place the foregoing 3 must make their usual changes with regard to one another (i. e. 6) which receiving a new variety from the position of the added letter all along, must make at the end 24 changes, i. e. $6 \times 4 = 1 \times 2 \times 3 \times 4$. Now we have for these changes, thus far multiplied, the natural digits, as mentioned in the rule, and as the same reasoning might be continued to any number of things, its truth is sufficiently illustrated.

Practical Questions.

1. How many changes may be rung upon 12 bells, and how long would they be in ringing, but once over, supposing ten changes might be rung in a minute, and the year to contain 365 days, 6 hours?

Answer 479001600 changes, and the time would be 91y. 3w. 5d. 6h.

2. A young scholar, but an arithmetician, coming into a town for the convenience of a good library, demands of a gentleman with whom he lodged, what his diet would cost him for a year? The gentleman asked him 10l. the scholar answered, he was not certain what time he might stay, and would know, what he must give him for his diet so long as he could place his family, consisting of 6 persons besides himself, every day at dinner in a different position. The gentleman considering of it, and thinking it would not be long, tells him he would allow him his diet so long for 5 pounds; to which the scholar assents. The question is what he gave for his table per annum? Answer His time 13 years 308 days, or 10 months 15 days, which makes but about 7s. 3d. a year.

*The Rule of Contraries.**Definition.*

*“BY this are answered * several of those questions, where of an unknown number or quantity 'tis asserted, that if (in reference to some of the common operations of arithmetic) it be ordered in such a manner, and what results in such other manner, &c. the last result will be equal to some certain number or quantity given.”*

+ Rule.

“Take the given number, and with it perform an operation just contrary to that last mentioned in the question, and with the result, another operation.”

* Some examples may appear to belong to this rule, which cannot easily be solved by it, and they will in general be found of a kind, wherein what is to be added or subtracted is some number of times, or part of, the unknown number, or some other derived from it, in the course of the question. When this is the case, recourse must be had to Position, the article immediately following.

† The reason on a little examination will be very evident from the practice.

This method of reversing operations (with others such like) is so very often applied in almost every mathematical enquiry, that the enforcing of it ought not to be

operation contrary to what was spoke of immediately before the last, and so on, running from the latter end of the question to the beginning; and when you have reversed every operation mentioned, the final result is the answer required.—'Tis almost needless to repeat, that addition and subtraction, multiplication and division, raising of powers and extracting roots, are respectively contrary operations, and mutually undo one another."

Examples.

1. A gentleman, having bought a house, and being disposed to try the knowledge of his son in arithmetic, told him, that if the number of pounds the house cost, were divided by 8,---then the quotient multiplied by 50,---next the square root taken of this product, and, lastly, 10 added to the result the sum would be 60 pounds. What did the house cost?

60 last result.

10 subtracted, the contrary of being added.

50 squared, the contrary of taking the root.

50

5|0) 250|0 divided by 50, the contrary of being multiplied by it.

50 multiplied by 8, the contrary of being divided by it.

8

£.400 the answer, as is easy to prove.

2. A lady being asked her age, in order to avoid a direct reply to what is reckoned so impertinent a question, answered, that if it was multiplied by 3, and also $\frac{2}{7}$ of the products multiplied by the same number, the square root of $\frac{2}{9}$ of the last result would be 4. Query her age?

R r 2

4 squared

be omitted, and, as the questions here collected (tho' not of much seeming utility) may perhaps furnish some useful hints relative to this end, 'tis hoped they will not be thought altogether unworthy the reader's consideration, and more especially when he is told, that one of our writers of algebra has taken up above 50 pages in solving questions of this class, and to whom several of these belong,

4 squared, the contrary of the root mentioned.

4

16 $\frac{2}{3}$ taken of this, the contrary of $\frac{2}{3}$.

9

2) 144

3) 72 divided by 3, the contrary of being multiplied by it,

24 $\frac{7}{2}$ taken of this, the contrary of $\frac{7}{2}$.

7

2) 168

3) 84 divided by 3, the contrary of being multiplied by it.

28 the answer.

Practical Questions.

CLASS I.

1. A general, upon taking a list of his army, found that, if from the square root of the number of men it contained there were subtracted 3196, and to this remainder added 2721, and from this result 1711 again subtracted, there would finally leave 99997814. Required the number of men in the army? Answer 1000 men.

2. A merchant broke for so many pounds, that, if they were in the first place multiplied by 4; and the product next divided by 6; and then the square root taken of the quotient, and lastly the root made less by 60, there would remain 40. What was the sum for which the merchant broke? Answer 15000l.

3. A running footman being out upon an errand was told, that, if he squared the distance he was to run---next multiplied the square by 4---next divided the product by 40---next added 500 to the quotient---next deducted 1400 from the sum, and finally extracted the square root of the remainder, the result would be 10. How many miles was the footman to run; Answer 100 miles.

CLASS II.

1. What number is that, to which, if you add $\frac{1}{11}$ of 12, more $\frac{1}{19}$ of 27, and from the total, subtract $\frac{1}{3}$ of $7\frac{1}{2}$, less $\frac{29}{30}$ of $1\frac{1}{4}$, the remainder shall be 8? Answer $6\frac{3911}{3016}$.

2. A

2. A gentleman returned a loser from the gaming-tables; when some of his acquaintance laughing at him for his folly, asked him how much he had lost; to which he answered, If you in the first place square the number of pounds I have lost---secondly divide the result by 4---thirdly multiply the quotient by 10,---fourthly add 3900 to the product---fifthly extract the square root of this sum, and lastly deduct 80 from the root, the remainder will be equal to 90. 'Tis required from this to know what the gentleman lost?

Answer 100l.

CLASS III.

A merry young fellow, in a small time, got the better of $\frac{1}{3}$ of his fortune; by advice of his friends, he then gave 2200l. for an Exempt's place in the guards; his profusion continued 'till he had no more than 880 guineas left, which he found by computation was just $\frac{3}{20}$ of his money, after the commission was bought: Pray what was his fortune?

Answer 10450l.

* POSITION.

Definition.

"THIS is a method of discovering by supposed numbers true answers to questions, in which there do not appear to be sufficient data; &c. to attempt the solution, by the common direct rules. 'Tis of two kinds called single and double as follows."

I. Single

* This rule has in several late arithmetical authors been omitted, because, say they, all questions solvable by it, may be more speedily, and better answered by a common smattering of algebra. But this, tho' true, does not seem to be dealing fairly with their pupils, who may a great many of them perhaps never be acquainted with that science, and, if so, they would certainly thank their instructor for giving them something, which might in a little degree supply its place, and contribute in like manner to their amusement. For the sciences should be made in some sort entertaining, or else very few will be tempted to pursue them; and this is tacitly agreed to by the practice of writers classed far above arithmeticians. Besides this, to have dealt consistently, other rules, that might have been better effected by algebra, should have been omitted, and then Progression (not to mention some other articles), as not very useful where it is taught, would have gone along with it. But it may be answered, that several of the particulars there laid down lead to what is very useful in the mathematics, and consequently a hint of their nature can never be unseasonably thrown in. I agree to it, and would have this another reason for retaining Position, as it may be applied sometimes to the solution of *affected* and *exponential* equations better than the method of series. Several other approximations it is very useful in, and if great names, and great subjects, may raise it in the opinion of the learner, we shall observe to him, that no less a man than Dr. Gregory, in his astronomy, has apply'd it in determining, by approximation, the trajectory of a comet, &c.

I. *Single Position.**Definition.*

"The rule here resolves all those questions, which give certain parts, sums of parts, and other results, or conditional circumstances of unknown required numbers, which would bear to one another, and to those unknown numbers they came from, a constant proportion, of what value or magnitude soever they were supposed."

* *Rule.*

"Assume a number at pleasure (called the position) for the unknown answer, and proceed with it according to the purport of the question, and if the result at the end be the same as that given in the question, you have fallen upon the answer, and nothing farther is required; but if it does not agree with it (as will chiefly be the case) you may find from this false result, the true one, by saying, as the false result is to the position which gave it, so is the true result of the question to the answer."

Examples.

1. Three persons A, B, and C upon enquiry discover, that A's age is double B's, and B's triple C's, and the sum of their ages to be 140 years: What was each person's age?

$$\left. \begin{array}{l} \text{Suppose A's age} = 60 \\ \text{Then B's} = \frac{60}{2} = 30 \\ \text{And C's} = \frac{30}{3} = 10 \\ \text{Sum} \quad \quad \quad 100 \end{array} \right\}$$

$$\text{Hence } 100 : 60 :: 140 : \frac{140 \times 3}{5} = 84 = \text{A's age.}$$

$$\text{Then } \frac{84}{2} = 42 = \text{B's age.}$$

$$\text{And } \frac{42}{3} = 14 = \text{C's age.}$$

$$\text{Proof } \underline{\underline{140}}$$

2. A

* This will be found to carry its own evidence along with it, from the perusal of a question or two.

2. A merchant bought 30 yards of taffety, and forty yards of fattin for 330 shillings, but every yard of fattin cost twice as much as a yard of taffety, what did a yard of this last mentioned article cost?

Suppose the taffety = 4s. per yard; then a yard of fattin = $2 \times 4 = 8$ s. per yard.

Hence $30 \times 4s + 40 \times 8s = 440s$. the price of the whole.

Therefore $440 : 4s :: 330 : \frac{330 \times 4}{440} = 3s$. the price of the fattin.

Proof $3s \times 30 + 6s \times 40 = 330s$.

Practical Questions.

CLASS I.

1. A gentleman bought a chaise, horse, and harness for 60l. the horse came to twice the price of the harness, and the chaise to twice the price of the horse and harness; what did he give for each?

Answer Horse £.13-6-8; harness £.6-13-4, and chaise 40l.

2. A, B, and C, determining to buy together a certain quantity of timber, worth 36l. agree that B shall pay $\frac{1}{3}$ more than A, and C $\frac{1}{4}$ more than B; I demand how much C paid?

Answer 15l.

3. Admit there are £.212-14-7 to be divided amongst a captain, 4 men and a boy; The captain to have a share and a half, the men each a share, and the boy $\frac{1}{3}$ of a share: What ought each person to have? Answer Each man £.36-9-4 $\frac{2}{7}$, the captain £.54-14-0 $\frac{3}{7}$, and the boy £.12-3-1 $\frac{3}{7}$.

CLASS II.

1. Four merchants have a sum of money to be divided among them, in such manner, that the first shall have $\frac{1}{3}$ of it; the second $\frac{1}{4}$; the third $\frac{1}{6}$; and the fourth the remainder which is 28l. what was the sum?

Answer 112l.

2. Three men A, B, and C put in money together, and gained 100l. of which A took up a certain sum, B took up twice as much as A, and C took up thrice as much as B; what did each take up a-part?

Answer A £.11-2-2 $\frac{1}{2}$, B £.22-4-5 $\frac{1}{4}$, C £.66-13-3 $\frac{3}{4}$.

CLASS III.

There is a cistern with three unequal cocks, containing 60 gallons of water; and if the greatest cock be opened the cistern will be empty in one hour, if the second cock be opened, it will be empty in two hours; if the third be open it will be empty in three hours. Now I demand in what time it will be empty, if all run together?

Answer 32.727' 3c.

II. Double

* II. Double Position.

Definition.

"This rule answers the † most part of those questions, in which are given certain conditions or relations of unknown required numbers, with the final result of certain operations, &c. which include (along with what is above intimated) some fixed independent numbers also given, and which consequently will not bear (as likewise any affected by them) a constant proportion to the unknown numbers of what value or magnitude soever they are supposed."

‡ Rule.

"Make two suppositions, and proceed with them according to the tenor of the question, as before. Then (in case you have not hit upon the answer by chance) see how much the results of these assumed numbers (or positions) exceed, or fall short of what they should have been, and mark the differences with + or - accordingly for (what are called) errors. Next multiply the positions and errors alternately, i. e. the first position into the second error, and the second position into the first error, and if the errors are both alike, or are both distinguished by the same sign, divide the difference of the products, by the difference of the errors, but if they are of different kinds, divide the sum of the products by the sum of the errors, and the quotient is the answer."

Examples.

1. An impertinent fellow, on asking another man what money he had in his pocket, got this for answer; as much more, as I have, half as much more, with 7s. 6d. would just make 5l. pray what had he?

Suppose

* If the reader should not be able at the first to distinguish from the definitions to which rule questions of this nature belong, it will not be amiss for him to know, that any question which can be solved by the single rule may also be answered by the double one.

† All questions, that appear to fall under this rule, cannot be solved by it, as will be found by experience, and those that fail are generally in some part of them circumstanced as follows. 1st. A given number is to be divided by the unknown (or supposed) number, or some part of it. 2d. The number sought, or some part of it, is to be raised to some given power. 3d. Some parts of the required number are to be multiplied together, or, lastly, some root of the number sought or part of it, is to be extracted.---To these observations we may add the following universal method of discovering when the rule will succeed, and when not, *viz.* Make three or more suppositions with equidifferent numbers, and if the errors be also equidifferent, the answer may be found by the rule, otherwise 'tis impossible as the question stands.

‡ The demonstration at the end of the article,

Suppose 1st. 23s. Then $23 \times 2 + \frac{23}{2} + 7s. 6d. = 65s.$ Hence (5l. =) 100s. - 65s. = 35 + is the first error.

Again, suppose 41s. Then $41 \times 2 + \frac{41}{2} + 7s. 6d. = 110s.$ Hence 110 - 100 = 10 - is the second error.

The parts concerned may now stand orderly thus

1st. position 23	41 2d. position,
$\begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array}$	
1st. error 35 +	10 - 2d. error.

Hence $\frac{23 \times 10 + 41 \times 35}{35 + 10} = 37s.$ the answer.

Proof $37 \times 2 + \frac{37}{2} + 7s. 6d. = 100$ shillings.

2. 30 ells of lockram and canvas were sold for 51s. the canvas was rated at 18d. per ell, and the lockram at 2s. how many ells of each sort were there?

Suppose 1st. 10 ells of canvas, then must $30 - 10 = 20$ be the ells of lockram.

$\begin{array}{r} \text{s. d. s.} \\ \text{Hence } 10 \times 1 - 6 = 15 \\ 20 \times 2 - 0 = 40 \\ \hline \text{Amount } 55 \end{array}$	$\left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Consequently } 55 - 51 = 4 + \text{ is the first error.}$
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Suppose again, 6 ells of canvas, then must $30 - 6 = 24$ be the ells of lockram.

$\begin{array}{r} \text{s. d. s.} \\ \text{Hence } 6 \times 1 - 6 = 9 \\ 24 \times 2 - 0 = 48 \\ \hline \text{Amount } 57 \end{array}$	$\left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Consequently } 57 - 51 = 6 + \text{ is the 2d. error.}$
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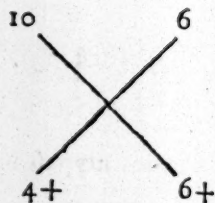
So

Therefore

Therefore $\frac{10 \times 6 - 4 \times 6}{6 - 4} = 18$ the ells of canvas,

And $30 - 18 = 12$ the ells of lockram.

Proof $18 \times 1s. 6d. + 12 \times 2s. = 51s.$



3. Three persons discoursed thus of their age; quoth A I am 18 years of age, quoth B I am as old as A and $\frac{1}{2}$ C, then quoth C I am just as old you both; required from this the ages of B and C?

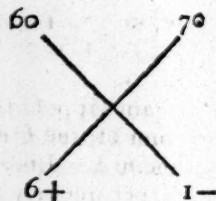
Suppose C's age 60. Then $\frac{60}{2} + 18 = 48 = B$'s age, but $18 + 48 = 66$, which should have been 60, hence $66 - 60 = 6+$ is the first error.

Again, suppose C's age 70. Then $\frac{70}{2} + 18 = 53 = B$'s age; but $18 + 53 = 71$, hence $71 - 70 = 1-$ is the second error.

Therefore $\frac{6 \times 70 - 60 \times 1}{6 - 1} = 72$ C's age.

Consequently $\frac{72}{2} + 18 = 54$ B's age.

Proof $18 + 54 = 72$.



4. There is a certain fish, whose head is 9 inches long, and his tail is as long as his head and half his body, and his body is as long as both head and tail. The whole length of the fish is demanded?

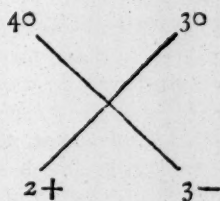
1st. Suppose the body $= 40$ inches, then $\frac{40}{2} + 9 = 29 =$ the length of the tail; but per question $29 + 9 = 38 =$ the length of the body, hence $40 - 38 = 2+$ is the first error.

2d. Suppose the body again $= 30$, then $\frac{30}{2} + 9 = 24 =$ the tail, but this gives $24 + 9 = 33$ for the body. Hence $33 - 30 = 3-$ is the second error.

Therefore

Therefore $\frac{40 \times 3 + 30 \times 2}{2 + 3} = 36$ the body.

Consequently $36 + 9 + \frac{32}{2} + 9 = 72$ is the answer.



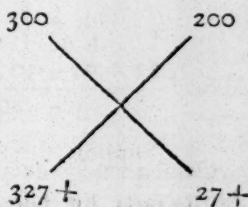
5. Three merchants, from three different fairs, meet together at an inn, where they reckon up their gains, and find them the sum of 780 crowns. Moreover, if you add the gain of the first, and second together, and subtract the gain of the third from the sum, there remains the gain of the first + 82 crowns; but if you add the gain of the second and third, and from the sum subtract the gain of the first, there remains the gain of the third - 43 crowns: What was the gain of each?

Suppose the gain of the third = 300 crowns; then $780 - 300 = 480$ = the sum of the gain of the first and second: Therefore $480 - 300 = 180$ is the gain of the first + 82 crowns, hence $180 - 82 = 98$ = the gain of the first: Hence also $480 - 98 = 382$ = the gain of the second. But per question $382 + 300 - 98 = 584$ = the gain of the third - 43, therefore $584 + 43 = 627$ is the gain of the third, consequently $627 - 300 = 327$ + is the first error.

Again suppose the third = 200 crowns. Then $780 - 200 = 580$ = the sum of the first + 82 crowns, hence $580 - 82 = 498$ = the gain of the first. Likewise $580 - 498 = 82$ = the gain of the second. But (per question) $82 + 200 - 498 = 184$ is the gain of the third - 43, hence $184 + 43 = 227$ is the gain of the third; therefore $227 - 200 = 27$ + is the second error.

Hence $\frac{327 \times 200 - 27 \times 300}{327 - 27} = 191$ the 3d.

man's share, from which in the same manner as above the first and second man's share will be found to be 316, and 273 respectively.



6. Hiero king of Sicily, having ordered a crown of pure gold to be made, suspected on its completion, that the workman had adulterated it with silver, and accordingly appointed Archimedes to discover the fraud, if possible. After revolving several methods in his mind, it occurred to him, that in bathing. &c. as much water is displaced by the immersion of the body, as is equal to it in magnitude;
Ss 2 and

and that silver being a lighter metal than gold, the same weight of it must exclude a greater quantity of water. He therefore got two pieces of these metals, each of the same weight with the crown, and on putting all three into a vessel full of water, found the quantities displaced respectively, and from thence the precise quantities of gold and silver, that were mixed in the crown. Now *Vitruvius*, who writes this history, does not mention the particulars of the process; but supposing the crown, and other masses to weigh each 80 ounces, and that when they were put into water, the gold expelled 7 cubic inches of water, the crown 8.04, and the silver 12.2: 'Tis required from this to ascertain, how much silver the workman had foisted into the composition?

First, suppose 300z. of silver; then there must be 500z. of gold.

Hence $80 : 12.2 :: 30 : 4.575$ inches the silver would exclude.
and $80 : 7 :: 50 : 4.375$ inches the gold would exclude.

8.95 inches the mixture would exclude.

Therefore $8.95 - 8.04 = .91 +$ is the first error.

Again, suppose 200z. of silver, then there must be 600z. of gold.

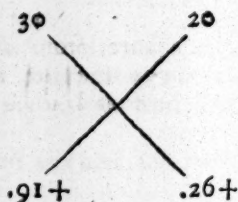
Hence $80 : 12.2 :: 20 : 3.05$ inches the silver would displace.
and $80 : 7 :: 60 : 5.25$ inches the gold would displace.

8.3 inches the mixture would displace.

Therefore $8.3 - 8.04 = .26 +$ is the second error.

Hence $\frac{20 \times .91 - 30 \times .26}{.91 - .26} = 160z.$ of silver.

Consequently the quantity of gold was 640z.
the truth of all which may be easily proved
as above.



Scholium.

Some questions, which, according to what has been observed, are exceptionable in these rules, may be changed to others, or reduced

to different considerations, which then fall within their limits; an instance or two of this, with a hint relating to extending them farther, may be as follows.

In the first example of the rule just illustrated, where 'tis said "as much money as a certain sum, and half as much, with 7*s*. 6*d*. will make 100*s*." by reason of the fixed part 7*s*. 6*d*. double position is used, but if we take 92*s*. 6*d*. the difference between 100*s*. and 7*s*. 6*d*. and consider the question to be "as much and half as much as a certain sum makes 92*s*. 6*d*." it may be answered by the single rule. For 'tis evident, that 7*s*. 6*d*. added to the result of the question, considered as above, will be the answer required. Also in this question, "The square of a person's age increased by its $\frac{1}{2}$, and then lessened by $\frac{1}{3}$ gives 891," by reason of the second power mentioned, it cannot be answered by the rule as it stands, but if stated thus, "A certain number increased by $\frac{1}{2}$ and then lessened by $\frac{1}{3}$ gives 891," 'tis evident the square root of the answer to this (i. e. 729) must be the age required (i. e. 27). Again in this example " $\frac{3}{4}$ of an old man's age multiplied by $\frac{2}{3}$ of it gives a product equal to 1080" the rule will fail by what was observed in the notes, but (as the products of these parts are to one another, as the square of the unknown number of what value or magnitude soever it is supposed, as is well known to geometricians) we may find the answer (60) if after the errors are found, as usual, we square the positions, and work with these squares in their stead, and extract the square root of what results. And in general we may observe (with Mr. *Welsh*), that when parts of an unknown number are given to be multiplied together (or powers raised of it or some part) and the result is a fixed number, the answer may be found in the same manner, as above, if the index of the power and root there mentioned be taken equal to the number of factors concerned.

We mentioned in the note page 309, that this rule was very useful in approximating to some answers, which were exceeding difficult to come at by any other means; and here we shall just deliver the method, as 'tis easy enough to understand and apply it, without formal illustration. * In any enquiry, where 'tis possible to prove any answer to be right, when discovered, "make two positions, as near the truth as you are able to guess, and from them deduce an answer, as directed in the rule: Then take the nearer of the two last positions, and the result above gained, as two other positions, and in like manner from them deduce another answer: Proceed thus, 'till the answer is as accurate, as you would have it, remembering, that at every operation it will sensibly approximate nearer and nearer the truth."

Practical

* See an instance of this kind in *Hill's arithmetic*, page 343.

Practical Questions.

CLASS I.

1. Two persons discoursing of their revenues, says A, if B would yield him a post he has of 25*l.* a year, their revenues would be equal: Says B, if A would give him a place, he holds of 2*l.* per annum, the revenue of B would be double that of A. Quere their revenues?

Answer A's is 116*l.* and B's 166*l.*

2. A labourer had 576 pence for threshing 60 quarters of corn (*viz.*) wheat and barley; for the wheat he had 12 pence per quar er, and for the barley he had 6 pence. Required, from this, how many quarters of each he threshed? Answer 36 of wheat, and 24 of barley.

3. A, B, and C would divide 100*l.* among them, so as that B may have 3*l.* more than A; and C 4*l.* more than B: How much must each man have? Answer A 30*l.* B 33*l.* and C 37*l.*

4. A hogshead filled with two sorts of wine, the one at 6*s.* and the other at 5*s.* per gallon, came to £.16-14. Required how much of each sort was put in? Answer 19*gal.* at 6*s.* and 44 at 5*s.*

5. A man lying at the point of death, left to his three sons all his estate in money; *viz.* to F, $\frac{1}{2}$ wanting 50*l.* to G, $\frac{1}{3}$, and to H the rest, which was 10*l.* less than the share of G; I demand the sum left, and each man's part? Answer, sum left 360*l.* whereof F had 130*l.* G 120*l.* and H 110*l.*

6. One man said to another, I think you had this year 2000 lambs: So I had, replied he, but what with paying the tithe of them, and their several losses, they are much abated: For at one time I lost half as many as I have now, at another time $\frac{1}{3}$, and another time $\frac{1}{4}$ as many: The question is, how many had he left?

Answer 864.

CLASS II.

1. A gentleman distributing money among some poor people found he wanted 10*s.* to be able to give 5*s.* to each, therefore he gives each 4*s.* only, and finds that he has 5*s.* left? Required, the number of shillings and poor people?

Answer 65*s.* and 15 poor people.

2. A merchant buying cloth finds, that if he takes 12 pieces, he shall want 42*l.* to pay for them; but if takes 9 pieces, then he will have 84*l.* too much: How much money did he carry with him to the market, how much did a piece cost, and what number did he buy, supposing he laid out what he had? Answer He had 462*l.* which will pay for 11 pieces at 42*l.* per piece?

3. A

3. A beam $\frac{1}{3}$ in the water $\frac{1}{2}$ in the mud, and 10 feet above the surface. I demand the length of such a beam?

Answer $21\frac{3}{7}$ feet.

4. Two merchants A and B, go partners in a certain adventure; and such is each man's share, that if to the sum of them you add 160*l*. the whole will be double A's share; but if from the difference of their shares be taken away 60*l*. the remainder will be half of B's share. Required what each man advanced?

Answer A 360*l*. B 200*l*.

5. A gentleman bought a cloak of a salesman, which cost £.3-10, and desiring to know what the salesman gained by it, he answered $\frac{1}{4}$ of what it cost him: Pray what was that?

Answer 56*s*.

CLASS III.

1. A gentleman has an orchard of fruit trees, one half of the trees bearing apples, one fourth pears, one sixth plumbs, and 50 of them bearing cherries; how many fruit trees in all grow in the said garden?

Answer 600.

2. A goldsmith hath three silver cups with a superb cover of 18 ounces, and the second cup weighs half as much as the first, and third cups. Now if the cover be put upon the first cup, it weighs as much as all the three cups, and if joined to the second, its weight will be double the third added to the first. But if it be placed upon the third cup, it weighs twice as much as both the first and second: Required from this the weight of each cup?

Answer the 1*st*. 6, the 2*d*. 8. and the 3*d*. 10 ounces.

3. Three merchants put into stock each an unknown sum; but this is known; that what the first put in added to half what the other two put in, makes 17*l*. what the second put in added to the third part of what the two others put in, makes also 17*l*. and what the third put in added to the $\frac{1}{4}$ of the other two, makes also 17*l*. How much did each of them put in?

Answer the 1*st*. 5, the 2*d*. 11, and the 3*d*. 13*l*.

4. There are 2700*l*. to be divided amongst three persons A, B, and C, in such proportion, that if from twice the share of B you subtract $\frac{2}{3}$ of itself, the remainder will be equal to the share of A, and $\frac{2}{3}$ of $\frac{1}{4}$ of the share of A, be subtracted from the sum of A, and B's share, the remainder will be equal to C's share. Now I demand each person's share of the said gain?

Answer A's share 810*l*. B's £.607-10, and C's £.1282-10.

A demon.

A demonstration of the rule of double position.

In order to this we must first shew the truth of the other following rule, from which ours is drawn, and in which form it is delivered in several authors (*viz.*)

“As the difference of the errors, if alike, or sum, if unlike,
Is to the difference of the positions;

So is either error, to a fourth number; which added to, or subtracted from, the supposition it came from, just as it was too big, or too little, gives the answer.”

As the positions may be both too little, or both too big, or one too little and the other too big, the demonstration resolves itself into the three following cases.

C A S E 1.

When both the positions are too little.

In what is here delivered we will confine ourselves to example 1st. where 'tis said; “As much more money as a certain sum, half as much more, with 7*s.* 6*d.* will just make 100*s.* Quere this certain unknown sum?”

As quantities of any kind may be represented by lines, in *Fig. VIII.* let ac = the money required, $mg = 100*s.*$ given, and $mn = 7.5*s.*$ (or 7*s.* 6*d.*) fixt. Also let ab (33) and de (35) be the two positions, both short of the truth (or ac). Call bc , ef (what the two positions are short of the truth) *errors of the positions*, and let ig , hg be the common *errors of the results* respectively (found as per the rule, i. e. the result from the position ab (33) reaches from m to i , and that of de (35) from m to h). Farther as mn is a given fixt quantity, it may be conceived omitted, and to have no other effect in the question, than to lessen the line mg , of 100*s.* and to leave the question in this form; “As much more money as a certain sum, and half as much more makes $(100 - 7.5 =) 92.5$. (or ng): Quere the unknown sum of money.” This being conceived, 'tis evident, that whatever proportion the positions ab (23), de (35), and the errors of the positions bc (14), ef (2) bear to one another respectively, the same proportion must the results of the positions ni (82.5), nh (87.5) and the errors of the result ig (10), hg (5) bear also to one another; as the latter are still some *times, parts of a time, or times, &c.* of the former, according to the nature of the question, i. e. (instancing in the errors of the positions and results, as all is effected among them) 'tis as bc (4) = hf :

$kf : ef (2) :: ig (10) : hg (5)$. Again it must be, also, as $ih (5) : ke (2) :: hg (5) : ef (2)$ OR $gi (10) : bc (4)$. But $ih (15)$ is the difference of the errors of the results, $ke (2)$ the difference of the positions, $hg (5)$ and $ig (10)$ the two errors commonly so called, and $ef (2)$, $bc (4)$ what the two positions are short of the truth. Hence as the proportion thus gained, agrees with the rule above given, 'tis shewn to be right; and the reader may farther observe, that the reason of having two positions is to get the fixt line $ke (2)$, or their difference) in order to have something certain to refer to from the proportional part $hi (5)$, or the difference of the errors of the results) in a stating intended to discover the errors of the positions ef , bc , wanted, &c.

Thus we have made the reason of one case of a common rule for working double position, pretty evident to the arithmetician; and tho' what follows, is the sort of reasoning used by algebraists, yet 'tis so easy an instance, that there cannot be much difficulty in conceiving it also.

From the above demonstrated rule in the form of a proportion to gain ours.

Take the characters at the ends of the adjoining cross for the positions and errors, as they stand (y being bigger than x) and then work the above stating in symbols, saying as

$$m-n : y-x :: m \text{ (one of the errors)} : \frac{m \times y - x}{m-n} =$$

$$\frac{m \times y - m \times x}{m-y} = \text{(per the rule) what is to be ad-}$$

ded to the position x for the answer. Hence

$$x + \frac{m \times y - m \times x}{m-n} \text{ is the answer. Now the above}$$

expression is a mixt number (x being the whole number, and the other part the fraction) which if we reduce symbolically into

$$\text{an improper fraction will stand thus } \frac{x \times m - n + m \times y - m \times x}{m-n}$$

$$\frac{m \times x - n \times x + m \times y - m \times x}{m-n}; \text{ but in the numerator of this expression}$$

we find $m \times x$ pointed out by the signs to be both added and subtracted; hence these terms destroy one another, and all that remain

$$\text{are } \frac{m \times y - n \times x}{m-n}, \text{ which equivalent expression to the above for the}$$

answer, put into words as imported by the letters and signs, agrees exactly with the rule page 312. Hence 'tis shewn also to be right.

Tt

CASE

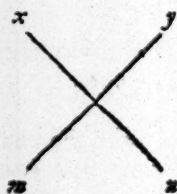
CASE 2.

When both positions are too big.

Let *ac* Fig. IX. = the money required $mg = 100s$. $mn = 7.5s$. *ab* (45) and *de* (53) be the two positions both too big. Then will *cb* (= *fk*) and *fe* be the errors of the positions, and *gi* (20) *gh* (40) the errors of the results, which lines, for reasons similar to the above, are respectively proportionable to one another. Hence it will be as *ih* (20) : *ke* (8) :: *gh* (40) : *fe* (16) OR :: *gi* (20) : *cb* (8). Therefore as the lineal portions referred to by the letters in the above proportion are denominated agreeable to the words in the rule, it is shewn to be right.

Ours is drawn from it just as before.

Thus (*y* being the bigger position) $n-m : y-x :: m : \frac{my-mx}{n-m}$
 = what is to be taken from *x*. Hence $x - \frac{my-mx}{n-m}$
 is the answer, which reduced to an improper fraction, as before, is $\frac{nx-mx-my+mx}{n-m} = \frac{nx-my}{n-m}$,
 an expression agreeing with the rule, &c.



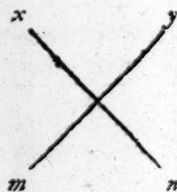
CASE 3.

When one position is too little and the other too big.

Let *ac* Fig. X. = the answer required, $mg = 100s$. $mn = 7.5s$. *ab* (23) be the first position, and too little, *de* (53) the other, and too big, *kf* (= *bc*) and *fe* the errors of the positions, and *gi* (10), *gh* (40) the errors of the result of a contrary kind. Then in like manner, as was shewn above, this proportion is easily derived, as *ih* (50) : *ke* (20) :: *gh* (40) : *fe* (16), and :: *gi* (10) : *bc* (4 = *kf*). [*fe* being here what the bigger position is too much and *bc* what the other is too little]. Hence as the terms of the above proportion agree with the rule its truth is manifest in this case, &c.

Ours is thus derived.

By the proportion (*x* being too little and *y* too big), as $m+n : y-x :: m : \frac{my-mx}{m+n}$. Hence $x + \frac{my-mx}{m+n} = \frac{mx+nx+my-mx}{m+n} = \frac{nx+my}{m+n}$ is an expression agreeing with our rule, &c.



Tho'

Tho' the above rule is demonstrated only for one example, and there are a great variety of forms of questions besides it, yet if the errors of the positions and results be proportionable, as above explained, this reasoning must extend itself equally to any other so circumstanced; but if not, it appears also that the rule cannot be applied. Now that the exceptions mentioned in the note, page 312, are of a sort which will not agree with this particular, every one a little acquainted with geometry will conceive; which hint is all the satisfaction, that can properly be here given with regard to the reasons for pointing them out.

Interest by Decimals.

Simple Interest.

HAVING already defined the terms, page 153, we shall enter immediately upon the usual problems, as follows.

Problem I.

"Principal, rate and time given to find the amount."

* Rule.

*"Find the interest of one pound for a year at the given rate, called
T t 2 * the*

* The reason is evident; for the interest of 1*l.* for 1 year (or the ratio of the rate) multiplied by the time given, must be the interest of one pound for that time, to which adding 1*l.* we have the amount of 1*l.* for the rate and time proposed; consequently, their sum multiplied by the principal mentioned in the question must give the amount required.

If the interest be only wanted, 'tis evident, that it may be found by multiplying the ratio of the rate by the time, and that product by the principal. Thus in the above example $4.5 \times .05 \times 254.5 = 75.2625 = \text{£}.57-5-3$ is the interest.

Tho' by the above rule the interest of any sum may be found for a proposed number of days, yet as they are something tedious to reduce to the decimal of a year in order to suit the rule, the following method is sometimes used in this case in its stead. In a little table, as below, is put down the interest of 1*l.* for one day at the usual rates; and then the tabular number agreeing with the proposed rate, the number of days, and principal multiplied together continually must be the interest sought.

R. per C.		R. per C.	
$2\frac{1}{2}$	.00006849	4	.00010959
3	.00008220	$4\frac{1}{2}$	.00012329
$3\frac{1}{2}$	.00009589	5	.00013696

Thus

* the ratio of the rate: *Then to the product of the ratio of the rate multiplied by the time, add unity; that sum multiplied by the principal gives the amount.*"

Examples.

1. What is the amount of £.254-10, for $4\frac{1}{2}$ years at 5 per cent per annum?

100*l.* : 5 :: 1 : .05 the ratio of the rate.

Then $4.5 \times .05 + 1 \times 254.5 = 311.7625 = \text{£}.311-15-3$ the answer.

2. What will £.280-10 amount to in 3 years and 148 days at 5 per cent per annum?

Answer £.328-5-2 $\frac{1}{4}$.

Problem II.

"The amount, rate, and time given to find the principal."

+ Rule.

"To the ratio of the rate multiplied by the time, add unity; divide the amount by the result, and the quotient is the principal."

Examples.

1. What principal put to interest will amount to £.311-15-3 in $4\frac{1}{2}$ years at 5 per cent per annum?

100*l.*

Thus, Required the interest of £.25-10 for 250 days at $4\frac{1}{2}$ per cent per annum? Interest for 1 day by the table is .00012329. Hence, $00012329 \times 250 \times 25.5 = .78591 = 15s. 8\frac{1}{2}d.$

The interests for 1 day are easily } thus $\left\{ \begin{array}{l} d. \quad \text{£}. \quad d. \\ 365 : .025 :: 1 : .00006849 \\ 965 : .03 :: 1 : .00008220 \end{array} \right\} \text{ \&c.}$
found from the ratios of the rates.

* The ratios of the rates may be easily known by inspection, from attending to a few instances: Thus 6, $5\frac{1}{2}$, 5, $4\frac{1}{2}$, 4, $3\frac{1}{2}$ &c. per cent, give .06, .055, .05, .045, .04, .035 &c. for corresponding ratios of rates.

† The reason is manifest from the last. For as per it, the amount is the product of the factor $4.5 \times .05 + 1$, into the principal, so 'tis plain that the given amount divided by $4.5 \times .05 + 1$, or one factor, must give the other or principal sought.---It will readily occur, that this problem is the same, as the rule of discount, page 161.

100*l.* : 5 :: 1 : .05 the ratio of the rate.

Then $\frac{311.7625}{4.5 \times .05 + 1} = 254.5 = \text{£.}254-10$ the answer.

2. What principal put to interest will amount to $\text{£.}367-13-5\frac{3}{4}$ in 7 years at $4\frac{1}{2}$ per cent per annum? Answer $\text{£.}279-12$.

Problem III.

"Principal, time, and amount given to find the rate."

* Rule.

"From the amount subtract the principal, the remainder divided by the product of the principal and time gives the ratio of the rate (from which the rate is easily found by inspection, or an obvious stating)."

Examples.

1. At what rate per cent will $\text{£.}254-10$, amount to $\text{£.}311-15-3$ in $4\frac{1}{2}$ years?

$\frac{311.7625 - 254.5}{254.5 \times 4.5} = .05$ the ratio of the rate. (Hence 1*l.* : .05 :: 100 : 5 is the rate required).

2. At what rate per cent will $\text{£.}327-17$ amount to $\text{£.}385-4-5\frac{1}{2}$ in 5 years? Answer $3\frac{1}{2}$.

Problem IV.

"Principal, amount, and rate given to find the time."

† Rule.

* Taking the principal from the amount leaves the interest, and as it is equal (per observation in the note problem I.) to the product of the ratio of the rate, time, and principal, multiplied continually (i. e. $254.5 \times 4.5 \times .05$) 'tis plain, that dividing the remainder just mentioned, or interest, by the product of two of the factors (i. e. 254.5×4.5) principal and time, must quote (.05) the ratio of the rate, the other factor or answer required.

* Rule.

"The difference of the amount and principal, divided by the product of the principal and rate, gives the time."

Examples.

1. In what time will £.254-10 amount to £.311-15-3 at 5 per cent per annum?

$$\frac{311.7625 - 254.5}{254.5 \times .05} = 4.5 \text{ years the answer.}$$

2. In what time will £.926-12 amount to £.1130-9-0 $\frac{1}{2}$ at 4 per cent per annum? Answer 5 $\frac{1}{2}$ years.

Problem V.

"To calculate the amount of an annuity in arrears as simple interest."

Definition.

"An annuity is a sum of money payable (upon some account) every year for a certain number of years, or for ever, and it is said to be in arrears, when the debtor keeps it in his hands, for a time agreed on, with intent to pay the whole at last with interest for every year, after it falls due: The total of the several years, with the interest due, is called the amount of the annuity forborn from that time."

† Rule.

"Multiply the sum of the natural series of numbers 1, 2, 3, &c. taken to the number of years less one, into one year's interest of the annuity (found as mentioned in note to problem I. by multiplying the ratio of the rate into the annuity) for the whole interest due upon the annuity, and to what results add the product of the annuity and time (or whole annuities) for the amount required."

Examples.

* The difference of the amount and principal being the interest, as mentioned in the last, and that equal to the product of the principal, rate, and time, 'tis plain, if that interest be divided by the product of the principal and rate, the quotient must be the time, which is what the rule directs.

† 'Tis pretty evident, that whatever be the time of forbearance, the first payment due will continue at interest one year less than that time; the second payment,

Examples.

1. If a salary of £.75-10 is forborn 6 years, at 5 per cent, what would it amount to?

(1.) $1+2+3+4+5=15$ the sum of the series.

(2.) $75.5 \times .5 = 3.775$ one years interest of the annuity.

Hence (3.) $3.775 \times 15 + 75.5 \times 6 = 509.625 = \text{£.}509-12-6$ the answer required.

2. If 250*l.* yearly pension be forborn 7 years, what will it amount to in that time at 4 per cent? Answer 1960*l.*

3. There is a house to be let upon a lease for 5 years at £.60-10 per annum; what would be the amount for the whole time at $4\frac{1}{2}$ per cent? Answer £.329-14-6.

Problem VI.

"Principal, rate, and time given to find the amount at compound interest."

What compound interest is has been already defined, page 159.

** Rule.*

*"Raise the amount of 1*l.* for one year at the rate given (called the ratio of the amount) to a power whose index is equal to the time (for the amount of 1*l.* for the time given) and multiply it by the given principal, for the amount required."*

Examples.

ment, two years less; the third three years, &c. till the last, upon which there is no interest due. Hence the annuity is at interest for the sum of these times, 1, 2, 3, &c. to the number of years, less one. Consequently the sum of a series thus circumstanced, multiplied by one years interest, must be the interest due from first to last, and this with the whole annuities or one yearly income, multiplied by the time of forbearance, must be the amount.

If the annuity be due half yearly or quarterly, 'tis evident that, if we look upon the number of years, or quarters, in the given time, as complete years, and take $\frac{1}{2}$ and $\frac{1}{4}$ of the above mentioned ratios of the rates, for new ratios of the rates respectively, these cases may also be answered by the rule just demonstrated.

* This will soon be pretty evident: For it must always hold, that as any principal is to its amount, so is any other principal to its amount, &c. Hence (in example 1st.) as $1P : 1.05a :: 1.05$ (the second years principal) : 1.05×1.05 , the second years amount. Again $1P : 1.05a :: 1.05 \times 1.05$ (the second years principal) : $1.05 \times 1.05 \times 1.05$ the third years amount, &c. Now as the first number
in

Examples.

1. What will £.36-10 amount to in 6 years at 5 per cent compound interest?

First the amount of 1*l.* for one year at 5 per cent is = 1.05 (i. e. 100*l.* : 105 :: 1 : 1.05). Hence $1.05^6 \times 36.5 = 1.34009 \times 36.5 = 48.913 = \text{£.}48-18-3$ the answer.

2. What

in these statings is still an unit, and the factors in each answer all alike, 'tis plain, that the amount of 1*l.* at the end of any time, is only a power of the first years amount of 1*l.* (or ratio of the amount) whose index is equal to the number of years: Consequently this multiplied by any number of pounds principal (as 36.5*l.*) must be the amount of it for the same time.

As the continual multiplying of the ratio of the amount by itself will, where the number of years is large, be very troublesome, it appears, that powers of these ratios of amounts, at different rates, carried on to a certain extent of years, and disposed in tables, would be very useful (if proved to be correct), as then "the tabular number against any year, multiplied by any given principal, would be its amount at once for that rate and time." This would be one of the many tables calculated for these affairs to be found in treatises of this subject, and the following specimen, carried on to 15 years, may not be without its use in this place, as the above problem is one pretty frequently wanted.---The rule for its use is judged too obvious to need illustration.

A table of the amount of 1*l.* at compound interest, for the years, and at the rates therein specified.

Years,	3 per cent,	3½ per cent,	4 per cent.	4½ per cent,	5 per cent,
1	1.03000	1.03500	1.04000	1.04500	1.05000
2	1.06090	1.07123	1.08160	1.09203	1.10250
3	1.09273	1.10872	1.12486	1.14117	1.15763
4	1.12551	1.14752	1.16986	1.19252	1.21550
5	1.15927	1.18769	1.21665	1.24618	1.27628
6	1.19405	1.22926	1.26532	1.30226	1.34010
7	1.22987	1.27228	1.31593	1.36086	1.40710
8	1.26677	1.31681	1.36857	1.42210	1.47746
9	1.30477	1.36290	1.42331	1.48610	1.55133
10	1.34392	1.41060	1.48024	1.55297	1.62899
11	1.38423	1.45997	1.53945	1.62285	1.71034
12	1.42576	1.51107	1.60103	1.69588	1.79586
13	1.46853	1.56396	1.66507	1.77220	1.88565
14	1.51259	1.61870	1.73168	1.85195	1.97993
15	1.55797	1.67535	1.80094	1.93528	2.07893

2. What will £.210-7-6 amount to in 3 years, at 5 per cent per annum?
Answer £.243-10-8½.

3. What will 450*l.* amount to in 5 years at 4 per cent per annum?
Answer £.547-9-10½.

Scholium.

The four first problems solve all questions that relate to simple interest, but the two last are selected out of a great number, that might be proposed relating to *annuities, leases in reversion, purchasing of freehold estates, &c.* For the subject of interest (taking in the speculative part as well as the useful) is very extensive, and many of its enquiries are exceeding difficult to answer by pure arithmetic. Therefore having just shewn the use of decimals in a few serviceable and common problems, we shall take leave of the subject, by recommending those who may find it necessary to be farther acquainted with this branch of arithmetic to those treatises professedly intended to illustrate it, observing that in them (besides the theory and other superior methods of calculation, as by logarithms, &c.) are commonly delivered tables with their use, so contrived as to render the business very easy and expeditious. And as there is always a hazard of mistakes being made in any table containing a large number of figures (as several of these do) it farther behoves the practitioner to make himself so far acquainted with the theory, and method of constructing them, as to be able of himself to examine their truth, &c. before he relies on them in calculation. And as all this would take up too much room in a treatise of this kind, and the particulars omitted are not frequently wanted, we conclude that what is laid down above is as much, as could be consistently here expected.

The Nature and Algorithm

OF

RATIOS.

IN the mathematics, when applied to philosophical problems, or to the discovery of certain properties of nature, &c. its enquiries are generally conducted by, and end in, expressions, which indicate a particular immutable relation (or *ratio* as 'tis mostly termed) of some one thing to another concerned in the process. And as such attainments as these, are perhaps the most useful, and striking ends of this kind of learning, the following sketch of what relates to a

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practical

practical management of the above intimated expressions, &c. must be well worth the perusal of the student, before he enters upon more learned discussions of things of this nature.

Notation, &c. of Ratios.

Definitions.

“**R**ATIO is the mutual relation two numbers (or other homogeneous quantities) are in with respect to magnitude.

Antecedent (as has been already taught) is the number compared in the ratio, and that to which it is compared is called the consequent; and this comparison is intimated by two points (:). Thus the ratio of 2 to 5 is wrote $2 : 5$, the ratio of 7 to 3 is wrote $7 : 3$, &c. 2 and 7 being antecedents, and 5 and 3 consequents.

* Again, as ratio relates to the magnitude of the numbers compared, or what part of a time, or how often the antecedent contains the consequent, this relation may always be discovered by dividing the antecedent by the consequent. Thus $2 : 5$ may be wrote $\frac{2}{5}$, and $7 : 3$, $\frac{7}{3}$, &c. which method of expressing ratios is followed by many authors; and to make the whole as intelligible as possible, we have carried on both notations together in most places of what follows.

Farther, when ratios are applied, as the two first terms of a proportion, in order to alter another given number in the same ratio (or to find a fourth proportional) they may be considered of two kinds, as having two different effects, i. e. ratios of increase and ratios of decrease. And it appears from this distinction, that when the antecedent is greater than the consequent, a ratio of decrease is implied; but when it is less, there is intimated a ratio of increase; and it will be found hereafter in their composition,

* We might have taken the quotient of the consequent by the antecedent for the ratio (as in G. P. page 296), it being indifferent whether way be used, as to the ends proposed, on a right consideration. Thus $2 : 5$, $7 : 3$ &c. might be wrote $\frac{5}{2}$, $\frac{3}{7}$; and then as the above method denotes a ratio of the antecedents containing the consequent, so in this, 'tis to be understood a ratio of the antecedents being contained in the consequent.

To this observation we shall add, that the fractional expressions arising from the above two methods are often called *reciprocals* of one another, by which is understood, that the value of the one is as much below unity as the other is above it. Thus $\frac{2}{5}$ and $\frac{5}{2}$ are reciprocal fractions; for per the rule of three, we have $\frac{2}{5} : 1 :: 1 : \frac{5}{2}$, &c. Hence the terms of any fraction ($\frac{a}{b}$) inverted gives ($\frac{b}{a}$) its reciprocal.

position, that these two kinds will counteract one another, like the debtor and creditor sides of a shop-book.

* From what has been said of ratios it appears, that they may be compared one to another, as to magnitude, and the standard is said to be this; "That ratio is greater, equal to, and less than another, whose antecedent hath a greater, an equal, or a less proportion to its consequent, than the other antecedent hath to its consequent." Thus $7 : 5$ is a greater ratio than $7 : 6$, because 7 in comparison of 5 has more magnitude (or hath a greater proportion to 5) than in comparison with 6; and $5 : 7$ is a less ratio than $5 : 6$, as 5 compared with 7 has less magnitude, than when compared with 6.---When the antecedents (or consequents) thus compared are not alike, they may be one easily reduced to the value of the other by the rule of three, which is too obvious to need illustration.

Now with regard to the fractional method of expressing ratios, it will be found, that one ratio is greater, equal to, or less than another, according as the fraction expressing the first is greater, equal to, or less than the fraction expressing the latter. † Thus $8 : 6$ is greater than $9 : 7$ because $\frac{8}{6}$ as a fraction is greater than $\frac{9}{7}$; the first being (when both are reduced to a C. D.) $=\frac{4}{3}$, and the other $\frac{9}{7}$, &c."

Theorem I.

"In a series of quantities of any kind whatever encreasing, or decreasing from first to last, the ratio of the extremes is compounded of the ratio of all the intermediate terms. Thus in 16. 12. 10. 3.--- $16 : 3$ (the extremes) is compounded of the ratios of $16 : 12$, of $12 : 10$, and of $10 : 3$."

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Demonstration.

* Perhaps the following method of discovering the difference of the magnitudes of two ratios may be as easy to remember, as any: "Of two ratios, that is the greater, which would produce a less fourth proportional than the other to a proposed number in a stating." Thus $6 : 4$, $5 : 4$, $4 : 4$, $3 : 4$, and $2 : 4$ are ratios bigger than one another in the order they stand, because they give each in the same order a less fourth proportional to the number 12 in a stating, than that succeeding it.

$$\text{i. e. } \left\{ \begin{array}{l} 6 : 4 :: 12 : 8 \\ 5 : 4 :: 12 : 9\frac{3}{5} \\ 4 : 4 :: 12 : 12 \\ 3 : 4 :: 12 : 16 \\ 2 : 4 :: 12 : 24 \end{array} \right\} \&c.$$

† Tho' this method may serve very well to compare two ratios, as to greater and less, yet it must not be inferred, that the precise value of the excess of one ratio above the other is pointed out by the difference of the fractions; as it will be hereafter shewn that the algorithm of these relative quantities is very different from that of vulgar fractions.

Demonstration.

The truth of this will readily appear by actually trying it. Thus to reduce 16 to a lower number in the ratio of 16 : 12, the first ratio in the series, we must say, as 16 : 12 :: 16 : 12; again to reduce this 12 farther down in the proportion of 12 : 10, we say, as 12 : 10 :: 12 : 10; again to reduce this last result 10 still lower in proportion to the next, or last ratio, 10 : 3, we say as 10 : 3 :: 10 : 3. Now as the two ratios, contained in every proportion, are the very same, and nothing is done in all this, but a bare running through the ratios, as first mentioned, what is said above is manifest in this instance, nor could the reasoning used here fail in any other.

Theorem II.

"In a series of G. P. as 1. 2. 4. 8. 16. 32, &c. the ratio of the third term 4 to the first one (i. e. 4 : 1) is double, or in a duplicate ratio of the second term 2 to the first (i. e. 2 : 1). Also the ratio of 8 : 1 is treble, or in a triplicate ratio of 2 : 1, &c. Likewise, on the contrary, the ratio of 2 : 1 is but one half, or in a subduplicate ratio, of 4 : 1, and one third, or in a subtriplicate ratio, of 8 : 1, &c. agreeably to the relative distances these antecedents (2, 4, 8, &c.) are from the first term of the series."

Demonstration.

Take the first term 1, and increase it by the ratio 1 : 2, and we have 1 : 2 :: 1 : 2. Again encrease this result 2 in the same ratio 1 : 2, and we have 1 : 2 :: 2 : 4. Hence 4 : 1 is double 2 : 1. Again encreasing 4 in the same ratio 1 : 2, we have 1 : 2 :: 4 : 8, which is treble the ratio of 1 : 2. Hence as the numbers gained by this reasoning (which is equally applicable to any other series) agree with the terms pointed out in the progression, by the first part of the theorem, it is so far true, and if this be true the converse must be true; hence the whole is true.

These things premised, we shall next proceed to the algorithm.

Problem

Problem I.

“Addition of Ratios.”

* Rule.

“*M*ULTIPLY all the antecedents together for an antecedent, and all the consequents for a consequent, of the ratio of the sum required.”

Examples.

1. Add $2 : 3$ and $2 : 1$ together?--- $2 \times 2 = 4$, and $3 \times 1 = 3$, hence $4 : 3$ is the answer. OR $\frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$ is the answer.

2. Add $3 : 2$, $5 : 8$, and $7 : 11$ together?--- $3 \times 5 \times 7 = 105$, and $2 \times 8 \times 11 = 176$, hence $105 : 176$ is the answer. OR $\frac{3}{2} \times \frac{5}{8} \times \frac{7}{11} = \frac{105}{176}$ is the answer.

3. Add $2 : 3$, $3 : 2$, and $4 : 1$ together?--- $2 \times 3 \times 4 = 24$, and $3 \times 2 \times 1 = 6$, hence $24 : 6 = 4 : 1$ is the answer (the same as the last ratio given, $2 : 3$ and $3 : 2$ destroying one another). OR $\frac{2}{3} \times \frac{3}{2} \times \frac{4}{1} = \frac{24}{6}$ is the answer.

Problem

* In the first example, if the ratio had been continued (as 'tis called, i. e. the consequent 3 of the first ratio, the same as the antecedent 2 of the second) the sum would have been, per theorem I. the ratio of the antecedent of the first ratio to the consequent of the second. But as they are not, we must, by proportion make them so, and then strike out the intermediate terms for the sum. Thus if 2 becomes 3, what will 1 become, i. e. $\frac{2}{3}$; therefore we may make the ratio continued thus 2. 3. $\frac{3}{2}$, (i. e. $2 : 3$ and $3 : \frac{3}{2}$) and the sum of them $2 : \frac{3}{2}$ is consequently the sum of $2 : 3$, and $2 : 1$. But $2 : \frac{3}{2} = 4 : 3$ (from multiplying both terms by 2) which ratio is the same, as that given by the rule. By this method any number of ratios may be reduced to a continued series, and added in proof of the rule, if a farther proof be thought needful. OR, we may illustrate the business thus: In a continued series, 16. 12. 10. 3, if we take it to pieces, and represent the ratios at every step in this form $\frac{16}{12}$, $\frac{12}{10}$, $\frac{10}{3}$, and then sum them by the rule we have $\frac{16}{12} \times \frac{12}{10} \times \frac{10}{3}$ for the answer; but as numbers equal to the intermediate terms (12 and 10) fall in both the numerators and denominators, the same number of times, they will have no effect in the multiplication; hence $\frac{16}{3} = 16 : 3$ is the sum, which agreeing with what was said in theorem I. shews the rule to be true in another light.---Addition of ratios is often called *composition*.

Problem II.

“Subtraction of Ratios.”

* Rule.

“**I**NVERT the terms of the ratio to be subtracted, and then the product of the antecedents, and consequents give an antecedent and consequent of the difference required.”

Examples.

1. From $3 : 2$ take $4 : 5$? --- $3 \times 5 = 15$, and $2 \times 4 = 8$, hence $15 : 8$ is the answer. OR $\frac{3}{2} \times \frac{5}{4} = \frac{15}{8}$ is the answer.

2. From $8 : 1$ take $6 : 7$? --- $8 \times 7 = 56$, and $1 \times 6 = 6$, hence $56 : 6 = 28 : 3$ is the answer. OR $\frac{8}{1} \times \frac{7}{6} = \frac{56}{6} = \frac{28}{3}$ is the answer.

3. From $5 : 6$ take $6 : 5$? --- $5 \times 5 = 25$, and $6 \times 6 = 36$, hence $25 : 36$ is the answer. OR $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$ is the answer.

Problem III.

“Multiplication of Ratios.”

† Rule.

“**R**AISE both terms of the ratio to a power, whose index is equal to the units in the multiplier for the ratio of the product.”

Examples.

* If multiplication add ratios, division must of consequence subtract them; but the rule for dividing fractions (and the truth of representing ratios by fractions must by this time be very evident) is to invert the terms of the divisor, and then multiply it and the dividend together for the quotient. Hence $3 : 2 - 4 : 5$ (in the first example) is the same as $\frac{3}{2} \div \frac{4}{5} =$ per above $\frac{3}{2} \times \frac{5}{4} = \frac{15}{8}$, as before. And for farther satisfaction this sum $\frac{15}{8}$ added to the subtractor $\frac{4}{5}$ gives $\frac{3}{2}$ the subtrahend (i. e. $\frac{15}{8} \times \frac{4}{5} = \frac{60}{40} = \frac{3}{2}$). We may observe also, that as multiplying antecedents and consequents together gives the sum of two ratios, so the antecedent and consequent of the subtrahend, divided by the antecedent, and consequent of the subtractor respectively, must give their difference; hence we have this expression for the difference of the above ratios, i. e. $\frac{3}{4} : \frac{2}{5}$, which reduced to a C. D. is $\frac{15}{20} : \frac{8}{20} = 15 : 8$ the same as before, and this latter way is often used.

† This is evident, for to multiply any thing by 2, 3, 4, &c. is to add it so many times

Examples.

1. Double the ratio of $2 : 3$? ---- $2 \times 2 = 4$, and $3 \times 3 = 9$, hence $4 : 9$ is the answer. OR $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ is the answer.

2. Treble the ratio of $5 : 4$? ---- $5 \times 5 \times 5 = 125$, and $4 \times 4 \times 4 = 64$, hence $125 : 64$ is the answer. OR $\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = \frac{125}{64}$ is the answer.

3. Required a ratio five times as big as $3 : 2$? ---- $2 \times 2 \times 2 \times 2 \times 2 = 32$, and $3 \times 3 \times 3 \times 3 \times 3 = 243$, hence $243 : 32$ is the answer. OR $\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{243}{32}$ is the answer.

*Problem IV.**“ Division of Ratios.”*** Rule.*

EXTRACT a root of the two terms, whose index is equal to the units in the given divisor, for the terms of the ratio of the quotient.”

Examples.

1. Required a ratio the one half of $9 : 4$? ---- $\sqrt{9} = 3$, and $\sqrt{4} = 2$, hence $3 : 2$ is the answer. OR $\sqrt{\frac{9}{4}} = \frac{3}{2}$ is the answer.

2. Required a ratio one third of $125 : 64$? ---- $\sqrt[3]{125} = 5$, and $\sqrt[3]{64} = 4$, hence $5 : 4$ is the answer. OR $\sqrt[3]{\frac{125}{64}} = \frac{5}{4}$ is the answer.

3. Required a ratio equal to $\frac{1}{6}$ th. of $729 : 64$? ---- $\sqrt[6]{729} = 3$, and $\sqrt[6]{64} = 2$, hence $3 : 2$ is the answer. OR $\sqrt[6]{\frac{729}{64}} = \frac{3}{2}$ is the answer.

Scholium.

times to itself as the multiplying digits imply; but ratios are added by the multiplication of the antecedents, and consequents; hence, as they are still the same, in this case their sum must be a power of both the terms, whose index is equal to the number of times they are supposed to be taken.

* This is evident, being the reverse of multiplication, &c. And we may observe, that as the roots of the greatest part of numbers cannot be taken exactly, it will be very seldom we can give in rational numbers expressions for a required part of a ratio.

Scholium.

Having now got multiplication and division, we may alter any ratio in any proportion, or take any fractional part of it. Thus suppose we want to alter the ratio of $27 : 8$ in the * proportion of $3 : 2$, or to take $\frac{2}{3}$ ds of it, the terms of the stating would be $3 \text{ — } 2 \text{ — } 27 : 8$. Now $27 : 8$ multiplied by 2 is $27^2 : 8^2 = 729 : 64$, which divided by 3 is $\sqrt[3]{729} : \sqrt[3]{64} = 9 : 4$ the answer. Again $\frac{3}{4}$ of $81 : 16$ is thus found $4 \text{ — } 3 \text{ — } 81 : 16 = \sqrt[4]{81^3} : \sqrt[4]{16^3} = 27 : 8$ the answer, &c.

† *Of the Composition and Resolution*

O F

R A T I O S.

WE have observed before, that what is termed the composition of ratios is effected by their addition: And on the contrary, we are now chiefly to shew, that the resolution (as 'tis called) or the decompounding of ratios depends upon the application of subtraction. For as it is required in a resolution, that the parts or members of it, if compounded again, should make up exactly the ratio given to be resolved, 'tis very evident, that if we take a ratio (or the sum of any number of ratios) arbitrarily assigned, (or any other way determined upon) out of the given one, what remains will, with the other, or others, used in the process, be strictly the parts of the resolution required. Thus $8 : 3$ may be decompounded into $7 : 5$, and $40 : 21$, by assuming $7 : 5$ arbitrarily and finding $40 : 21$ by subtraction. Also $9 : 5$ may be resolved into $4 : 3$, $5 : 4$, and $27 : 25$, in which $4 : 3$, $5 : 4$ are taken arbitrarily and $27 : 25$, found by taking their sum from $9 : 5$, &c. Farther as ratios of encrease, and ratios of decrease act contrary to one another, one, or more of the ratios pitched upon in the resolution of that given, may even have more magnitude, than that

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* The reader will perhaps have observed before now, that the term *proportion* is sometimes used as equivalent to *ratio*, but as this little deviation from the strict sense given of it, page 300, cannot be attended with any bad consequences (what it refers to still indicating its meaning) it is generally given into in treating of these things, for the sake of variety in the expression, &c.

† Several instances, where the more useful of these two particulars, or the composition of ratios, is applied, may be seen in the rule following, called *Plural proportion*, which was in a great measure omitted till now, for the sake of the mutual light these two subjects may chance to throw upon one another.

to be resolved; as the rule will always provide a yoke-fellow of an opposite nature to qualify, or bring the other, or others (in composition) down to a proper magnitude. Thus $5 : 3$ may be resolved, as above, into $9 : 2$ (a bigger ratio than $5 : 3$) and $10 : 27$, &c.

Yet it may be added to this business of resolution, that if any set of terms be wrote in the manner of a series between the terms of a given ratio, it will be compounded into all the intermediate ratios (per theorem I.) Thus, if between $10 : 3$, 9 , 8 , 7 , 5 , 4 be wrote, $10 : 3$ is resolved into $10 : 9$, $9 : 8$, $8 : 7$, $7 : 5$, $5 : 4$, and $4 : 3$, &c."

Common Properties

O F

PROPORTIONALITY.

TOWARDS the better understanding of the very useful and extensive rule of proportion, it will be worth the reader's while to enlarge his notions of it by attending to the following particulars.

Definition.

"Proportion (as has been already observed) is a similitude of ratios; or according to Euclid, four numbers are said to be proportionable, when the first is the same multiple part or parts of the second, that the third is of the fourth. Now as we can easily know what multiple part or parts one number is of another, by placing them fractionwise (i. e. finding their quotient by division) it follows, that of four numbers if a fraction, whose numerator is the first number, and denominator the second, is equal to another whose numerator is the third number, and denominator the fourth, these four numbers are proportionable. Thus $3 : 4 :: 9 : 12$ are proportionable, because $\frac{3}{4} = \frac{9}{12}$, and besides this consideration, these expressions may be looked upon as ratios, and then the terms are proportionable by the first part of the definition.

We have already demonstrated (page 299) that when four numbers are proportionable, the product of the mean terms is equal to the product of the extremes; therefore this property may also be looked upon as another certain mark of proportion."

These things premised, the common properties of proportionality are as follows.

Xx

1. If

1. If four numbers be proportionable, as $2 : 4 :: 3 : 6$, then will the second have the same proportion to the first, that the fourth hath to the third, i. e. $4 : 2 :: 6 : 3$, because the products of the extremes and means are equal, i. e. $2 \times 6 = 4 \times 3$, and this is called taking the terms *inversely.

2. If four numbers are proportionable, as $2 : 4 :: 3 : 6$, then will the first be to the third, as the second is to the fourth, i. e. $2 : 3 :: 4 : 6$, because the products of the extremes and means are equal, i. e. $2 \times 6 = 3 \times 4$, and this is called taking the terms alternately.

3. If four numbers are proportionable, as $2 : 4 :: 3 : 6$, then will the first be to the sum of the first and second, as the third is to the sum of the third and fourth, i. e. $2 : 2+4 :: 3 : 3+6$, because the products of the extremes and means are equal, i. e. $2 \times 3+6 = 2+4 \times 3$, and this is called compounding the terms.

4. If four numbers are proportionable, as $2 : 4 :: 3 : 6$, then will the first be to the difference between the first and second, as the third is to the difference between the third and fourth, i. e. $2 : 4-2 :: 3 : 6-3$, because the products of the extremes and means are equal, i. e. $2 \times 6-3 = 4-2 \times 3$, and this is called taking the terms dividedly.

5. If four numbers are proportionable, as $2 : 4 :: 3 : 6$, then will the sum of the first and second, be to the difference between the first and second, as the sum of the third and fourth, is to the difference between the third and fourth, i. e. $2+4 : 4-2 :: 3+6 : 6-3$, because the products of the extremes and means are equal, i. e. $2+4 \times 6-3 = 4-2 \times 3+2$, and this is called taking the terms mixtly.

6. If four numbers are proportionable, as $2 : 4 :: 3 : 6$, then if the two first terms (or two last terms) be multiplied by any common number (5) the products will still be proportionable to the other terms, i. e. $2 \times 5 : 4 \times 5 :: 3 : 6$, because the products of the extremes and means are equal, i. e. $2 \times 5 \times 6 = 4 \times 5 \times 3$, and this is called multiplying the terms.

Corollary. Hence if all the terms were multiplied by the same number, the products would be proportionable, and also if the two first and two last were multiplied by different numbers, the products would be proportionable.

7. If four numbers are proportionable, as $2 : 4 :: 3 : 6$, then if the two first terms (or two last terms) be divided by any common number (2) the quotients will be proportionable to the other terms, i. e. $\frac{2}{2} : \frac{4}{2} :: 3 : 6$, because the product of the extremes and means are equal, i. e. $\frac{2}{2} \times 6 = 3 \times \frac{4}{2}$, and this is called dividing the terms.

Corol-

* It should be observed that *inverse* here means only a taking of the terms of the ratios backwards, and not what the same word implies in the article reciprocal proportion, page 94.

Corollary. Hence if all the terms were divided by the same number, the quotients would still be proportionable, and if the two first and two last, were divided by different numbers, the quotients would be proportionable.

Several other properties might be added, but these may be sufficient for the learner in this place. And as there is a regularity in ordering of the terms in each article, which may perhaps be better delivered by the eye to the memory than by the judgement, we have expressed them for that purpose in symbols, in the table below, and taken, in one part of it, the first letters of the alphabet for four terms in general, as they will perhaps help to make still stronger impressions on the imagination, than particular digits."

If four quantities are proportionable. as $3 : 9 :: 8 : 24$, or $a : b :: c : d$. Then

1. inversely.	$9 : 3 :: 24 : 8$	}	{	$b : a :: d : c$
2. alternately.	$3 : 8 :: 9 : 24$			$a : c :: b : d$
3. compoundedly	$3 : 3 + 9 :: 8 : 8 + 24$			$a : a + b :: c : c + d$
4. dividedly.	$3 : 9 - 3 :: 8 : 24 - 8$			$a : b - a :: c : d - c$
5. mixtly.	$3 + 9 : 9 - 3 :: 8 + 24 : 24 - 8$			$b + a : b - a :: d + c : d - c$
6. by multip.	$5 \times 3 : 5 \times 9 :: 8 : 24$			$r \times a : r \times b :: c : d$
7. by division.	$\frac{3}{5} : \frac{9}{5} :: 8 : 24$			$\frac{a}{r} : \frac{b}{r} :: c : d$

Here $r = 5$ or any other number.

Plural Proportion.

Definition.

"THE questions here answered are of the nature of those in the golden rule, only that five, seven, nine or any odd number of terms are given, in order to find another of a denomination agreeing with some one of those proposed."

* Rule.

"Having found, from a perusal of the question, a distinction of the terms into those of supposition, and demand (which is very easy), put down the odd one, which is of the same kind with the answer. Next on the right hand of it place in a perpendicular column (in any order) all the terms of demand, and on the left of it in like manner all the terms of supposition, observing that the latter class correspond, as to denomination, ex-

X x 2

actly

* For the reason of the rule, see the latter end of the article.

ally with the former. Then beginning with the highest, or any other pair of corresponding terms, imagine them, with the above named middle term wrote betwixt them, a stating in the single rule of three, and consider, whether according to its nature (in that light) the answer ought to be greater, or less, than the middle term; if greater, mark the greater of the extremes thus $\times$, but if less, mark the less of the extremes; and in this manner proceed with every pair of the corresponding terms. This done, multiply continually the middle term, and all the terms marked with this sign $\times$ for a dividend, and all the other terms for a divisor, and the quotient thence arising will be the answer."

This method is general, and answers any question, that can be proposed of the nature of those that follow.---It is almost needless to observe, that when any pair of corresponding terms contain odd denominations, they must be both reduced to the lowest mentioned in either of them, and that the method of striking out similar factors in the divisor and dividend (as in page 186) may be used here to very good purpose.

Examples.

1. If 12 men build a wall 30 feet long, 6 feet high, and 3 feet thick in 15 days: In how many days will 60 men raise a wall 300 feet long, 8 feet high, and 6 feet thick?

Terms of supposit.	{	Men	12 $\times$	15 days	$\times$	60 men	{	Terms of demand.
		Feet long	30			300 feet long		
		Feet high	6			8 feet high		
		Feet thick	3			6 feet thick		

$$\text{Then } \frac{15 \times 12 \times 300 \times 8 \times 6}{60 \times 30 \times 6 \times 3} = \frac{5 \times 10 \times 8}{5} = 80 \text{ days the answer.}$$

2. If 100*l.* yield 5*l.* interest in one year, how much interest will 850*l.* yield in 3 years and 8 months?

Principal	100 <i>l.</i>	5 <i>l.</i> interest	$\times$ 850 <i>l.</i> principal.
One year in thirds	$= 3$		$\times 11$ thirds in $3\frac{2}{3}$ years.

$$\text{Then } \frac{5 \times 850 \times 11}{100 \times 3} = \text{£. } 155-16-8.$$

3. If 12 pennyworth of wine will satisfy 8 persons at a meal, when wine is at 6*d.* per quart, how many persons will 20 pennyworth of wine satisfy, when wine is sold at 4*d.* per quart?

Pennyworth

Pennyworth 12 8 men $\times 20$ pennyworth
 Pence per quart $6 \times$ 4 pence per quart

$$\frac{6 \times 8 \times 20}{12 \times 4} = 20 \text{ men the answer.}$$

4. If when a bushel of wheat costs 3s. 4d. the penny loaf weighs 12 ounces, I demand the weight of a loaf worth 9 pence, when the bushel costs 10 shillings?

Price 3s. 4d. $\overset{d.}{=} 40 \times$ 12oz. $\overset{d.}{100} = 10$ shillings.
 Loaf value $= 1$ $\times 9 =$ loaf value.

$$\frac{40 \times 12 \times 9}{1 \times 120} = 36 \text{ ounces the answer.}$$

5. If when the tun of wine is worth 30l. 20l. worth will serve a ship's company of 336 men for four days, at a pint to each per day: How long will 500l. worth serve a crew of 250 men at $1\frac{1}{2}$ pint to each man a day, when the tun is worth 24l?

£ per tun 30 $\times$ 24 £ per tun
 £ value drank 20 $\times 500$ £ value drank
 No. of men 336 $\times$ 4d. 250 No. of men
 Half pints per day 2 $\times$ 3 half pints $= 1\frac{1}{2}$

$$\frac{30 \times 336 \times 2 \times 4 \times 500}{20 \times 24 \times 250 \times 3} = 112 \text{ days the answer.}$$

6. If 15 men eat 13s. worth of bread in 6 days, when wheat is at 12s. per boll, how many days will 30 men take, at that rate, to eat 43s. 4d. worth of bread, when wheat is at 10s. per boll?

Men - - - 15 $\times$ 30 men
 Value eat 13s $= 156d.$ 6 days $\times 520d. = 43s. 4d.$ value eat,
 Price per boll 12s. $\times$ 10s. price per boll

$$\frac{15 \times 12 \times 6 \times 520}{156 \times 30 \times 10} = 12 \text{ days the answer.}$$

7. * If 54 workmen can build a fort in 18 days, when the day is 17 hours long; in how many days will 68 workmen build the same, when the day is but 9 hours long?

Men

* This question, having five terms given to find a sixth, is one of those; that fall under what is called *the double rule of three* in many authors; but as the method of solution there used is founded on a supposition, that there can be no more, than one inverse proportion in such questions (and this plainly contains two) it appears that the rule hinted at is not so general, as might be wished.

Men $54 \times$ 18 days 68 men
Hours $17 \times$ 9 hours

$$\frac{54 \times 17 \times 18}{9 \times 68} = 27 \text{ days the answer.}$$

8. If 248 men in $5\frac{1}{2}$ days dig a trench of $23\frac{1}{4}$ yards long, $2\frac{1}{3}$ deep, and $3\frac{2}{3}$ wide: What length of trench of $3\frac{1}{2}$ yards deep, and $5\frac{2}{3}$ wide, can be dug by 24 men in 198 days?

Men 248 $\times 24$
Days $5\frac{1}{2}$ $\times 198$
Depth $2\frac{1}{3} \times$ $23\frac{1}{4}$ length $3\frac{1}{2}$
Width $3\frac{2}{3} \times$ $5\frac{2}{3}$

Then $\frac{3\frac{2}{3} \times 2\frac{1}{3} \times 23\frac{1}{4} \times 24 \times 198}{5\frac{1}{2} \times 3\frac{1}{2} \times 5 \times \frac{1}{2} 248} =$ (per reducing mixt numbers to improper fractions, &c.) $\frac{\frac{11}{3} \times \frac{5}{2} \times \frac{93}{4} \times 24 \times 198}{\frac{11}{2} \times \frac{7}{2} \times \frac{11}{2} \times \frac{1}{2} \times 248} =$ (per placing the factors together that would join in an actual division of the higher set of terms by the lower) $\frac{5 \times 11 \times 2 \times 7 \times 2 \times 93 \times 24 \times 198}{3 \times 28 \times 3 \times 7 \times 4 \times 11 \times 248} =$ (when reduced) $35\frac{3}{4}$ yards in length the answer.

Scholium.

We must take notice to the reader, before we conclude this article, that any question in plural proportion may be answered by two or more statings in the single rule of three, which method (tho' something more tedious) he may use for farther certainty, if he should be at a loss about the truth of any solution by the general rule. Thus the last question, answered by single proportions, will stand as below, in which process, it may be observed, the middle number of every stating is of the same name as that required, and (after the first) 'tis also the answer of the preceding stating. viz.

len.
Men 248 : $23\frac{1}{4}$:: 24 : $2\frac{3}{4}$ Direct.
Days $5\frac{1}{2}$: $2\frac{3}{4}$:: 198 : 81 Direct.
Depth $2\frac{1}{3}$: 81 :: $3\frac{1}{2}$: 54 Inverse.
Width $3\frac{2}{3}$: 54 :: $5\frac{2}{3}$: $35\frac{3}{4}$ Inverse.

Hence $35\frac{3}{4}$ is the answer required.

As to the order of taking the pairs of terms of different denominations in the statings, the reader will find, by trying an example or two,

two, that 'tis arbitrary, as also is the method of keeping the middle number of the same name, with that of the answer, though it is perhaps as easy as any other.

Practical Questions.

CLASS I.

1. If 8 men deserve 2*l.* wages for 5 days work, how much will 32 men deserve for 24 days, at the same rate? Answer £. 38-8.
2. Suppose the weight of a beam whose length is 4 feet, breadth 3 feet, and height 2 feet, be 240*lb.* what will the weight of another beam be, whose length is 10 feet, breadth 4 feet, and height one foot? Answer 400*lb.*
3. If the carriage of 60 hundred weight, 20 miles, cost £. 14-10, what will 15*C.* weight cost carrying 30 miles, after that rate of carriage? Answer £. 5-8-9.
4. If 10 cannon in one day spend 40 barrels of powder, how many barrels will 24 cannon spend in 30 days, at that rate? Answer 2880.
5. If 15 ells of stuff 3 quarters wide cost 37*s.* 6*d.* what will 40 ells of the same stuff cost, being yard wide? Answer £. 6-13-4.
6. If 40 acres of grass be mowed by 8 men in 7 days, how many acres can be mowed by 24 men in 28 days? Answer 480.

CLASS II.

1. How many yards of bays 3 quarters wide will suffice to line 1000 soldiers coats, each containing $2\frac{1}{2}$ yards of cloth, of 5 quarters wide? Answer 4166 $\frac{2}{3}$.
2. If 10 bushels of oats be enough for 18 horses, 20 days; how many bushels will serve 60 horses 36 days? Answer 60.
3. An usurer put out 86*l.* to receive interest for the same, and when it had continued 8 months, he received for principal, and interest £. 88-17-4. I demand at what rate per cent per annum he received interest? Answer 5 per cent.
4. Suppose 2000 men, besieged in a town, with provision for 3 weeks, allowing each man 20 ounces per day, were to be reinforced with 2000 men more, and were resolved to make the aforesaid provision last them 6 weeks; how many ounces must each man have per day? Answer 5 ounces.

CLASS III.

1. If 16 men can build a wall 50 feet long, 7 feet high, and 3 feet thick in 18 days, when the day is 12 hours long, in how many days will 48 men build a wall 2000 feet long, 9 feet high, and 4 thick, when they work 8 hours per day? Answer 61 $\frac{1}{2}$ days.
2. A

2. A draper bought 27 pieces of cloth, each $24\frac{1}{2}$ yards long, and 7 quarters wide, at 14s. 8d. a yard: How many pieces of such cloth, each $31\frac{1}{4}$ yards long, and 5 quarters wide, may be bought for 1375*l*?

Answer 84 pieces.

A demonstration of the preceding rule.

This will readily appear from the method of compounding ratios. For in example the first, tho' we place 15 days (the odd term) in the middle of the supposed statings, yet (per alternating the terms) we can easily imagine it all along in the third place, and then will the corresponding terms of supposition and demand be so many ratios, i. e. 12 : 60, 30 : 300, 6 : 8, and 3 : 6, OR $\frac{12}{60}$, $\frac{30}{300}$, $\frac{6}{8}$, and $\frac{3}{6}$. And as the third term (15) must be affected by a ratio of their sum, 'tis evident were all the ratios direct (or implied an encrease or decrease of the third term, according as the second term is bigger or less than the first) the products of the terms of supposition and demand, as they stand, would be the ratio of the sum wanted (i. e. $\frac{12}{60} \times \frac{30}{300} \times \frac{6}{8} \times \frac{3}{6}$) and agreeably to this, the signs ($\times$) by the rule would all fall upon the terms of demand; hence it is so far right. But as some of the statings may be of the inverse kind (or where more requires less and less more) 'tis evident, that, in one of those, *what* the two terms of the ratio intimate, as to encrease or decrease, must be contradicted; which might easily be done by making them change places; but to answer the same end (as it might perhaps in some sort puzzle the learner by a mixture of terms of supposition and demand) the rule directs to mark it accordingly, and in the multiplication only to have its place, as a transposed one. This then accounts for the marks falling in inverse ratios on the opposite side to that of direct ones. And as, by what is done above, the whole of the question is now reduced to three terms, 'tis very evident, from the nature of working the common rule of three, that the odd term should be multiplied into the product of the terms of demand (or new made consequent from the added ratios) before the answer can be gained by dividing by the first term (or new made antecedent).----Hence every step of the rule is shewn to be right.

End of the second and last part.



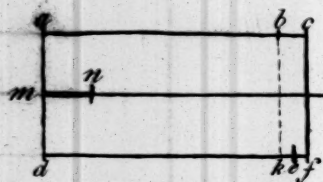


Fig. VIII.

Fig. II.

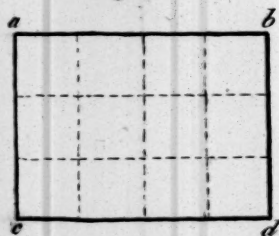


Fig. III.

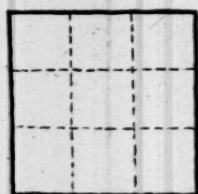


Fig. VI

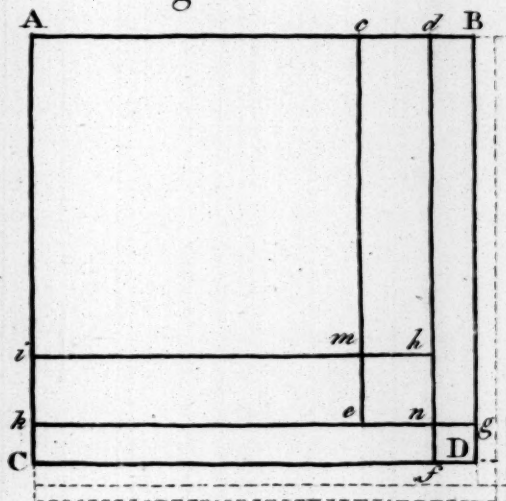


Fig. IV.

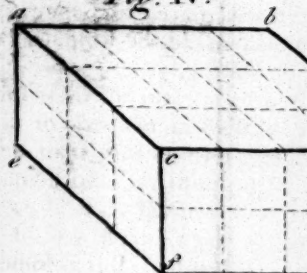


Fig. V.

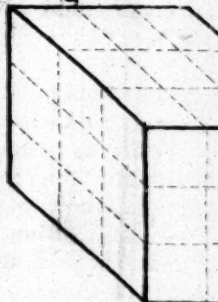


Fig. IX.



To fold out at the End

Fig. I.



Fig. VII.

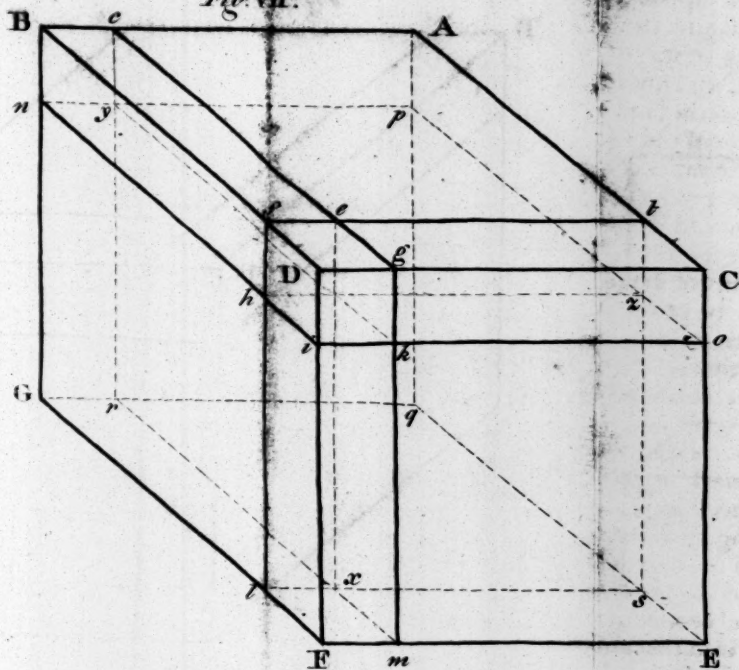
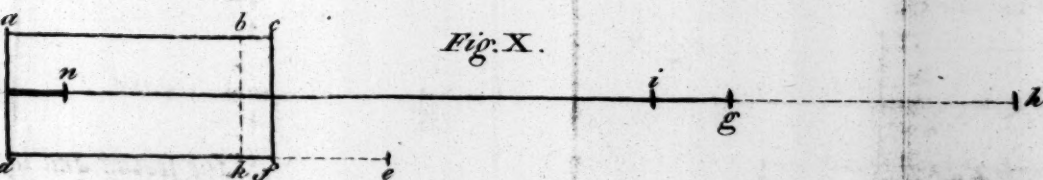
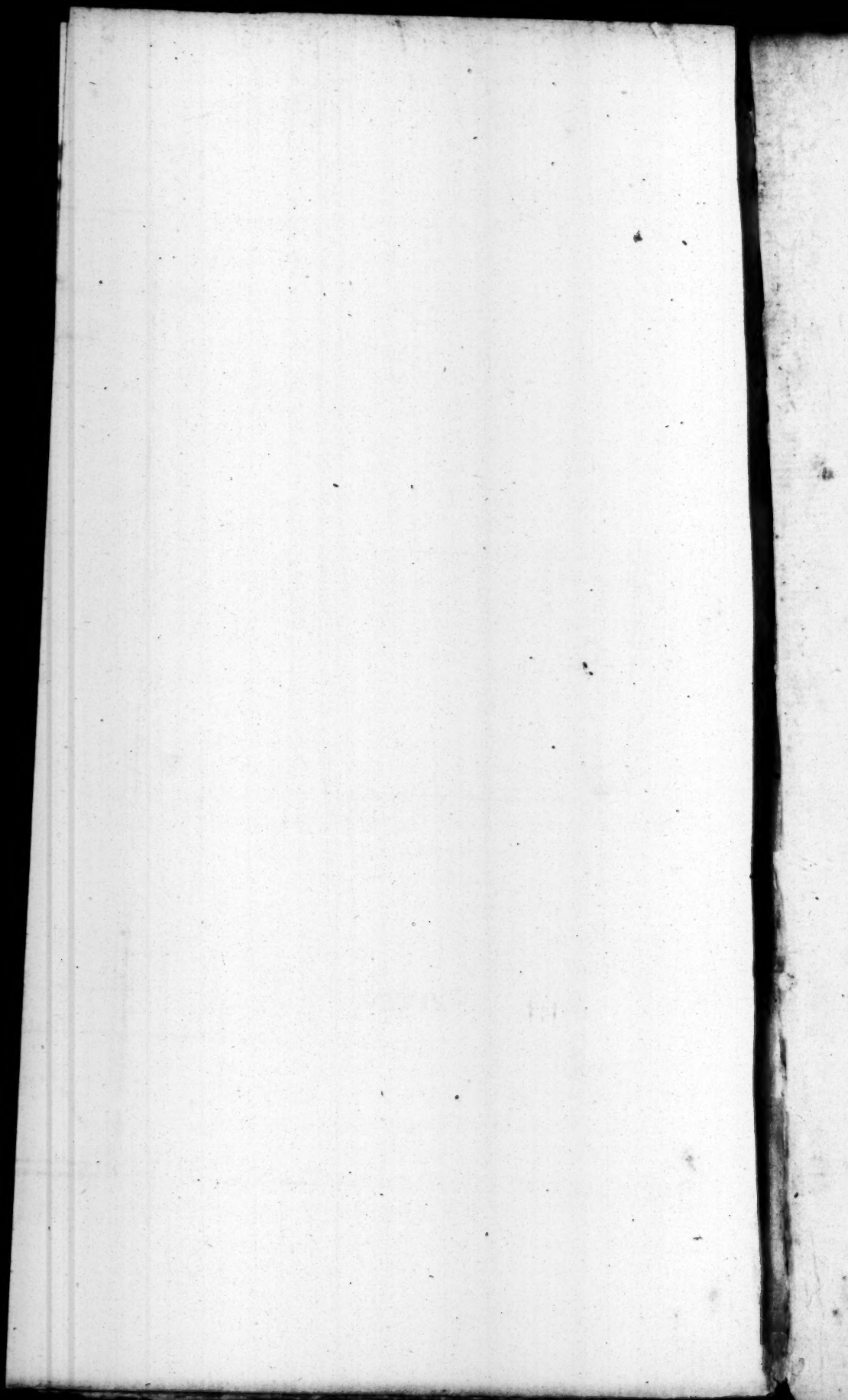


Fig. X.





The following errors (together with some inaccuracies of pointing too trifling to particularize) the reader is desired to correct; as, also, any other oversight of the same nature he may yet meet with unpointed out. Note, *t* intimates the lines were reckoned from the top, and *b* from the bottom.

ERRATA.

PAGE 10th. line 5th. *b*. read, 2 and 1 make 3, and. 11. 1 13 *b* r. £. 46-7-6. 12. 1 10 *b* dele and. 13. 1 13 *t* r. adjoining. 15. 1 21 *t* r. carried. 16. 1 17 *t* r. are units. 20. 1 5 *t* r. when added. 23. 1 10 *t* r. 604150. 24. 1 5 *b* r. 37567962432. 30. 1 5 *b* r. agreeably. 32. 1 4 *t* r. succeeding. 35. 1 4 *t* r. preceding. 38. 1 14 *t* r. foregoing. 40. 1 6 *t* r. separation. 44. 1 10 *t* r. times 9. 45. 1 4 *b* r. bushels. 46. 1 14 *t* r. currants. 47. 1 11 *b* r. IV. Do. 1 10 *r*. corns. 48. 1 9 *t* r. butt. 58. 1 14 *b* r. cloves. 63. 1 2 *t* r. weights. 64. 1 16 *b* r. and. 65. 1 1 *t* r. eighths. 72. 1 17 *b* r. IX. 73. 1 15 *b* r. 1576322. 75. 1 2 *b* r. Of 364*lb*. 78. 1 6 *b* dele he. 80. 1 4 *t* r. concurring. 81. 1 1 *t* r. numbers. 92. 1 20 *t* r. footman. Do. 1 12 *b* r. 4½*d*. Do. 1 3 *b* r. 6*s*. 3*d*. 94. 1 8 *t* r. required. Do. 1 19 *t* dele in. 96. 1 11 *t* r. garrison. Do. 1 25 *t* r. are equal. 99. 1 3 *b* r. take. 100. 1*s* 3 and 4 *t* r. 2763. 112. 1 2 *b* r. 34*d*. 113. 1 3 *t* r. 16*s*. 7½*d*. 117. 1 12 *t* r. 14*C*. 12*Q*r. 12½*lb*. Second futtle. 118. 1 6 *t* r. 52*C*. 2*Q*r*s*. 16½*lb*. Do. 1 7 *b* r. 9*s*. 6*d*. 129. 1 8 *b* r. £. 247-16-9¾. 131. 1 1 *t* r. buy. 135. 1 2 *b* r. consequently. 138. 1 11 *t* r. come. Do. 1 19 *t* r. 10½*lb*. 139. 1 4 *t* r. 15½*d*. 143. 1 5 *b* r. in most. 147. 1 7 *b* r. is each man's. 154. 1 2 *b* r. at 4¾. 167. 1 5 *b* r. the purpose. 175. 1 5 *b* r. Louis d'or. 185. 1 1 *t* r. Examples. 194. 1 6 *b* r. 10*s*. 206. 1 2 *t* r. hundredths. 207. 1 6 *t* r. to its. 208. 1 10 *t* r. 62113. Do. 1 4 *b* r. 918. 210. 1 12 *t* r. 5. 211. 1 8 *b* r. figures. 216. 1 5 *b* r. terms. 218. 1 2 *b* r. in the 219. 1 9 *b* r. a single. 224. 1 1 *b* r. single. 225. 1 2 *t* r. denominator. 236. 1 6 *b* r. stopped. 237. 1 10 *b*, 238. 1 6 *t* and 239. 1*s* 8 and 10 *t* r. hundredths. 239. 1 2 *t* r. nicety. 244. 1 12 *t* r. of a *C*. 246. 1. 1 *t* r. denomination, and 14 *t* r. off. 251. 1 8 *b* r. 455-5173. 258. 1 11 *b* r. as its. 259. 1 6 *t* r. gauging. 269. 1 4 *t* r. number. 280. 1 13 *b* dele the. 281. 1 15 *t* r. position. Do. 1 17 *t* dele in. 284. 1 16 and 285. 1 22 *t* r. planes. 299. 1 7 *b* r. means. Do. 1 1 *b* r. = 72. 305. 1 1 *t* r. limited. 307. 1 3 *b* r. product. 308. 1 19 *b* dele root. Do. 1 16 *b* r. 10000 men. 318. 1 5 *b* r. he takes. 319. 1 1 *t* dele the. Do. 1 5 *b* r. and if. 320. 1 7 *b* r. *ab* (33) Do. 1 6 *b* *bc* (4) 321. 1 2 *t* r. *ib* (5) 326. 1 13 *t* r. at simple.

NOTES.

Page 8 line 2 *b* r. results. 9. 1 6 *t* r. weights, 10. 1 5 *t* r. arising. 13. 1 1 *t* r. preceding. 17. 1 1 *t* r. than that. 17. 1 1 *b* r. preceding. 23. 1 9 *t* r. units. 32. 1 7 *t* r. affixing. 42. 1 1 *t* r. contrary. 98. 1 6 *t* r. at ½*d*. 99. 1 2 *b* r. trifling. 104. 1 1 *t* r. procedure. 135. 1 13 *b* dele it. 212. 1 3 *b* r. possible. Do. 1 1 *b* r. lowest terms. 213. 1 4 *b* dele of the. 224. 1 6 *b* r. dissimilar. 239. 1 2 and 3 *t* r. hundredths. 240. 1 2 *t* r. of 10ths. Do. 1 13 *t* r. prefixed. Do. 1 16 *t* r. equal to. 245. 1 6 *b* r. the rule. 257. 1 2 *b* r. foregoing. 273. 1 1 *b* r. 512.

To the Book-binder.

Place this table at the end, after the copper-plate.

